PHY385H1F – "Introductory Optics"

Term Test 2 November 6, 2012 Duration: 50 minutes

NAME:______________________________ **Student Number:**___________________.

Aids allowed: A pocket calculator with no communication ability. One 8.5x11" aid sheet, written on both sides. You may not communicate with anyone other than the invigilator during the test.

Possibly helpful information:

The ratio of the circumference to the diameter of a circle: $\pi = 3.14159$

The speed of light in a vacuum: $c = 3.00 \times 10^8$ m s^{-1}

Permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-12} \mathrm{m^{-3} kg^{-1} s^4 A^2}$

Permeability of free space: $\mu_0 = 1.26 \times 10^{-6}$ m kg s^{-2}

Planck's Constant: $h = 6.63 \times 10^{-34}$ J s

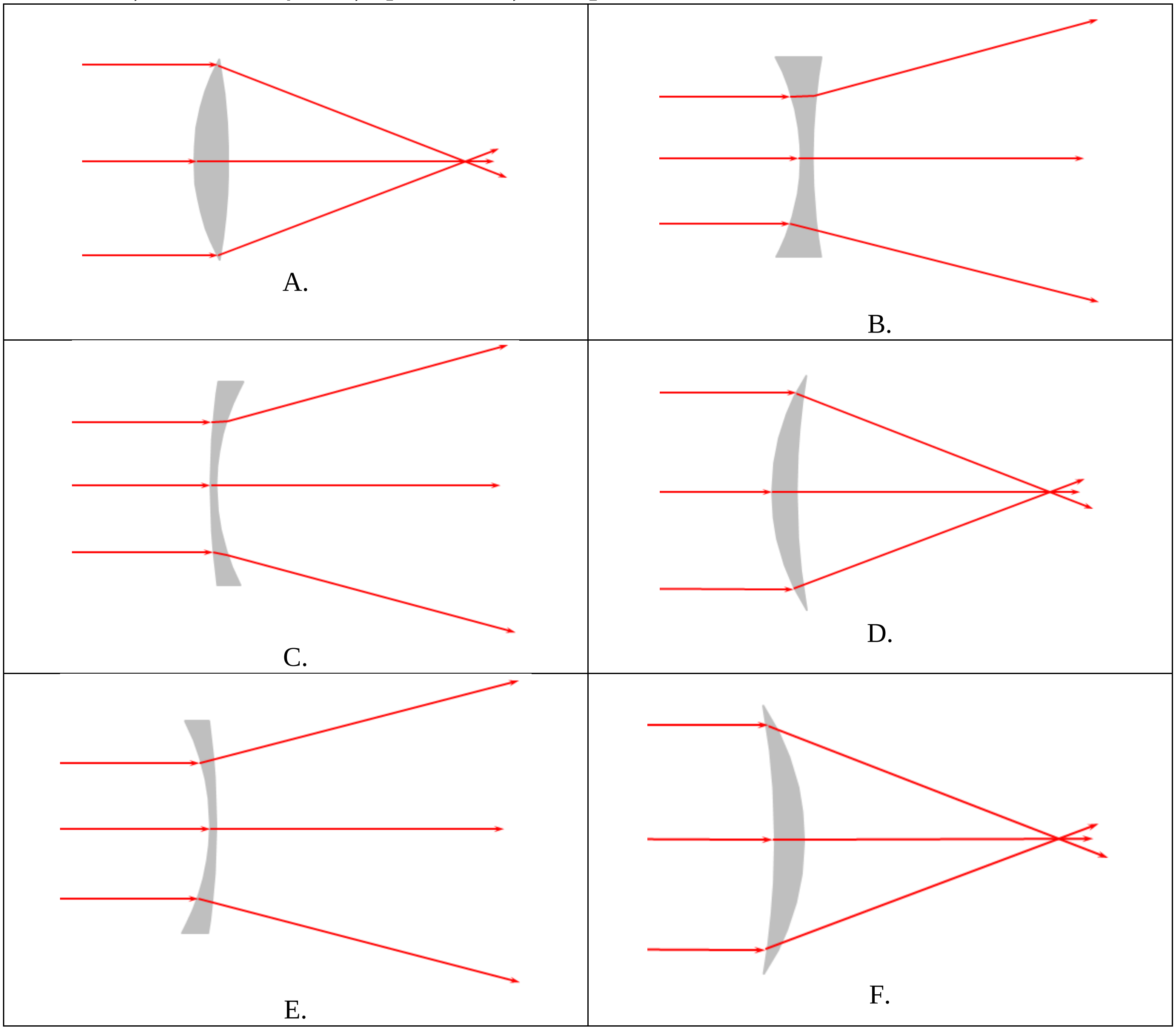
Boltzmann's Constant: $k = 1.38 \times 10^{-23}$ J/K

- Thin Lens Equation, standard form: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$
- Lensmaker's Formula: $\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, where n_l is the index of refraction of the lens which is in air, and, if the light enters from the left, R_1 and R_2 have the sign convention that they are positive if the centre of curvature C is to the right of the lens, and negative if C is to the left of the lens.
- Thin Lens Equation, Newtonian form: $x_o x_i = f^2$
- Transverse Magnification: $M_T \stackrel{\text{def}}{=} \frac{y_i}{y_o} = -\frac{s_i}{s_o}$
- Mirror Formula: $\frac{1}{s_o} + \frac{1}{s_i} = -\frac{2}{R}$
- Numerical Aperture for an optical fibre: NA = $n_i \sin\theta_{\max} = \sqrt{{n_f}^2 - {n_c}^2}$, where n_i is the index of the surrounding medium, n_f is the index of the core, and n_c is the index of the cladding
- Modal dispersion of a step-index optical fibre of length L: $\Delta t = \frac{L n_f}{c}\left(\frac{n_f}{n_c} - 1\right)$
- Magnifying power of a simple magnifier with image at ∞: $MP = \frac{25\text{ cm}}{f}$
- Magnifying power of a standard compound microscope with objective focal length f_o and eyepiece focal length f_e: $MP = \left(-\frac{16\text{ cm}}{f_o}\right)\left(\frac{25\text{ cm}}{f_e}\right)$
- Magnifying power of a telescope for distant objects: $MP = -\frac{f_o}{f_e}$
- Planck's Radiation Law for blackbodies: $I_\lambda = \frac{2\pi h c^2}{\lambda^5}\left[\frac{1}{\exp\left(\frac{hc}{\lambda k T}\right) - 1}\right]$
- Stefan Boltzmann law for blackbodies: $P = \sigma A T^4$, where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
- Wien Displacement law for blackbodies: $\lambda_{\max} T = 0.002898$ m K
- Maxwell-Boltzmann relative distribution of two energy states 1 and 2 where a transition between state 1 and 2 involves a photon of frequency ν: $\frac{N_2}{N_1} = \exp\left(\frac{-h\nu}{kT}\right)$
- Absorption rate for state $i \rightarrow j$: $R_{ab} = -B_{ij} N_i u_\nu$
- Stimulated emission rate for state $j \rightarrow i$: $R_{st} = -B_{ji} N_j u_\nu$
- Spontaneous emission rate for state $j \rightarrow i$: $R_{sp} = -A_{ji} N_j$

Multiple Choice Part (6 points)

Circle the letter of the best answer for each question.

1. The Lensmaker's Formula is Hecht Equation 5.17, and it is also on the front page of this test, along with the appropriate sign conventions for R_1 and R_2. Which one of the lenses, shown below in cross-section, could have n_l = 1.5, R_1 = +6.5 cm, and R_2 = +17 cm?



2. An object with a height y_o is held a distance $2f$ in front of a lens of positive focal length f. What is the height of the image, y_i? [Positive values correspond to an erect image, negative values correspond to an inverted image.]
   A. $2y_o$
   B. $-\frac{y_o}{2}$
   C. $-y_o$
   D. $-2y_o$
   E. No real image is formed in this case.

3. When light enters a camera, it first encounters a lens L1, then an aperture stop AS, then a second lens L2, and finally a third lens L3 before falling on the detector. The aperture stop has a much smaller diameter than any of the three lenses. What is the entrance pupil?
   A. The image of AS formed by L2 and L3.
   B. The image of AS formed by L1.
   C. The image of AS formed by L1, L2 and L3.
   D. AS
   E. L3

4. The Nikon Coolpix S800C 16MP camera has a focal length of 45 mm, and its focal ratio is f/3.2. What is the diameter of the lens?
   A. 1.4 cm
   B. 3.2 cm
   C. 4.5 cm
   D. 7.1 cm
   E. 14 cm

5. A small object is held a distance $R/2$ to the left of a shiny silver bowling ball of radius R. The ball acts like a mirror, and we call the point on the surface of the ball nearest to the object V. Where does the image of the small object form?
   A. $R/4$ to the left of V
   B. $R/4$ to the right of V
   C. $R/2$ to the left of V
   D. $R/2$ to the right of V
   E. R to the left of V
   F. R to the right of V

6. Two perfect blackbodies emit different spectra. The first blackbody has a temperature T_1 and a peak wavelength in the green region of 500 nm. The second blackbody has a temperature T_2 and a peak in the near infrared at 1000 nm. What is the ratio T_1/T_2?
    A. 2
    B. ½
    C. 4
    D. ¼
    E. 16
    F. $^1/_{16}$

Long Answer Part (14 points)

Please complete the following problems in the examination booklet provided. Show all your work, and if there is a final answer, draw a box around it to aid the marker.

LA1. **[3 points]** At maximum zoom, the Nikon Coolpix S800C 16MP camera has a focal length of 45 mm, and its focal ratio is f/3.2. The 16 megapixel detector in this camera is a rectangle whose longer length is 6.2 mm. If you are photographing an object that is 2.5 m in front of the camera using this zoom setting, what is the largest object for which you can fit all of the image on the detector?

LA2. Two positive thin lenses, L1 and L2, are separated by 5.0 cm. Their diameters are 6.0 and 4.0 cm, respectively, and their focal lengths are f_1 = 9.0 cm and f_2 = 3.0 cm. A very small, point-like object is placed on the optical axis 15 cm in front of L1, and a crisp image is formed a detector.

(a) **[2 points]** Where should the detector be placed, relative to L2?

(b) **[2 points]** An aperture stop 1.0 cm in diameter is placed between the two lenses, 2 cm in front of L2. What is the location and size of the exit pupil, relative to L2?

(c) **[2 points]** The exit pupil defines a cone of rays incident on the detector. What is the angular radius of this cone, θ_{max}?

(d) **[2 points]** If you wish to replace the detector with an optical fibre on the optical axis, and the core of the optical fibre has an index of 1.60, what is the highest value of the index of the cladding that will still accept all the light coming from the exit pupil of this system? [Assume the fibre is surrounded by air.]

LA3. **[3 points]** A beam of monochromatic light with frequency $\nu = 5.8 \times 10^{14}$ Hz travels through a thin transparent Neon gas of temperature T = 5400 K. When an outer Neon electron in excited energy state j drops to a lesser excited state i, it emits a photon of this same frequency, $\nu = 5.8 \times 10^{14}$ Hz. As the beam travels through this gas, what is the ratio R_{ab}/R_{st}, where R_{ab} is the rate of photon absorption causing electrons to be bumped up from i to j, and R_{st} is the rate of stimulated emission causing electrons to be bumped down from j to i?