PHY385H1F – "Introductory Optics"

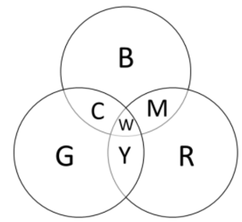
Term Test 1 | October 9, 2012 | Duration: 50 minutes

Version 1

NAME:______________________________ **Student Number:___________________.**

Aids allowed: A pocket calculator with no communication ability. One 8.5x11" aid sheet, written on both sides.
You may not communicate with anyone other than the invigilator during the test.

Possibly helpful information:



The ratio of the circumference to the diameter of a circle: $\pi = 3.14$
The speed of light in a vacuum is $c = 3.00 \times 10^{8}$ m s^{-1}
Permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-12} \mathrm{m}^{-3}\mathrm{kg}^{-1}\mathrm{s}^{4}\mathrm{A}^{2}$
Permeability of free space: $\mu_0 = 1.26 \times 10^{-6}$ m kg s^{-2}
Planck's Constant: $h = 6.63 \times 10^{-34}$ J s

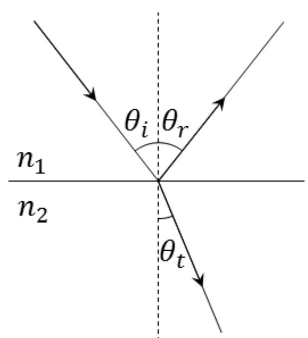
Common Prefixes: k = "kilo-" = 10^{3} c = "centi-" = 10^{-2} m = "milli-" = 10^{-3}
μ = "micro-" = 10^{-6} n = "nano-" = 10^{-9}

- Phase speed of a harmonic wave in terms of wavelength, λ, and frequency, f: $v_{\mathrm{ph}} = \lambda f$
- Phase speed of a harmonic wave in terms of propagation number, k, and angular frequency, ω: $v_{\mathrm{ph}} = \frac{\omega}{k}$
- Phase speed of an electromagnetic wave in terms of speed of light in a vacuum, c, and index of refraction, n: $v_{\mathrm{ph}} = \frac{c}{n}$

Irradiance of an electromagnetic wave of electric field amplitude, E_0: $I = \frac{c\epsilon_0}{2} {E_0}^2$
Irradiance of a spherically symmetric point source of power L at a distance r from the source: $I = \frac{L}{4\pi r^2}$
Photon energy: $E_{\mathrm{ph}} = h f$

Law of reflection: $\theta_r = \theta_i$

Snell's Law: $n_1 \sin\theta_i = n_2 \sin\theta_t$



Fresnel's Equations:

TE-mode: $r_\perp = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}$, $t_\perp = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t}$, $R_\perp = {r_\perp}^2$, $T_\perp = 1 - R_\perp$

TM-mode: $r_\parallel = \frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_1\cos\theta_t + n_2\cos\theta_i}$, $t_\perp = \frac{2n_1\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i}$, $R_\parallel = {r_\parallel}^2$, $T_\parallel = 1 - R_\parallel$

Multiple Choice Part (6 points)

Circle the letter of the best answer for each question.

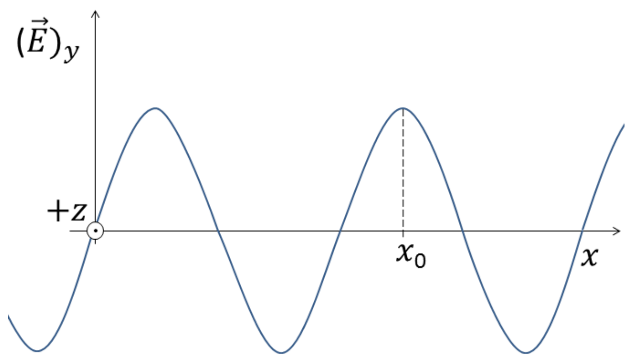
1. Any colour can be created by mixing a suitable amount of red (R), green (G) and blue (B) light. To obtain white light (W), the mixture is all three: R + G + B. Other colours may be obtained by subtracting R, G or B from W. Which of the following mixtures could produce yellow light?
   - A. R + B
   - B. W – R
   - C. W – B
   - D. W – G
   - E. B + G

2. What types of waves can be polarized?
   - A. Transverse waves
   - B. Longitudinal waves
   - C. Water waves
   - D. Both A and B
   - E. All of the above

3. An electromagnetic wave has the form:

$$\vec{E} = \vec{E}_0 e^{i(kx+\omega t)}$$

where $\vec{E}_0$ is parallel to the +y-axis, and k and ω are both positive numbers. It is shown at a particular time t at which the electric field is maximum in the +y direction at $x = x_0$. At this moment, what is magnetic field at $x = x_0$?



   - A. Zero
   - B. Maximum magnitude in the +x direction
   - C. Maximum magnitude in the –x direction
   - D. Maximum magnitude in the +y direction
   - E. Maximum magnitude in the –y direction
   - F. Maximum magnitude in the +z direction
   - G. Maximum magnitude in the –z direction

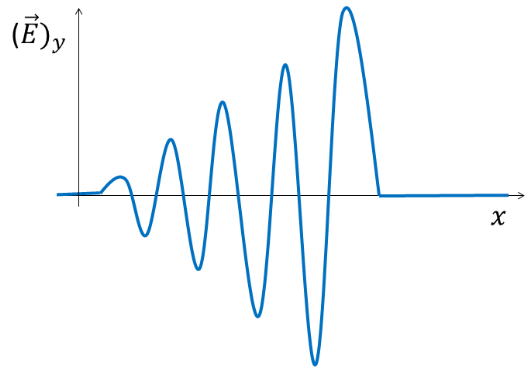
4. A spherical wave is created from a very small point source at the origin $x = y = z = 0$. The wave is symmetric around the origin as it travels outward. Consider the wave as it travels along $+x$-axis. The irradiance of this spherical wave is 1 W/m^2 at $x = 1$ m. At $x = 10$ m, what do you expect is the irradiance of the wave?
   A. 100 W/m^2
   B. 10 W/m^2
   C. 1 W/m^2
   D. 0.1 W/m^2
   E. 0.01 W/m^2

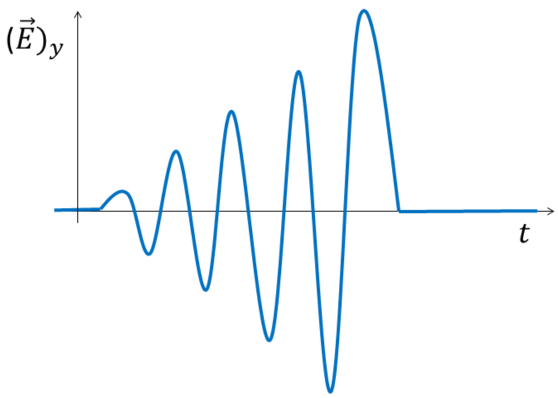
5. An electromagnetic wave of a single frequency shines onto a square detector. The irradiance is suddenly doubled, but the frequency of the wave remains the same. What property of the photons changes?
   A. The rate of photons striking the detector per second suddenly doubles.
   B. The energy of the individual photons suddenly doubles.
   C. Both the rate of photons striking the detector per second and the energy of the individual photons increase, so that the total energy flux doubles.
   D. The oscillation amplitude of the photons suddenly doubles.
   E. The oscillation amplitude of the photons suddenly increases by a factor of four.

6. To the right is a "snapshot graph" of a traveling electromagnetic wave pulse in which the component of the electric field in the y-direction is

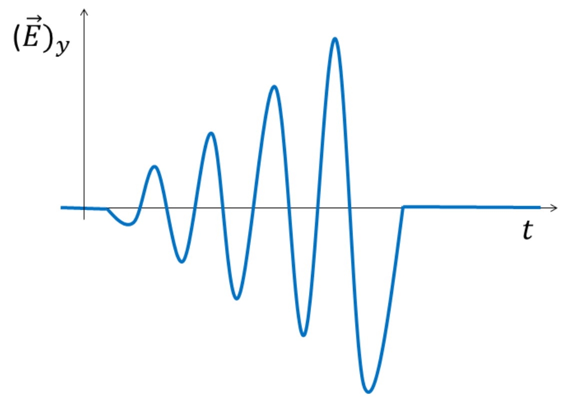
$$E_y = f\,(x - ct)$$

where f is a function with the shape shown, and c = the speed of light. The snapshot graph shows E_y at a specific time $t = t_0$. Which of the plots below best shows the correct shape of the corresponding "history graph" at some position $x = x_0$?

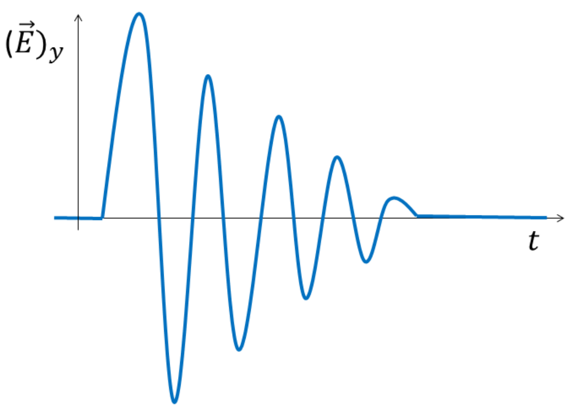




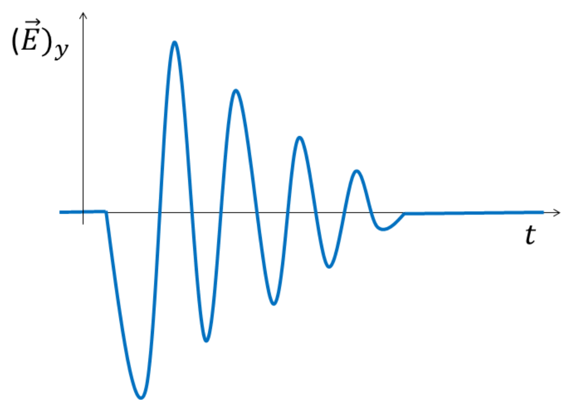
A.



B.



C.



D.

Long Answer Part (14 points)

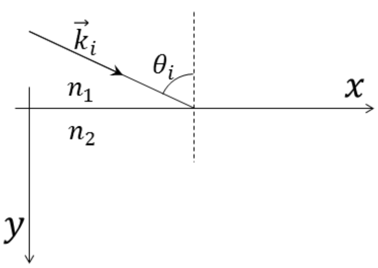
Please complete the following problems in the examination booklet provided. Show all your work, and if there is a final answer, draw a box around it to aid the marker.

LA1. **[1 point]** What is the frequency of green light, which has a wavelength in air of λ = 520 nm?

LA2. **[3 points]** On the surface of the Earth we receive about 1.37 kW of power per square meter from the Sun. Calculate the electric field amplitude associated with the sunlight (on the surface of the Earth), assuming that it is a harmonic wave (single frequency) with a wavelength of λ = 600.0 nm.

LA3. A beam of light with vacuum wavelength λ_0 = 600.0 nm starts in a medium with index of refraction n_1 = 1.50, and enters a medium with index of refraction n_2 = 1.00. The irradiance of the incident beam is 3.85 W/m^2. The incident beam is polarized so that the electric field oscillates perpendicular to the plane of incidence.

a. [**2 points**] If the angle of incidence is θ_i = 33.69º, what is the refraction angle of the transmitted beam θ_t?
b. [**2 points**] If the angle of incidence is θ_i = 33.69º, what is the irradiance of the reflected beam?
c. [**2 points**] If the angle of incidence is θ_i = 80º, what is the irradiance of the reflected beam?
d. [**4 points**] Set up a coordinate system as shown such that the plane of incidence is the *x*-*y* plane, the *x*-axis is along the n_1/n_2 boundary, and +*y* is perpendicular to the boundary increasing into the n_2 medium. In the first medium, the incident beam is a plane wave with propagation vector $\vec{k}_i = (k_i \sin\theta_i)\hat{\imath} + (k_i \cos\theta_i)\hat{\jmath}$ and electric field given by:



$$\vec{E}_i = \vec{E}_0 e^{i(k_i \sin\theta_i x + k_i \cos\theta_i y - \omega t)}$$

If the angle of incidence is θ_i = 80º, use Snell's Law to show that waves penetrate into the second medium with an electric field amplitude proportional to $e^{-\beta y}$. What is the decay constant β in m^{-1}?