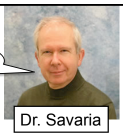
PHY131H1S - Class 18

Today:

- Gravitational Torque
- Rolling without slipping
- Rotational Kinetic Energy
- Static Equilibrium

Announcement


Dr. Savaria

- Today you should visit the course portal page and click on **Test 2 Instructions** on the left-menu to find out WHERE you are writing your test on Tuesday
- The rules and format for Test 2 are very similar to those for Test 1: you are allowed a hand-written aid-sheet and a calculator but no phones, there are 8 multiple choice plus a long answer, test is 80 minutes
- Main difference: Test 2 starts at **8:00pm Tuesday night**
- Remember tomorrow by 5:00pm is the deadline to register a time conflict with April Seeley in MP129
- Test 2 material is Chapters 5 through 12 plus what was covered in Practicals

---

Pre-class reading quiz on Chapter 12

A rigid body is in equilibrium if

A. $\vec{F}_{\text{net}} = 0$

B. $\vec{\tau}_{\text{net}} = 0$

C. neither A nor B.

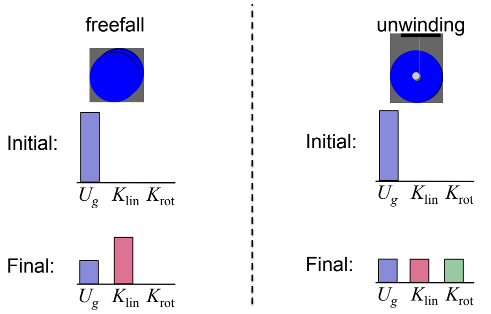
D. either A or B.

E. both A and B.

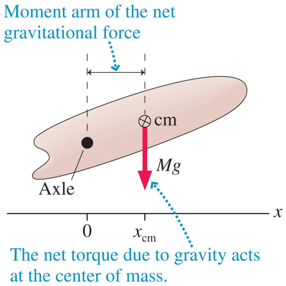
Last day I asked at the end of class:

- In Practicals this week you will hold the string of a yo-yo fixed as you drop it. As the yo-yo falls, the string unwinds and the yo-yo rotates. Does it fall faster or slower than 9.8 m/s^2?
- ANSWER:
- Slower
- The transformation of energy is U_g → kinetic; so *why* does it fall slower?
- ANSWER:
- In freefall, the transformation of energy is U_g all into linear kinetic energy ½mv^2. For the unwinding yo-yo, U_g is transformed into the sum of linear plus rotational kinetic energy, so it is *shared*.

- In Practicals this week you will hold the string of a yo-yo fixed as you drop it. As the yo-yo falls, the string unwinds and the yo-yo rotates. Why does it fall slower than 9.8 m/s^2?



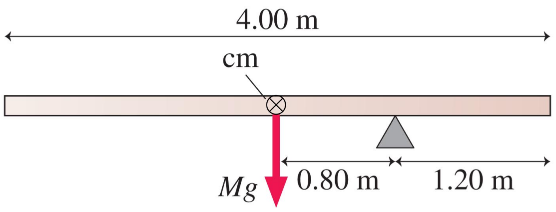
Gravitational Torque



When calculating the torque due to gravity, you may treat the object as if all its mass were concentrated at the centre of mass.

Example

A 4.00 m long, 500 kg steel beam is supported 1.20 m from the right end. What is the gravitational torque about the support?



Rolling without slipping: review

- No matter what the speed, four points on this car are always ***at rest!***
- Which points? The bottoms of the four tires!

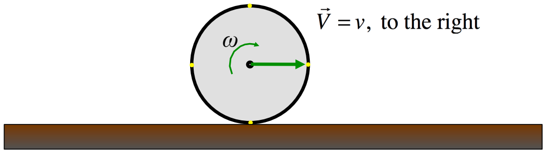
- A wheel rolls much like the treads of a tank.
- The bottom of the wheel is ***at rest*** relative to the ground as it rolls.

Rolling without slipping: review

S frame: the ground

S' frame: the axle

$\vec{V} = v$, to the right



The wheel rotates with angular speed ω.

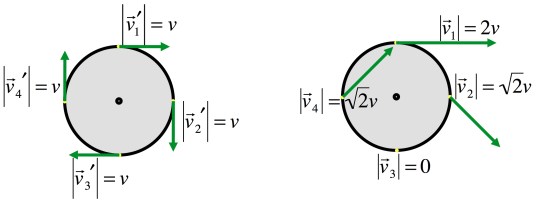
The tangential speed of a point on the rim is $v = \omega r$, relative to the axle.

In "rolling without slipping", the axle moves at speed v. This is the *S'* frame.

Rolling without slipping

S' frame: the axle

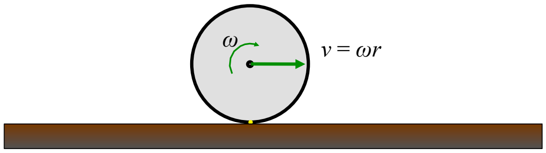
S frame: the ground



$\vec{V} = v$, to the right

$$\vec{v} = \vec{v}' + \vec{V}$$

Rolling without slipping



$v = \omega r$

The wheel rotates with angular speed ω.

The axle moves with linear speed $v = \omega r$., where r is the radius of the wheel.

Since the bottom point is always at rest, it is *static friction* which acts between the ground and the wheel.

The "Rolling Without Slipping" Constraints

When a round object rolls without slipping, the distance the axis, or centre of mass, travels is equal to the change in angular position times the radius of the object.

$$s = \theta R$$

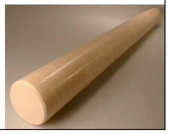
The speed of the centre of mass is

$$v = \omega R$$

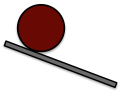
The acceleration of the centre of mass is

$$a = \alpha R$$

I recommend putting all of these on your aid sheet! - J.H.

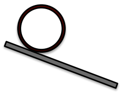


Rolling Without Slipping Race



- A solid disk and an empty ring both begin at the top of an incline, are released from rest, and allowed to roll without slipping down to the bottom. Which will win?

Predict:



A. Solid disk will win
B. Empty ring will win
C. Both will reach the bottom at about the same time.

Compare and Contrast Soup Cans

- About same mass
- About same radius and shape
- Thick paste, so when this can is rolling, the contents rotate along with the can as one solid object, like a solid cylinder
- Low viscosity liquid, so the can itself rolls while the liquid may just "slide" along.

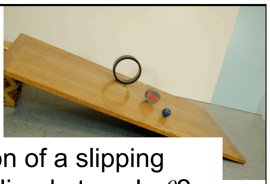
Soup Race

- Two soup cans begin at the top of an incline, are released from rest, and allowed to roll without slipping down to the bottom. Which will win?

Predict:

A. Cream of Mushroom will win
B. Chicken Broth will win
C. Both will reach the bottom at about the same time.

Examples:



1. What is the acceleration of a slipping object down a ramp inclined at angle θ? [assume no friction]
2. What is the acceleration of a solid disk rolling down a ramp inclined at angle θ? [assume rolling without slipping]
3. What is the acceleration of a hoop rolling down a ramp inclined at angle θ? [assume rolling without slipping]

Rotational Kinetic Energy

A rotating rigid body has kinetic energy because all atoms in the object are in motion. The kinetic energy due to rotation is called **rotational kinetic energy**.

$$K_{\text{rot}} = \frac{1}{2} I\omega^2$$

Example: A 0.50 kg basketball rolls along the ground at 1.0 m/s. What is its *total* kinetic energy? [linear plus rotational]

Linear / Rotational Analogy

	Linear	**Rotational Analogy**
	$\vec{s}$, $\vec{v}$, $\vec{a}$	θ, ω, α
	Force: $\vec{F}$	Torque: τ
	Mass: m	Moment of Inertia: I
Newton's 2nd law:	$\vec{a} = \frac{\vec{F}_{net}}{m}$	$\alpha = \frac{\tau_{net}}{I}$
Kinetic energy:	$K_{\text{cm}} = \frac{1}{2} mv^2$	$K_{\text{rot}} = \frac{1}{2} I\omega^2$

Summary of some Different Types of Energy:

- Kinetic Energy due to linear motion of centre of mass: $K = ½\, mv^2$
- Gravitational Potential Energy $U_g = mgh$
- Spring Potential Energy: $U_s = ½\, kx^2$
- Rotational Kinetic Energy: $K_{rot} = ½\, I\omega^2$
- Thermal Energy (often created by friction)
  - An object can possess any or all of the above.
  - One way of transferring energy to or out of an object is work:
- Work done by a constant force: $W = Fr\cos\theta$

Updated Conservation of Energy…

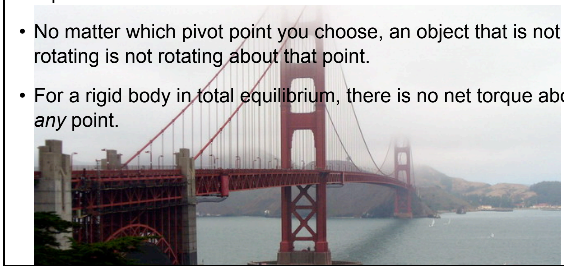
Conservation Laws

Energy is conserved for an isolated system.

- Pure rotation $E = K_{rot} + U_g = \frac{1}{2}I\omega^2 + Mgy_{cm}$
- Rolling $E = K_{rot} + K_{cm} + U_g = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{cm}{}^2 + Mgy_{cm}$

Equilibrium When Rotation is Possible

- The condition for a rigid body to be in *static equilibrium* is that there is no net force and no net torque.
- An important branch of engineering called *statics* analyzes buildings, dams, bridges, and other structures in total static equilibrium.
- No matter which pivot point you choose, an object that is not rotating is not rotating about that point.
- For a rigid body in total equilibrium, there is no net torque about *any* point.



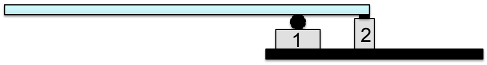
Static Equilibrium Problems

- In equilibrium, an object has no net force and no net torque.
- Draw an extended free-body diagram that shows where each force acts on the object.
- Set up x and y axes, and choose a rotation axis. All of these choices should be done to simplify your calculations.
- Each force has an x and y component and a torque. Sum all of these up.
- Three equations which you can use are:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

Static Equilibrium Question

A diving board has two supports, 1 and 2. What are the directions of the forces of these supports on the diving board?



A. Both forces are up
B. Both forces are down
C. F_1 is up, F_2 is down
D. F_1 is down, F_2 is up

Before Class 19 on Monday

- There is a MasteringPhysics problem set due on Friday! Please submit this before 11:59pm if you have not already done so.
- Please finish reading Chapter 12.
  - Something to think about: If a figure skater pulls in her arms while rotating, does her rotational speed *really* increase? If so, why?