PreClass Notes: Chapter 6

- From **Essential University Physics** 3rd Edition
- by Richard Wolfson, Middlebury College
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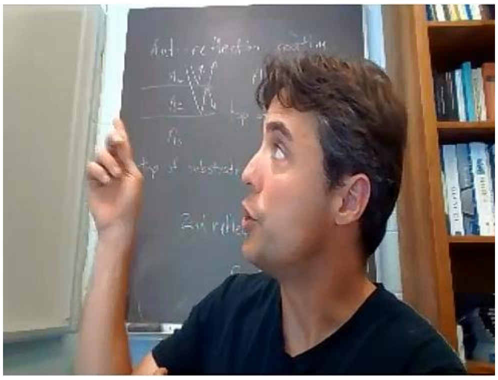
- Narration and extra little notes by Jason Harlow, University of Toronto
- This video is meant for University of Toronto students taking PHY131.

Outline

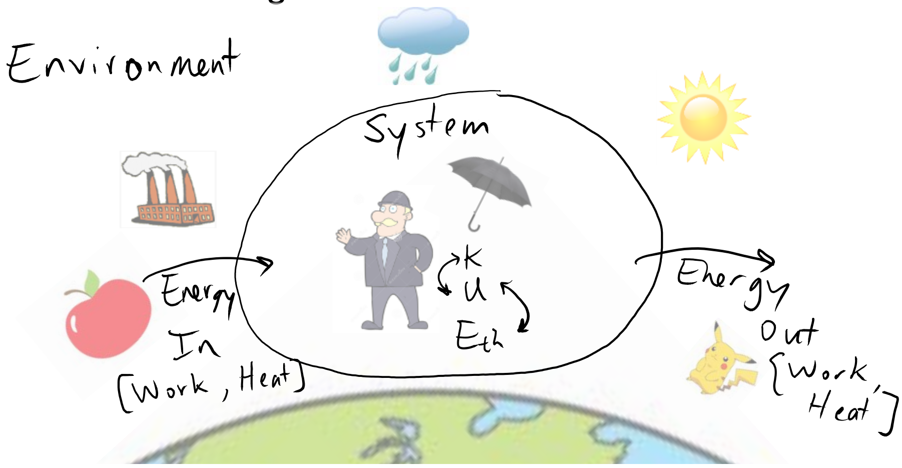
"Energy…is a fundamental aspect of the universe-a 'substance' akin to, and every bit as real as, matter itself." – R.Wolfson

- **6.1** Energy Overview
- **6.2** Introduction to Work
- **6.3** Work as an integral
- **6.4** Kinetic Energy
- **6.5** Power



Law of Conservation of Energy:

"Energy cannot be created or destroyed; it may be transformed from one form into another, but the total amount of energy never changes."



Sources of Energy

Sun

Examples:

- Sunlight evaporates water; water falls as rain; rain flows into rivers and into generator turbines; then back to the sea to repeat the cycle.
- Wind power turns generator turbines.

[Image from http://www.wallbacks.com/walls/sunset-sun-mountains-clouds-other.jpg]

Sources of Energy

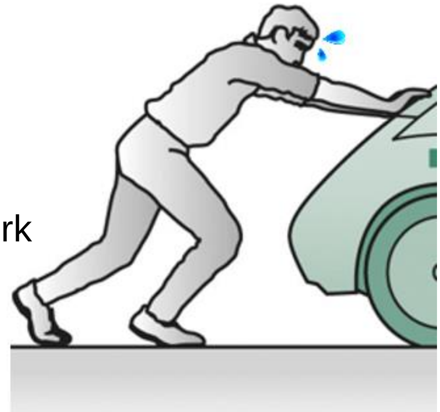
Example:
- Photovoltaic cells on rooftops catch the solar energy and convert it to electricity.

More energy from the Sun hits Earth in 1 hour than all of the energy consumed by humans in an entire year!

[Image from http://www.pennenergy.com/articles/pennenergy/2014/03/baywa-r-e-completes-forest-heath-solar-power-farm-in-england.html]

Work

- is force × distance.
- in equation form: $W = F_x \Delta x$.



Two things occur whenever work is done:
- application of force
- movement of something by that force

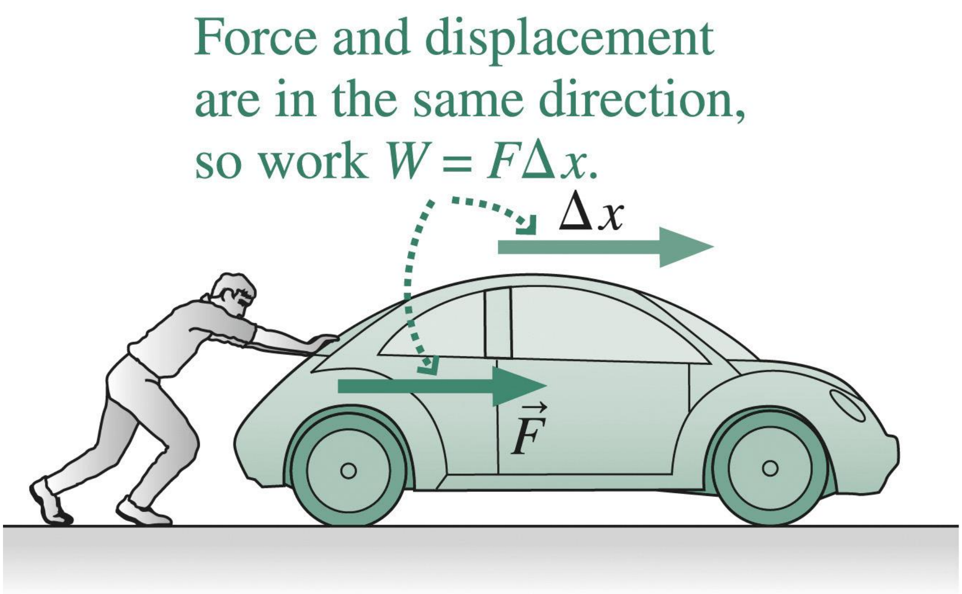
SI Unit of work:
newton-meter (N·m)
or joule (J)

Got it?

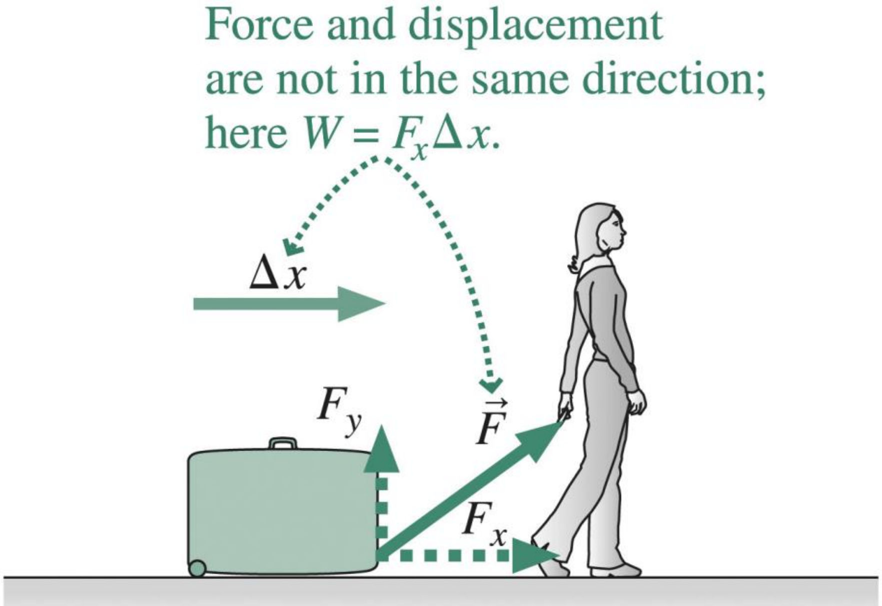
- If you push twice as hard against a stationary wall, the amount of work you do on the wall

A. quadruples.
B. doubles.
C. remains constant but non-zero.
D. remains constant at zero.
E. is halved.

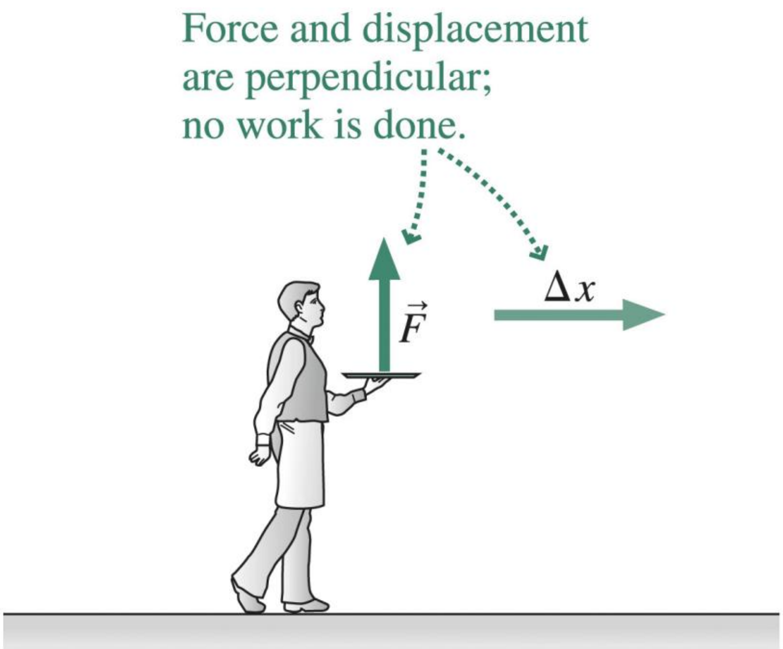
Doing Work



Work when force and displacement are not in the same direction

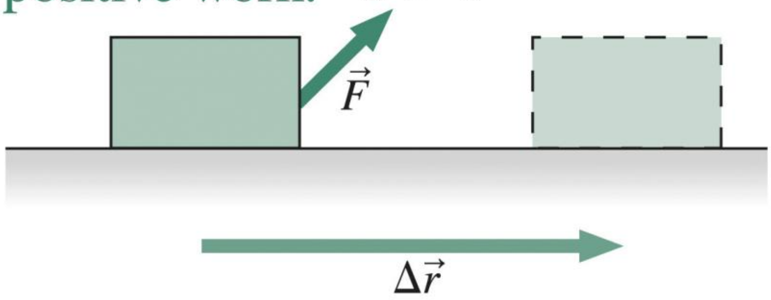


Zero Work



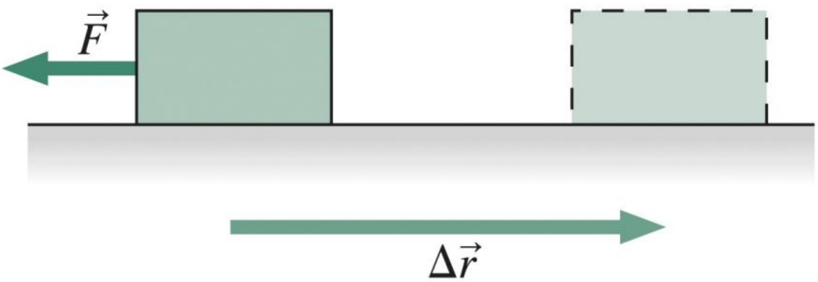
Positive Work

A force acting with a component in the same direction as the object's motion does positive work. $W > 0$



Negative Work

A force acting opposite the motion does negative work. $W < 0$



Got it?

- For which of the following situations is the net work done on a soccer ball positive?

A. You carry the ball out to the field, walking at constant speed.
B. You kick the stationary ball, starting it flying through the air.
C. The ball rolls along the field, gradually coming to a halt.

The Scalar Product

- Work is conveniently characterized using the *scalar product*, a way of combining two vectors to produce a scalar that depends on the vectors' magnitudes and the angle between them.
- The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

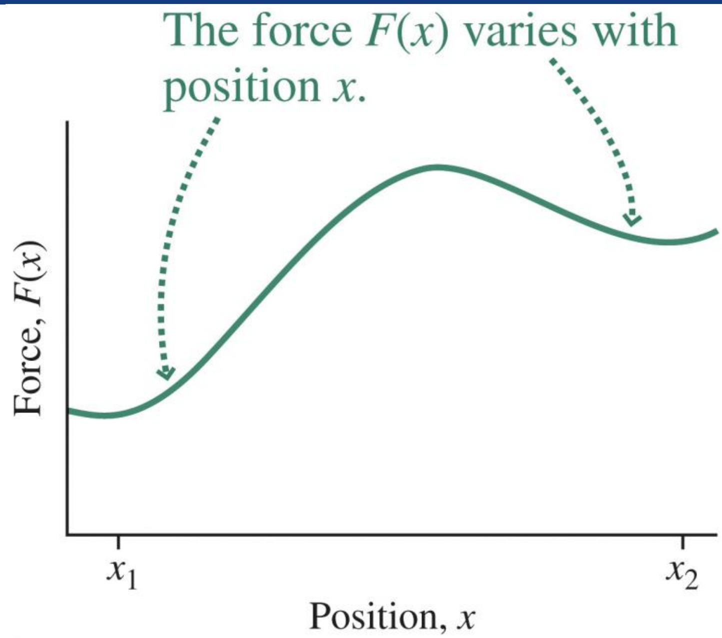
$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

where A and B are the magnitudes of the vectors and θ is the angle between them.

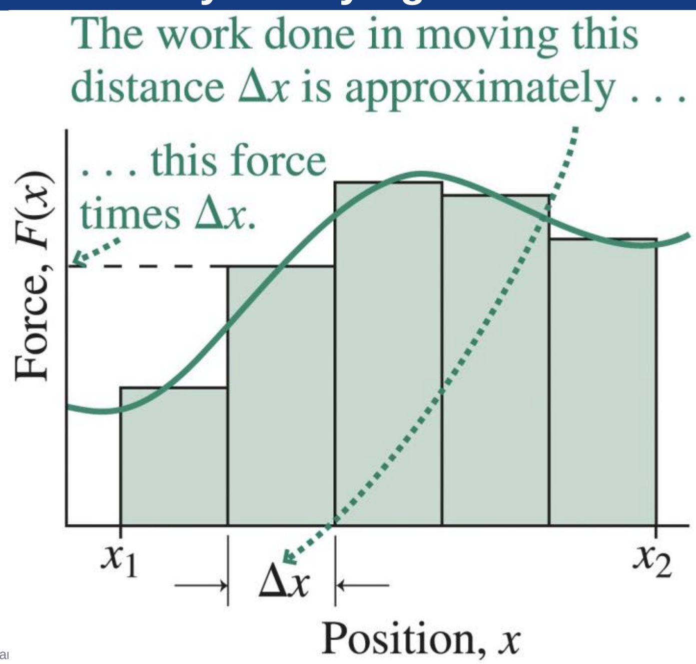
- Work is the scalar product of force with displacement:

$$\boxed{W = \vec{F} \cdot \Delta\vec{r}}$$

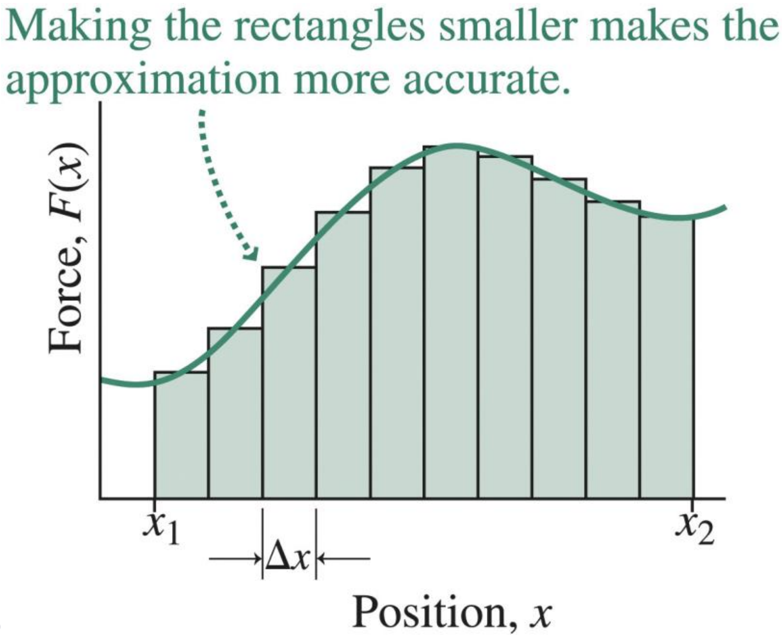
Work Done by a Varying Force



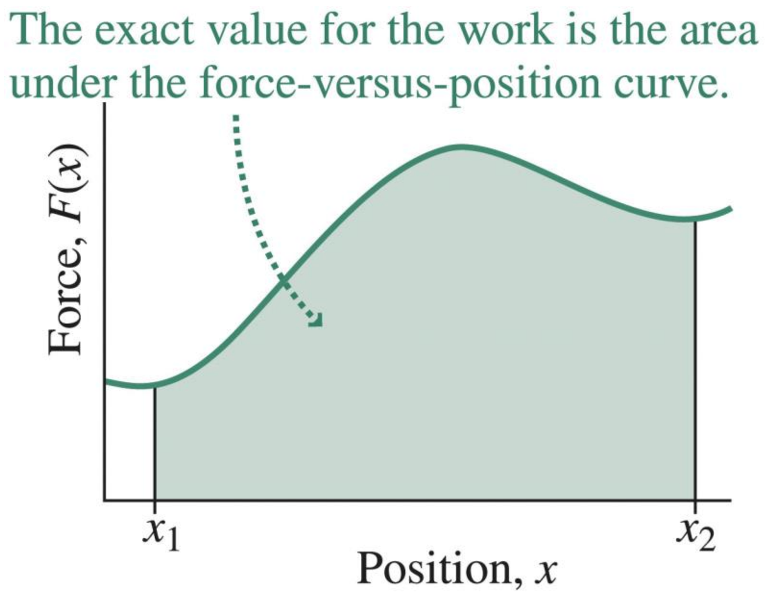
Work Done by a Varying Force



Work Done by a Varying Force



Work Done by a Varying Force



Integration

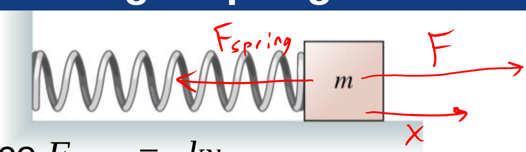
- The *definite integral* is the result of the limiting process in which the area is divided into ever smaller regions.
- Work as the integral of the force F over position x is written

$$W = \int_{x_1}^{x_2} F(x)\,dx$$

- Integration is the opposite of differentiation, so integrals of simple functions are readily evaluated. For a polynomial function $f(x) = x^n$, the integral is

$$\int_{x_1}^{x_2} x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_{x_1}^{x_2} = \frac{x_2^{n+1}}{n+1} - \frac{x_1^{n+1}}{n+1}$$

Work Done in Stretching a Spring



- A spring exerts a force $F_{\text{spring}} = -kx$.
- The agent stretching a spring exerts a force $F = +kx$, and the work the agent does is

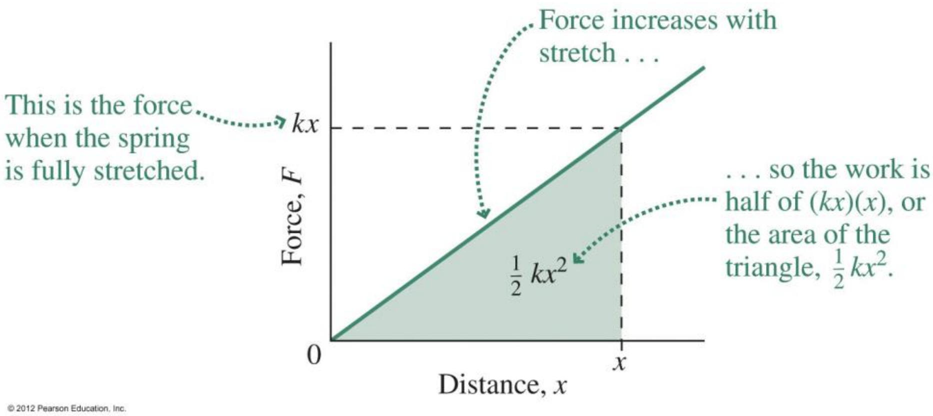
$$W = \int_0^x F(x)\,dx = \int_0^x kx\,dx = k\int_0^x x\,dx$$

$$W = k\left[\frac{x^2}{2}\right]_0^x$$

$$= \frac{1}{2}kx^2 - 0 = \boxed{\frac{1}{2}kx^2}$$

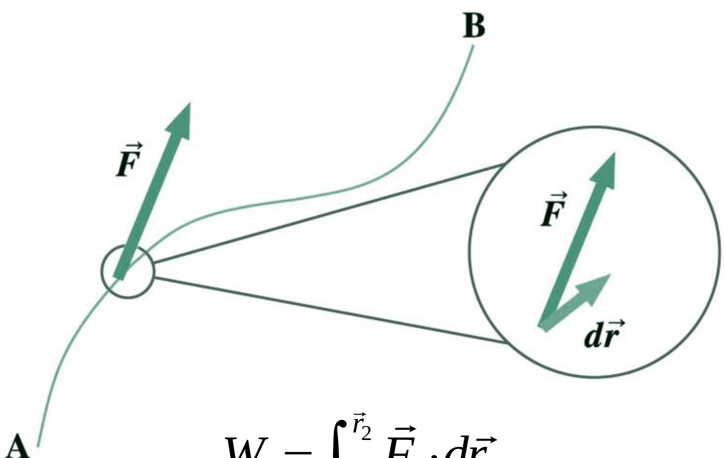
Work Done in Stretching a Spring

- In this case the work is the area under the triangular force-versus-distance curve:



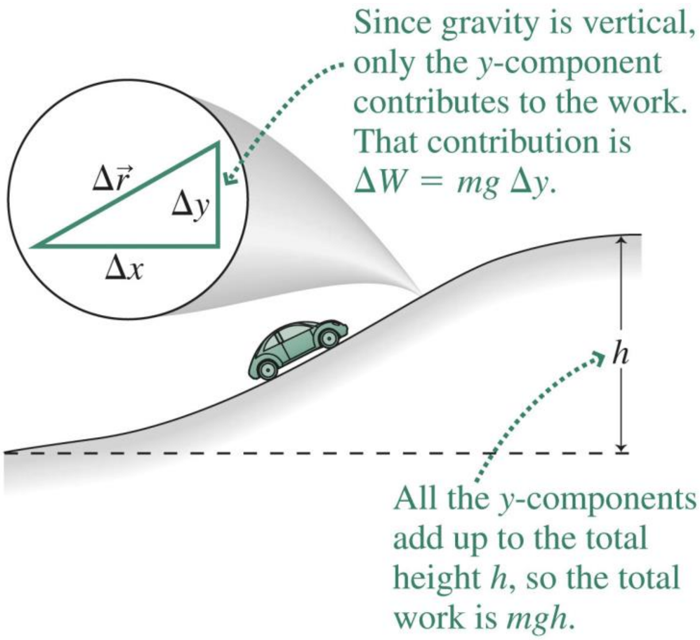
A Varying Force in Multiple Dimensions

- In the most general case, the integral for the work becomes a **line integral**, the limit of the sum of scalar products of infinitesimally small displacements with the force at each point.



$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

Work Done Against Gravity



Work Done Against Gravity

- The work done by an agent lifting an object of mass m against gravity depends only on the vertical distance h:

$$W = mgh$$

- The work is positive if the object is raised and negative if it's lowered.

Work and Net Work

- The work *you* do in moving an object involves only the force *you* apply:
  - But there may be other forces acting on the object as well.
  - The **net work** is the work done by all the forces acting—that is, the work done by the net force.
- Net work always changes the speed of an object.

The Work-Energy Theorem

- Applying Newton's second law to the net work done on an object results in the **work-energy theorem:**

$$W_{net} = \int F_{Net}\, dx = \int ma\, dx = \int m \frac{dv}{dt}\, dx$$

$$W_{net} = \int m \frac{dx}{dt}\, dv = \int m v\, dv$$

- Evaluating the last integral between initial and final velocities v_1 and v_2 gives

$$W_{net} = \int_{v_1}^{v_2} m v\, dv = \left[m \frac{v^2}{2} \right]_{v_1}^{v_2} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

- So the quantity $\frac{1}{2}mv^2$ changes only when net work is done on an object, and the change in this quantity is equal to the net work.

Definition: Kinetic Energy

- Energy of motion
- Depends on the mass of the object and square of its speed:

$$K = \frac{1}{2}mv^2$$

- If object speed is doubled ⇒ kinetic energy is quadrupled.

The Work-Energy Theorem

- The **work-energy theorem** states that the change in an object's kinetic energy is equal to the net work done on the object:

$$\Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = W_{\text{net}}$$

Power

- Measure of how fast work is done
- In equation form:

$$Power = \frac{work\ done}{time\ interval}$$

Unit of power

- joule per second, called the watt after James Watt, developer of the steam engine
  - 1 joule/second = 1 watt
  - 1 kilowatt = 1000 watts

[Image from http://www.spacex.com/news/2015/04/14/liftoff-falcon-9-and-dragon-begin-crs-6-mission-resupply-international-space-station]

Power

Examples:

- A worker uses more power running up the stairs than climbing the same stairs slowly.
- Twice the power of an engine can do twice the work of one engine in the same amount of time, or the same amount of work of one engine in half the time.

1 horsepower = 746 watts

[Image from http://shirabu-fiberglass.indonetwork.net/1295837/yamaha-outboard-motors.htm]

Power

- The work dW done by a force acting on an object that undergoes an infinitesimal displacement dr is

$$dW = \vec{F} \cdot d\vec{r}$$

- Dividing both sides by the associated time interval dt gives the power:

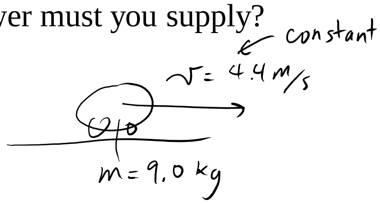
$$P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt}$$

- In terms of velocity:

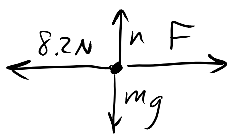
$$P = \vec{F} \cdot \vec{v}$$

Example 6.9

Riding your 9.0 kg bicycle at a steady 4.4 m/s, you experience an 8.2 N force from air resistance. If your mass is 66 kg, what power must you supply?



fbd of bike:



$a = 0 \Rightarrow F = +8.2\,N$

Power $= F \cdot v = 8.2(4.4)$

$P = 36\ N \cdot \frac{m}{s} = 36\ \frac{J}{s} = \boxed{36\ W}$