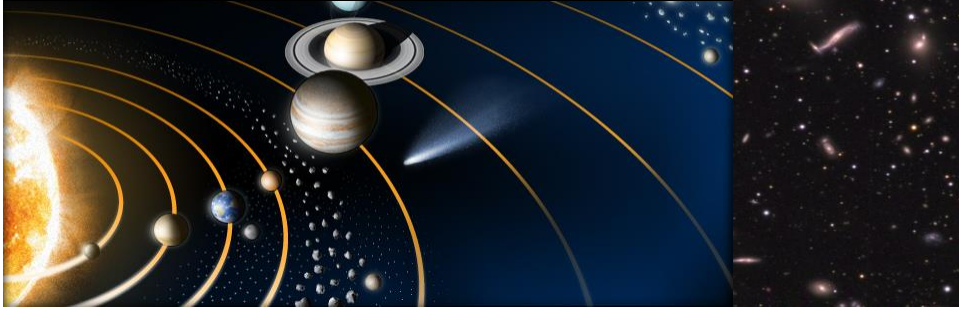# PHY131H1F  - Class 13

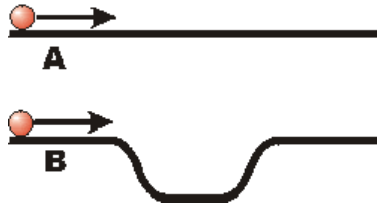**Today:**

Gravitational Potential Energy
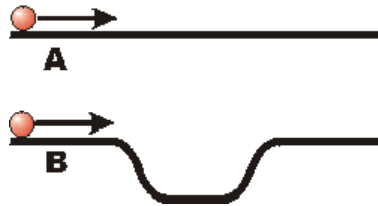
Newton's Law of Universal Gravitation

The Gravitational Field

Orbital Motion

Two balls are launched along a pair of tracks with equal velocities, as shown. Both balls reach the end of the track. *Predict*: Which ball will reach the end of the track first?

- A
- B
- C: They will reach the end of the track at the same time

Demo: Two balls were launched along a pair of tracks with equal velocities. Both balls reached the end of the track. *Observe*: Which ball reached the end of the track first?

- A
- B
- C: They reached the end of the track at the same time

### Class 13 Preclass Quiz on MasteringPhysics

- This was due this morning at 8:00am
- 80% of students got: Two objects attract each other gravitationally. If the distance between their centers decreases by a factor of 2, **the gravitational force increases by a factor of 4.**
- 86% of students got: Two objects attract each other gravitationally. If the mass of each object doubles, **the gravitational force increases by a factor of 4.**
- 93% of students got: Earth's orbit around the Sun is slightly elliptical. Thus, Earth actually gets closer to the Sun during part of the year. **Earth's orbital speed is greater when it is closer to the Sun than when it is farther from the Sun.**

## Class 13 Preclass Quiz Student Comments

- *"By the theory of conservation of energy, the total energy in the universe should stay consistent, how is that possible given that the universe is expanding?"*
- **Harlow answer:** Good question.  Two comments:
- 1. The universe is actually not expanding, it is just becoming less dense.  The distances between pairs of galaxies does increase with time as they all rush away from one another, but the whole universe is infinite in size, and always has been.  You can't get bigger than infinity!  In the big bang, the mass density was infinite, so any finite part of the universe was an infinitesimal point, but the whole thing was still infinite.
- 2. The conservation of energy principle applies to any closed, finite system within the universe.

## Class 13 Preclass Quiz Student Comments

- *"If the Earth has an orbital speed that's faster when it's near the sun, does that mean our summers are shorter than winters? Are we not being cheated?*
- **Harlow answer:** Good question.  Two comments:
- 1. Yes, we do spend more than 6 months of the year further than one semimajor axis from the Sun, and less than 6 months of the year closer than one semimajor axis, due to the increased orbital speed at perihelion.
- 2. The perihelion of Earth's orbit happens in early January.  That coincides with the winter in the Northern hemisphere and summer in the Southern hemisphere.  The distance between the Earth and Sun is **not** the cause of the seasons; it's almost a circular orbit, so it is negligible.  What is much more important is the 23° tilt of the rotation axis which causes the Northern hemisphere to receive more direct sunlight in the March-September months than otherwise.

## Class 13 Preclass Quiz Student Comments

- *"why the kinetic energy of circular orbit is always positive?"*
- **Harlow answer:** $K = \frac{1}{2}mv^2$ is always positive or zero for any object. You can't have negative K.
- *"Why do planets move in an elliptical orbit? If the sun is one focus, where/what is the other?"*
- **Harlow answer:** Newton's laws! There is nothing at the other focus, just empty space.
- *"Will there be a day when physicist discover an equation that can unite relativity and quantum mechanics?"*
- **Harlow answer:** I hope so! Right now it's pretty unsatisfying to have one theory for large scale structure (general relativity) and one theory for small scale structure (quantum mechanics), and no way to unite them.

## Class 13 Preclass Quiz Student Comments

- *"Gravity only attracts, never repels. If that's the case, what's stopping all celestial bodies from getting smushed together?"*
- **Harlow answer:** Initial velocity. Basically it's the big bang, and all the motion of the mass that still exists as a result of this.
- *"What would happen if an object is thrown into a bottomless pit (which goes across all the planet), would it be stuck in the center of the planet, would it go to the other side and come back?"*
- **Harlow answer:** The force of gravity would decrease as the object descended into the pit until it became zero at the centre of the earth. The object would then overshoot the centre, and go back "up" to the other side, until it stopped, and then went back down again, oscillating around the centre.

**Class 13 Preclass Quiz Student Comments**

- *"Hope everyone stays safe on this rainy day!!"*

- *"I was thinking about what would be a scary costume fo Halloween and came up with the conclusion of being a uoft midterm test.... yikes that's really scary"*

- *"The knack of flying is learning how to throw yourself at the ground and miss."*
- *-Douglas Adams*
- *(a little like orbits...)*

# Gravity

It was Newton who first recognized that **gravity is an attractive, long-range force between *any* two objects.**
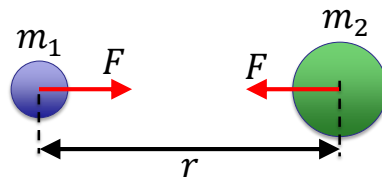


When two objects have masses $m_1$ and $m_2$ and centers are separated by distance $r$, each object attracts the other with a force given by Newton's law of gravity, as follows:
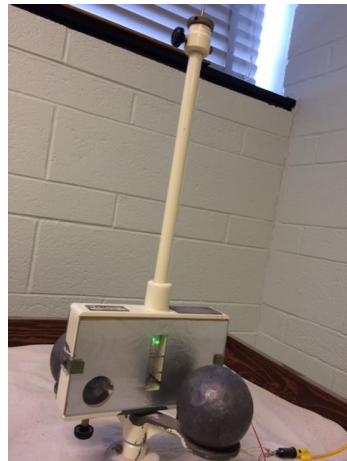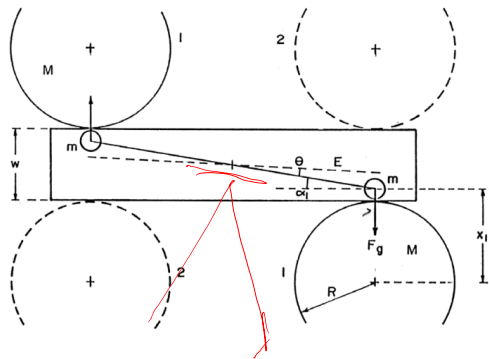
$$F = \frac{Gm_1 m_2}{r^2}$$

where $G = 6.67 \times 10^{-11}$ N m$^2$ kg$^{-2}$ is the Gravitational constant (the same everywhere in the universe).

- *"I found the metal ball thingy confusing. I didn't think the force of gravity between them would be that appreciable."*
- **Harlow answer:** Nano-newtons are not easy to measure, believe me.

# Cavendish Experiment

Done in second year labs (PHY224).



You end up measuring a force of about $10^{-8}$ N (10 nano-Newtons!) which is equivalent to the weight of 0.1 μg.

But it is doable in less than 2 weeks.

## Gravity Example

A mass, $m_1$, sits at the surface a giant spherical rock which is floating in space.

The giant rock has a mass of $m_2 = 6 \times 10^{24}$ kg and a radius of 6400 km.

(a) What is the force of gravity on the mass due to the giant rock, in terms of $m_1$?

(b) Can you think of a good name for this giant rock?

$m_2$

6400 km

$m_1$
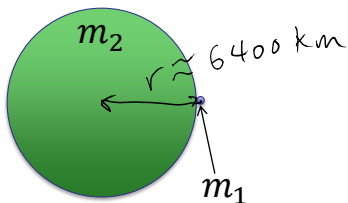
$$F = \frac{G\, m_1\, m_2}{r^2}$$

$$F = m_1 \left( \frac{G\, m_2}{r^2} \right)$$

$$= m_1 \left( \frac{6.67 \times 10^{-11} \left(6 \times 10^{24}\right)}{\left(6400 \times 10^3\right)^2} \right)$$

$$F = m_1 \,(9.8\,)$$

(b) "Earth"

# Gravity for Earthlings

If you happen to live on the surface of a large planet with radius $R$ and mass $M$, you can write the gravitational force more simply as:

$$\vec{F}_G = (mg, \text{ straight down}) \qquad \text{(gravitational force)}$$

where the quantity $g$ is defined to be:
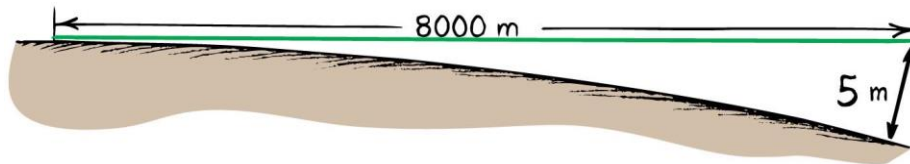
$$g = \frac{GM}{R^2}$$

At sea level, $g = 9.83$ m/s$^2$.
At 39 km altitude, $g = 9.71$ m/s$^2$.

# The Curvature of the Earth

- Earth surface drops a vertical distance of 5 meters for every 8000 meters tangent to the surface.
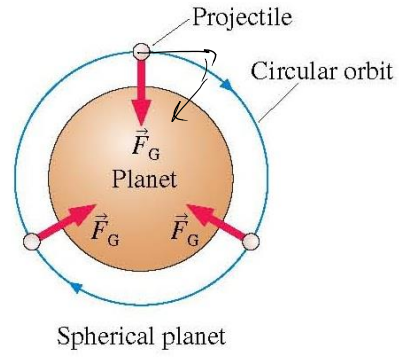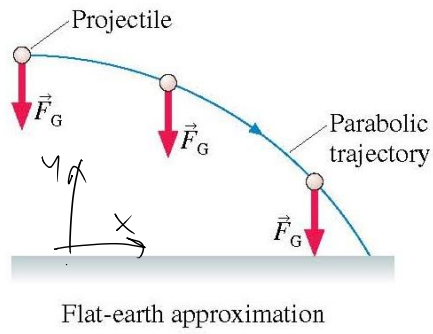


# Ball Launched Horizontally

- Consider a ball launched horizontally, so the initial $y$-component of the velocity is zero.
- How far down does it fall in 1 second?

$$\Delta y = \cancel{v_{0y}}^{0}\, t - \frac{1}{2}g t^2 = -\frac{1}{2}(9.8)t^2 = -4.9\,m \approx \boxed{5\,m\ down}$$

**Class 13 Preclass Quiz Student Comment**

- *"I don't really understand how an orbiting spacecraft is actually constantly in free fall...then wouldn't it fall down?"*
- Harlow answer: **As it falls, it is moving so fast sideways, that the Earth curves away** below it. It is moving at the exact right sideways speed so that the Earth is "curving away" at 9.8 m/s$^2$, so as it falls, it actually never gets closer to the surface!

Projectile

$\vec{F}_G$

$\vec{F}_G$

Parabolic trajectory

$\vec{F}_G$

Flat-earth approximation

Projectile

Circular orbit

$\vec{F}_G$
Planet

$\vec{F}_G$    $\vec{F}_G$

Spherical planet

Example

How fast would you have to drive in order to be "weightless" – ie, no normal force needed to support your car?

$\vec{v}$

$\vec{a} = \dfrac{v^2}{r}$

f.b.d.

$n = 0$

$mg$
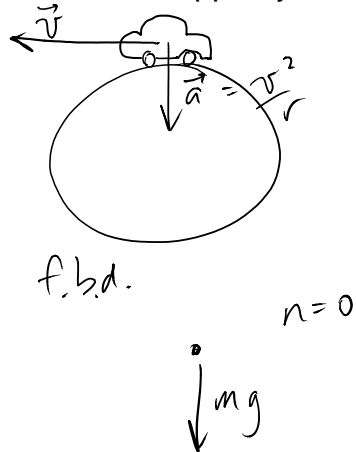
$F_{net} = mg = ma = \dfrac{mv^2}{r}$

$v = \sqrt{gr}$

$= \sqrt{(9.8)(6400 \times 10^3)}$

$v = 7900 \ m/s$

$v = 7.9 \ km/s$

Clicker Question

- Astronaut Kimiya Yui (@Astro_Kimiya) is currently living on the International Space Station, which orbits at 370 km above the surface of the Earth (low earth orbit).
- Assuming Yui has not changed his mass since moving to space, what is the force of gravity on Yui?

A. Zero
B. The same as the force of gravity on him while he was on earth.
C. A little bit less than the force of gravity on him while he was on earth.
D. Not exactly zero, but much, much less than the force of gravity on him while he was on earth.

## International Space Station

Orbit is drawn **to scale**
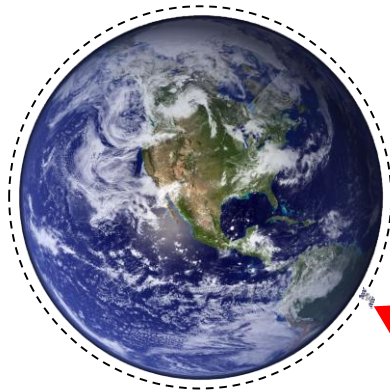


Yui *feels* weightless because he is in freefall!
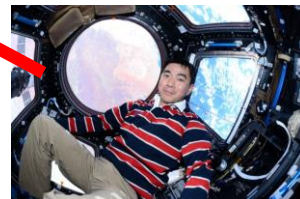
Radius of the Earth: 6400 km, $g$ = 9.8 m/s$^2$

Altitude of Space Station: 370 km, $g$ = 8.9 m/s$^2$ (about 10% less)

# Circular Orbits

An object moving in a circular orbit of radius $r$ at speed $v_{\text{orbit}}$ will have centripetal acceleration of

$$a_r = \frac{(v_{\text{orbit}})^2}{r} = g$$

That is, if an object moves parallel to the surface with the speed

$$v_{\text{orbit}} = \sqrt{rg}$$

then the free-fall acceleration provides exactly the centripetal acceleration needed for a circular orbit of radius $r$.

Near the surface of the Earth, this speed is about 8 km/s.

An object with any other speed will not follow a circular orbit.

Clicker Question

It costs upwards of $100 million to launch a communications satellite. What is the main reason why big companies do this?

A. To get outside Earth's gravitational pull so the satellite doesn't fall down
B. To get closer to the Sun in order to collect more solar power
C. To get away from air resistance so they can move fast and not burn up
D. To get away from radio interference on Earth
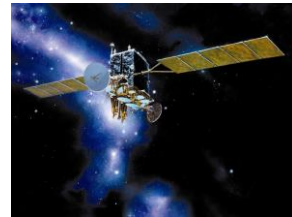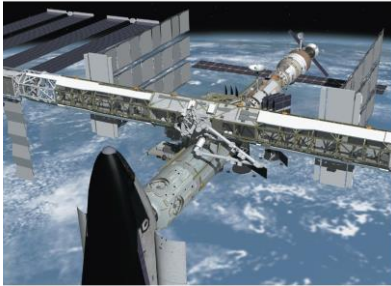E. To get far enough so they can communicate with the entire Earth at one time

# Circular Satellite Orbits



- Positioning: beyond Earth's atmosphere, where air resistance is almost totally absent
- Example: Low-earth orbit communications satellites are launched to altitudes of 150 kilometers or more, in order to be above air drag
- But even the ISS, as shown, experiences *some* air drag, which is compensated for with periodic upward boosts.

### The state of physics around 1650…

- In 1609 Galileo started observing the sky with a telescope.
- Around that same time, Kepler was investigating careful observations of the apparent positions of planets in the sky.
- It was determined that planets orbit the Sun, and that Earth was the third planet out from the Sun.
- Kepler noted that the shapes of the orbits of all the planets were not quite circles, but actually ellipses.

An ellipse is a mathematical shape.
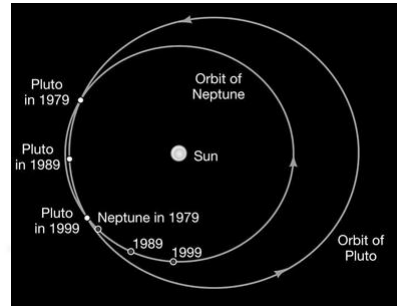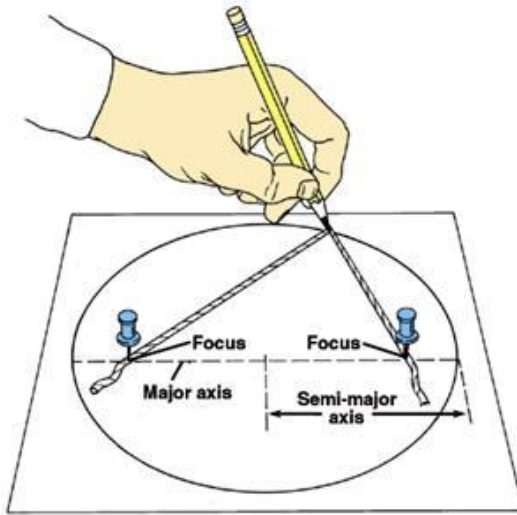




• The furthest distance from the centre of an ellipse to its edge is called the "semi-major axis"

• The eccentricity, $e$, tells you how squished the orbit is.

• A circle is a special case of an ellipse, when $e = 0$.

## Kepler's Laws of Planetary Motion

1. Planets, asteroids and comets move in orbits whose shapes are ellipses, with the sun at one focus of the ellipse. (Planetary orbits normally have low eccentricity: almost circular.)

2. A line drawn between the sun and a planet sweeps out equal areas during equal intervals of time. (They go faster when they are closer to the sun.)
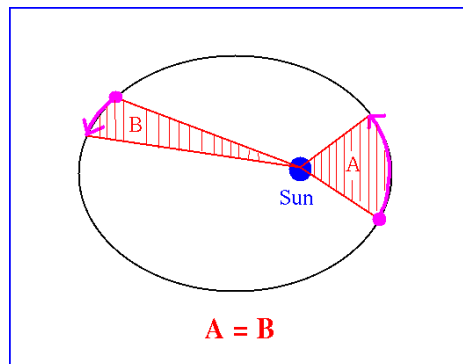


3. The square of a planet's orbital period is proportional to the cube of the semimajor-axis length. ($T^2 = C\,r^3$, where $C$ is some constant.)

• Kepler's Laws are empirical, like Hooke's Law, or the equation for kinetic friction or drag. They were written down in order to describe the observations. Kepler did not know "why" the planets moved in this way.

• Many scientists at the time, including Edmund Halley, believed that there was some kind of force from the Sun pulling the planets, asteroids and comets toward it.

• In 1687 Isaac Newton published one simple theory which explained all of Kepler's laws, as well as motion observed here on Earth:

  • The 3 Newton's Laws you already learned, plus:

  • "Newton's Law of Gravity"

# Projectile Motion and Orbits

- The "parabolic" trajectories of projectiles near Earth's surface are actually sections of elliptical orbits that intersect Earth.

- The trajectories are parabolic only in the approximation that we can neglect Earth's curvature and the variation in gravity with distance from Earth's center.

This section approximates a parabola.

Focus is Earth's center.

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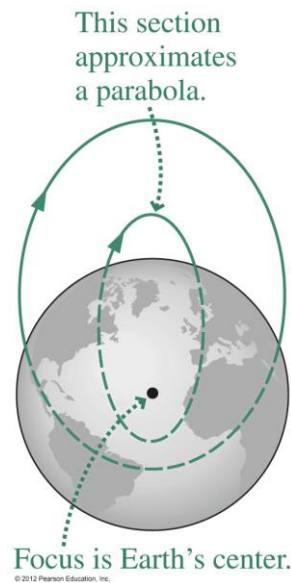
## Gravitational Potential Energy

When two isolated masses $m_1$ and $m_2$ interact over large distances, they have a gravitational potential energy of

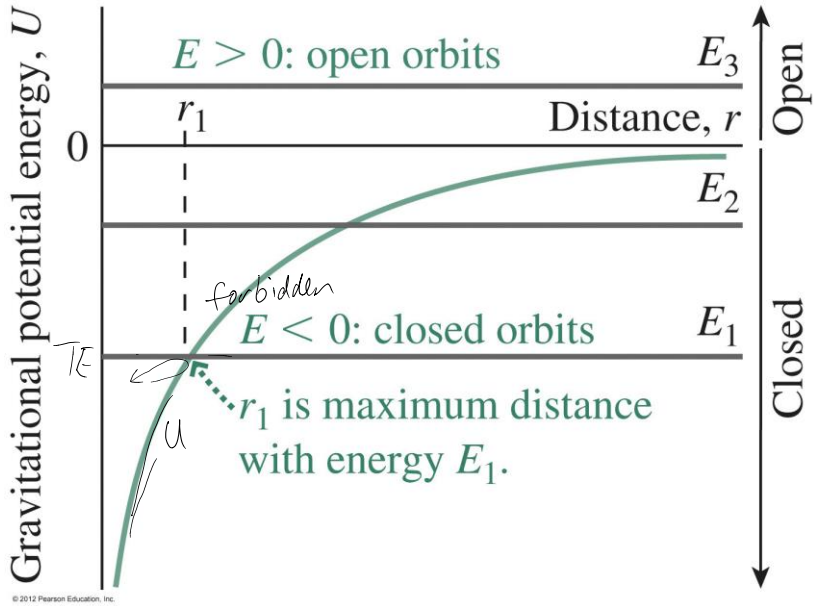$$U_g = -\frac{Gm_1m_2}{r}$$  $\approx$ mgh + constant
when h << r

where we have chosen the zero point of potential energy at $r = \infty$, where the masses will have no tendency, or potential, to move together.   U is always negative
Note that this equation gives the potential energy of masses $m_1$ and $m_2$ when their *centers* are separated by a distance $r$.

## Energy and Orbits

- By our definition of zero point at infinity, $U$ is always negative.
- $K$ is always positive.
- The sign of the total energy $E = K + U$ determines the type of orbit an object:
- $E < 0$: The object is in a bound, elliptical orbit.
  - Special cases include circular orbits and the straight-line paths of falling objects.
- $E > 0$: The orbit is unbound and hyperbolic.
- $E = 0$: The borderline case gives a parabolic orbit.

# Energy and Orbits



Gravitational potential energy, $U$

$E > 0$: open orbits — $E_3$ — Open

$r_1$    Distance, $r$

$0$

$E_2$

forbidden

$TE$

$E < 0$: closed orbits — $E_1$ — Closed

$U$

$r_1$ is maximum distance with energy $E_1$.

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# Energy and Orbits



Open (hyperbola)

$E > 0$

$E < 0$

$E = 0$

Closed (circle)

Borderline (parabola)

Closed (ellipse)

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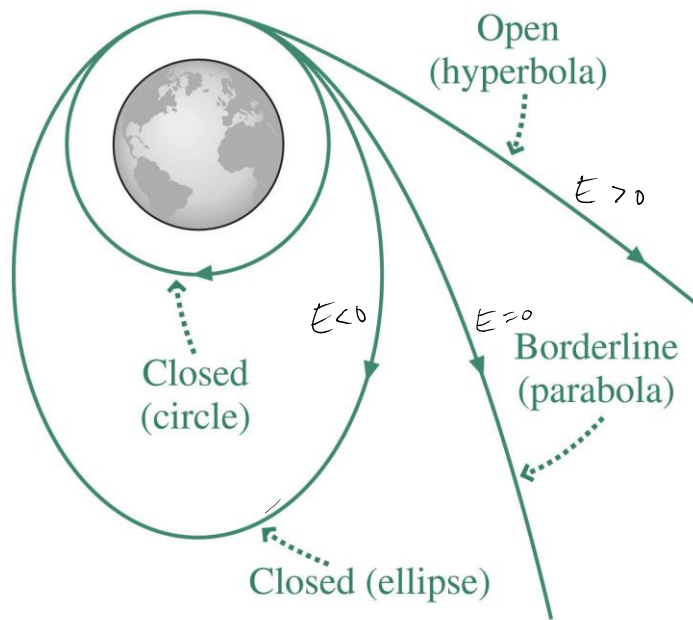
Last day I asked at the end of class:

- There is a saying, "What goes up must come down." Is this always true? How fast must you throw an object upwards so that it *never* comes down?
- ANSWER:
- When the total energy = 0, then, as *r* approaches infinity, *K* will approach zero. This is "escape speed":

$$E = K + U = \tfrac{1}{2}mv^2 - \frac{GM_{Earth}\cdot m}{r_{earth}} = 0$$

$$\tfrac{1}{2}mv^2 = \frac{GM_{Earth}\,m}{r_{Earth}}$$

$$v = \sqrt{\frac{2\,GM_{Earth}}{r_{Earth}}} = 11\ \frac{km}{s}$$

Escape speed.

- The escape speed near the Earth's surface is about 11 km/s.
- If the Earth's radius was less than 9 mm, then the escape speed would be greater than 300,000 km/s, which is greater than the speed of light.
- Since nothing can travel faster than light, nothing would be able to escape the Earth's surface, and the Earth would be what astronomers call a black hole.

## Clicker Question

- A moon is orbiting around Planet X. Which of the following statements is always true about its kinetic energy ($K$), and its gravitational potential energy ($U$)?

    A. $K < 0$ and $U < 0$
    B. $K < 0$ and $U > 0$
    C. $K > 0$ and $U < 0$
    D. $K > 0$ and $U > 0$
    E. $K < 0$ and $U = 0$

## Before Class 14 on Monday

- Remember MasteringPhysics.com **Problem Set 6** on Ch.7 is due Monday by 11:59pm.
- Please read the first two sections of Chapter 9 on Center of Mass and Conservation of Momentum
- Something to think about:
- How is it possible to clear the bar in a high jump if your center of mass does not reach to the height of the bar?



CM position at the peak of the jump

CM's trajectory