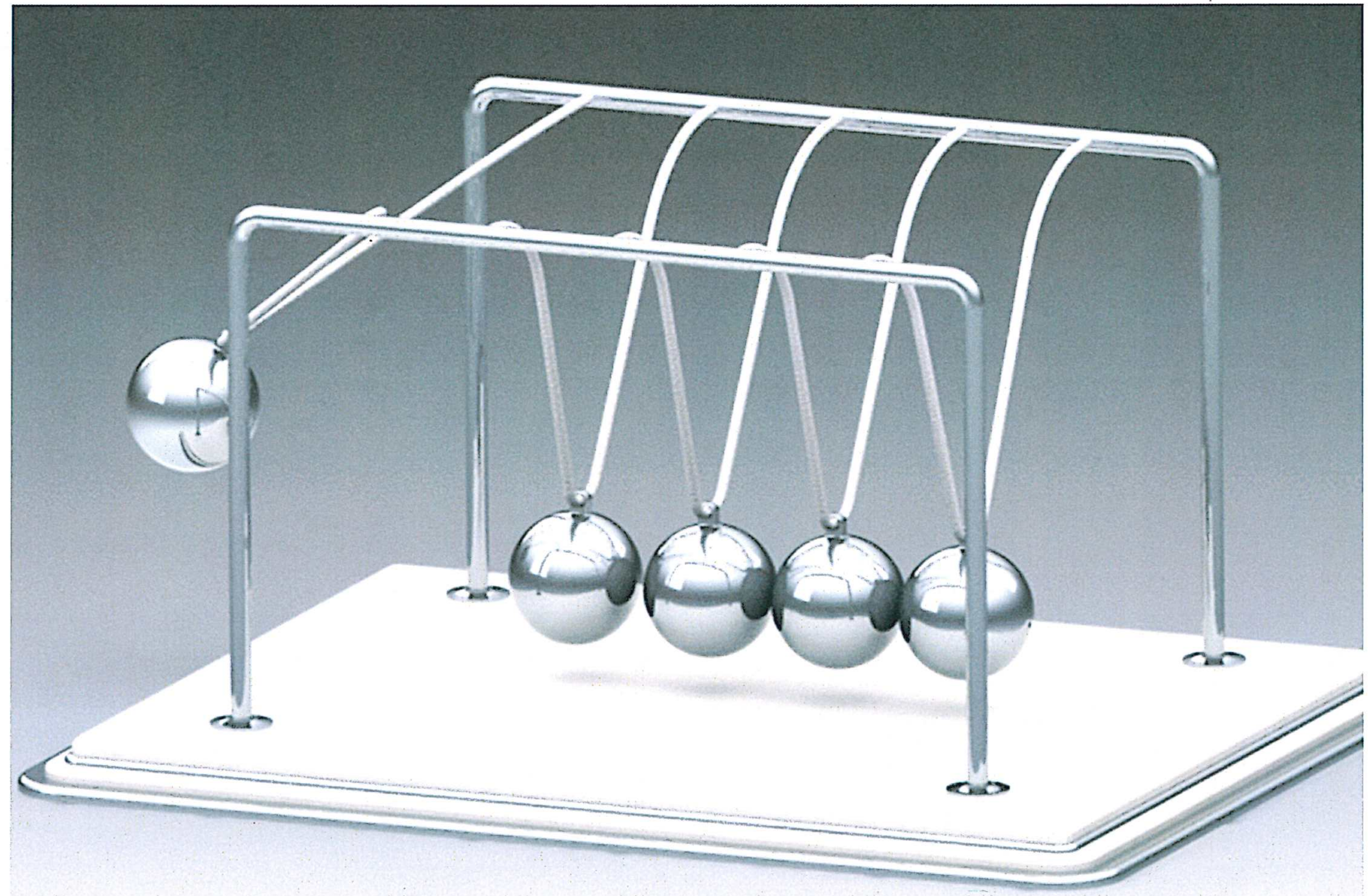
PHY131H1F - Class 15


[image from https://grabcad.com/library/newton-s-cradle-assembly-1]

Today, we are finishing Chapter 9 on Momentum:

Impulse and Momentum

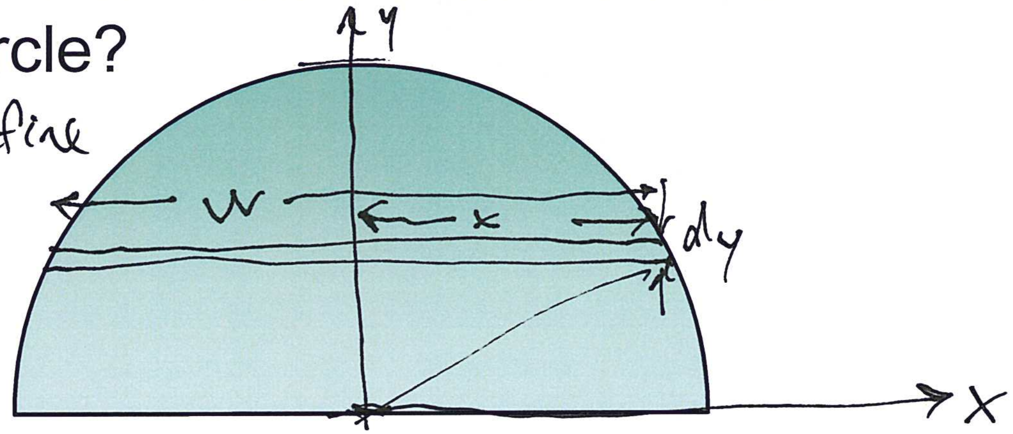
Energy in Collisions

Totally Inelastic Collisions

Elastic Collisions

Where is the centre of mass of a solid semicircle?

Define



$x_{cm} = 0$, by symmetry.

Use: $y_{cm} = \frac{1}{M}\int y\,dm$

Need a rectangle for dm that is at a single y-position.

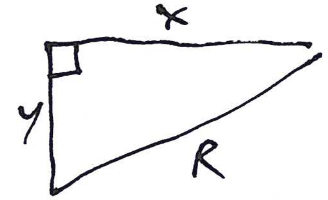
$dA = dy \cdot w$

Note: $w = 2x$

$\Rightarrow dA = 2x\,dy$

$$dm = dA\left(\text{mass per area}\right) = dA\left(\frac{M}{\pi R^2/2}\right)$$

$$dm = \frac{2M}{\pi R^2}\,dA = \frac{4M}{\pi R^2}\,x\,dy$$



$$x^2 + y^2 = R^2$$

$$\Rightarrow x = \sqrt{R^2 - y^2}$$

$$dm = \frac{4M}{\pi R^2}\sqrt{R^2 - y^2}\,dy$$

$$y_{cm} = \frac{1}{M}\int y\left[\frac{4M}{\pi R^2}\sqrt{R^2-y^2}\right]dy$$

$$= \frac{4}{\pi R^2}\int_0^R y\sqrt{R^2-y^2}\,dy$$

From an integral table:

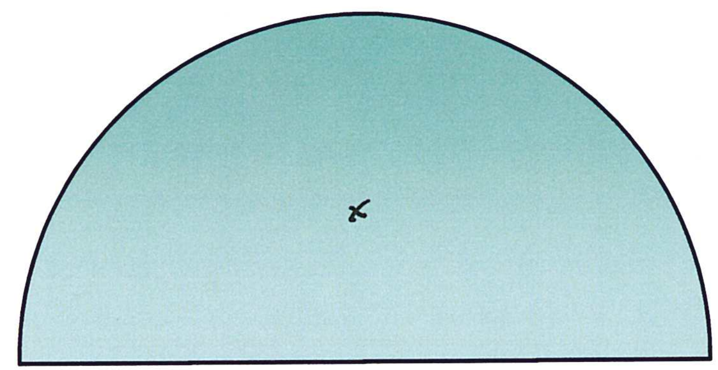
I would give this on a test

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\left(a^2-x^2\right)^{3/2}$$

$$y_{cm} = \frac{4}{\pi R^2}\left[-\frac{1}{3}\left(R^2-y^2\right)^{3/2}\right]_0^R$$

$$y_{cm} = \frac{4}{\pi R^2}\left(0 + \frac{1}{3}(R^2)^{3/2}\right)$$

$$y_{cm} = \frac{4}{3\pi R^2} R^3 = \frac{4}{3\pi} R$$



Pre-class 15 results: Q.1 (1 point)

In a perfectly ELASTIC collision between two perfectly rigid objects

A. the momentum of each object is conserved.

B. the kinetic energy of each object is conserved.

C. the momentum of the system is conserved but the kinetic energy of the system is not conserved.

D. both the momentum and the kinetic energy of the system are conserved.

E. the kinetic energy of the system is conserved, but the momentum of the system is not conserved.

560 responses, 90% correct

A. 3%
B. 3%
C. 3%
D. 90%
E. 1%

Pre-class 15 results: Q.2 (1 point)

In an INELASTIC collision between two objects

A. the momentum of each object is conserved.

B. the kinetic energy of each object is conserved.

C. the momentum of the system is conserved but the kinetic energy of the system is not conserved.

D. both the momentum and the kinetic energy of the system are conserved.

E. the kinetic energy of the system is conserved, but the momentum of the system is not conserved.

558 responses, 86% correct

A. 4%
B. 1%
C. 86%
D. 2%
E. 6%

Pre-class 15 results: Q.3 (1 point)

A shell explodes into two fragments, one fragment 25 times heavier than the other. If any gas from the explosion has negligible mass, then

A. the momentum change of the lighter fragment is 25 times as great as the momentum change of the heavier fragment.

B. the momentum change of the heavier fragment is 25 times as great as the momentum change of the lighter fragment.

C. the momentum change of the lighter fragment is exactly the same as the momentum change of the heavier fragment.

D. the kinetic energy change of the heavier fragment is 25 times as great as the kinetic energy change of the lighter fragment.

E. the kinetic energy change of the lighter fragment is 25 times as great as the kinetic energy change of the heavier fragment.

557 responses, 54% correct

A.	9%
B.	15%
C.	54%
D.	7%
E.	15%

Pre-class 15 results: Q.4 (7 points)

Do you have any questions or comments about today's reading and/or preclass video?

- *What's the difference between conservation of momentum and conservation of kinetic energy? If kinetic energy is conserved, is momentum then necessarily conserved too?*

Harlow answer: NO. These are totally different.

Conservation of energy ($E_f = E_i$) is when

a. No work is done on the system by external non-conservative forces
b. No heat is gained or lost from or to the environment

Conservation of momentum ($\vec{P}_f = \vec{P}_i$) is when there is no external net force on the system

- *What does the symbol "<<" refer to?*
- Harlow answer: "Much, much less than". So if Fnet,external << Finteraction, then the external net force is much, much less than the interaction forces, and we can use conservation of momentum.

Last day I asked at the end of class:

- Consider the two integrals below. What's the difference?

$$\vec{J} = \int \vec{F} dt$$

Impulse (Ch.9)

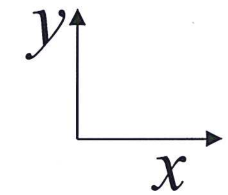
This is the force integrated over **time**, which gives the change in **momentum**. [Units: kg m / s]

$$W = \int \vec{F} \cdot d\vec{r}$$

Work (Ch. 6)

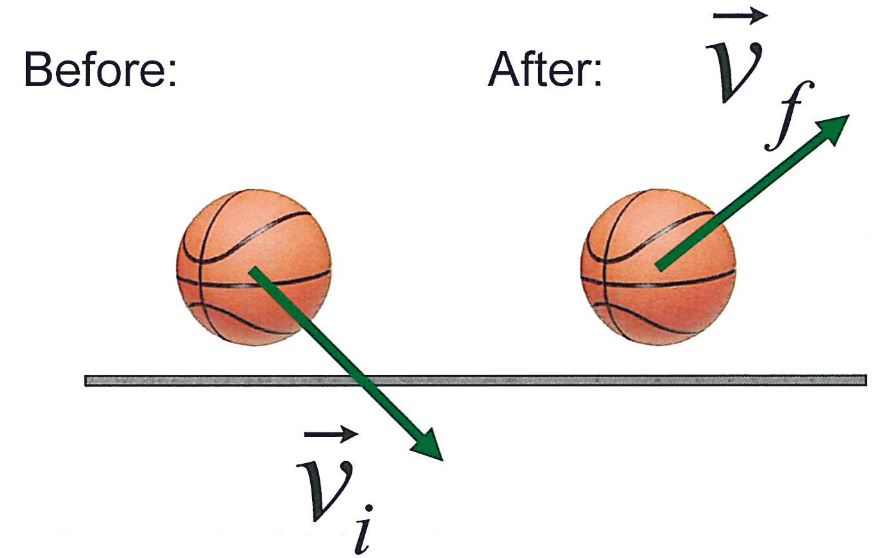
This is the force integrated over **distance**, which gives the change in **energy**. [Units: Joules]

$\vec{p} = m\vec{v}$ means $p_x = mv_x$ and $p_y = mv_y$.



A basketball with mass 0.1 kg is traveling down and to the right with $v_{xi} = +5$ m/s, and $v_{yi} = -5$ m/s.

It hits the horizontal ground, and then is traveling up and to the right with $v_{xf} = +5$ m/s, and $v_{yf} = +4$ m/s.



LC Question 1 What is the change in the x-component of the ball's momentum? 0

use units $\frac{kg \cdot m}{s}$ (but only enter the number)

LC Question 2 What is the change in the y-component of the ball's momentum?

$\Delta p_y = p_{yf} - p_{yi}$

$\Delta p_y = 0.1(v_{fy} - v_{iy})$

$= 0.1(4 - (-5)) = 0.1(9)$

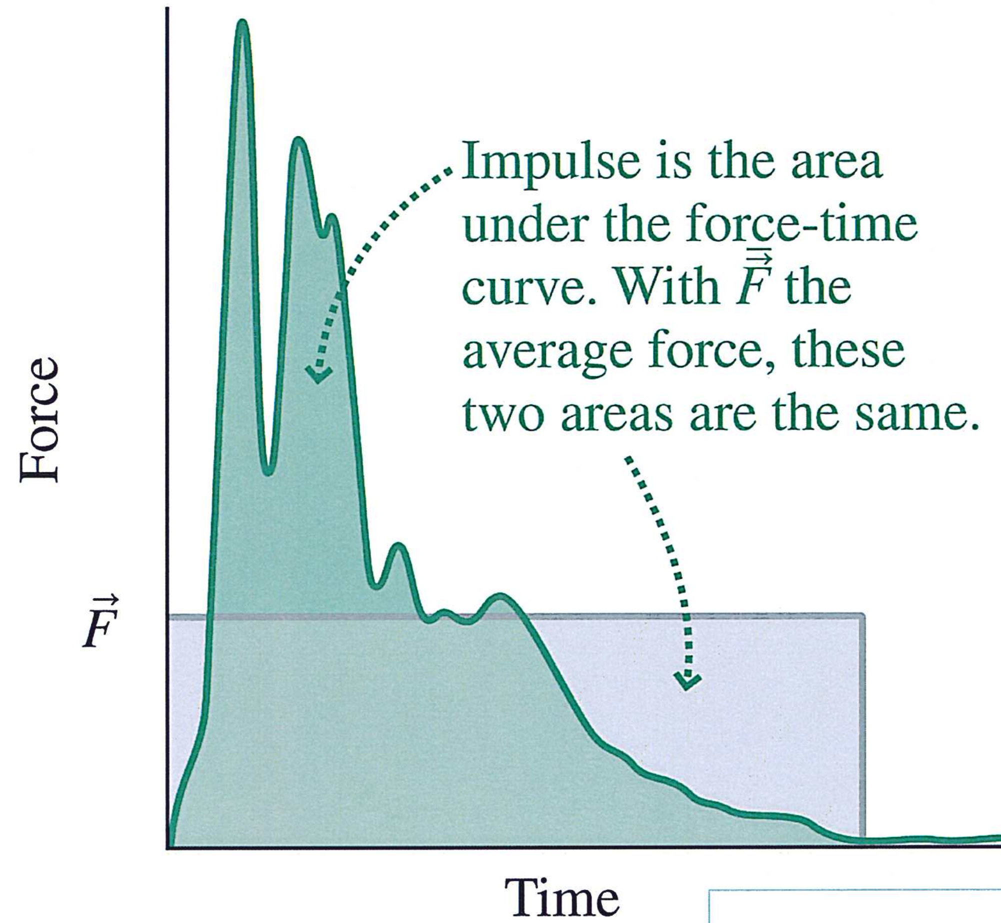
$\Delta p_y = 0.9 \ kg\frac{m}{s}$

Impulse

The *impulse* upon a particle is:

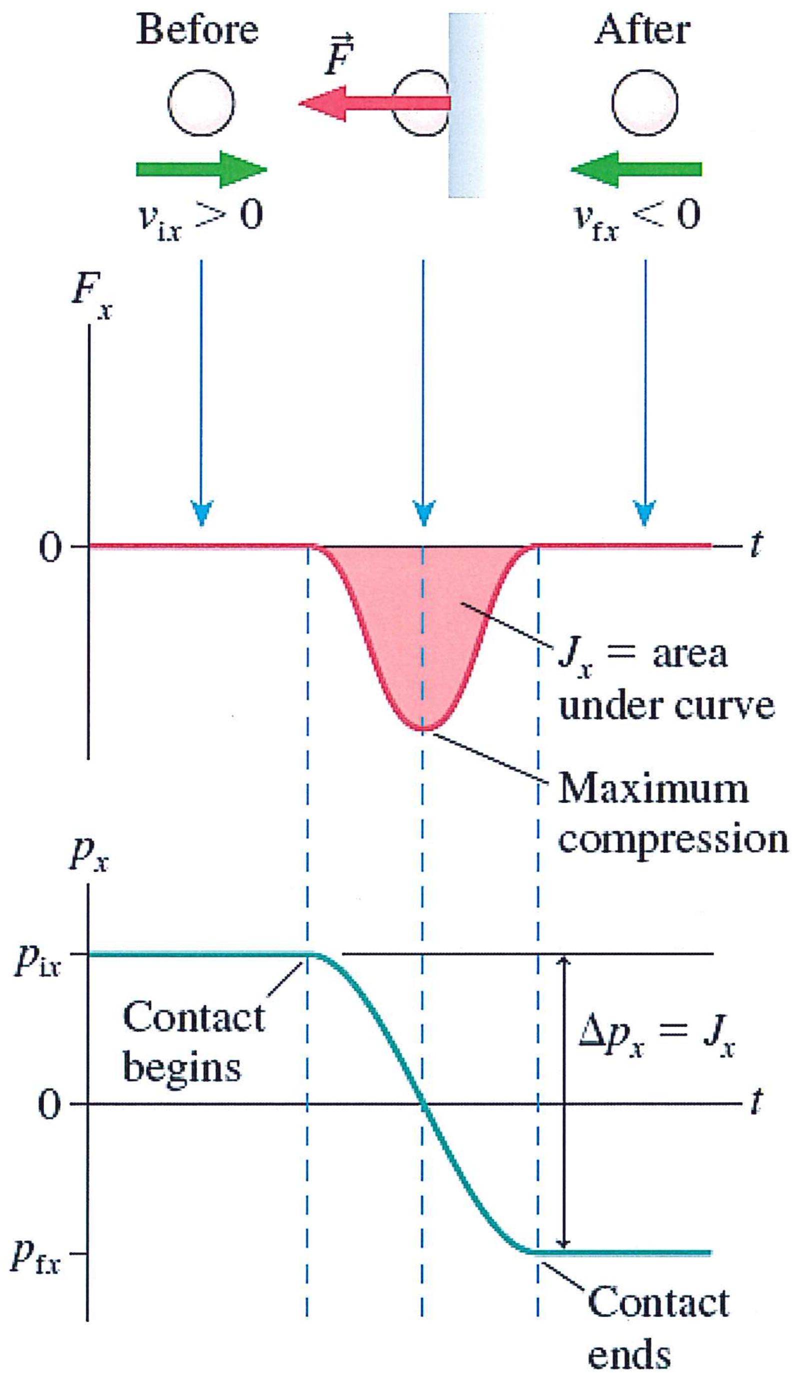
$$J_x = \int_{t_1}^{t_2} F_x dt$$

$$J_y = \int_{t_1}^{t_2} F_y dt$$



- Impulse has units of N s, but you should be able to show that N s are equivalent to kg m/s.
- The **impulse-momentum theorem** states that the change in a particle's momentum is equal to the impulse on it:

$$\Delta p_x = J_x \qquad \Delta p_y = J_y$$


Before
$\vec{F}$
After
$v_{ix} > 0$
$v_{fx} < 0$
F_x
0
t
J_x = area under curve
Maximum compression
p_x
p_{ix}
Contact begins
0
t
$\Delta p_x = J_x$
p_{fx}
Contact ends

A 0.50 kg cart rolls to the right at +1.2 m/s.

It collides with a force sensor.

A plot of force versus time (with positive force defined as towards the right) gives an area of –1.0 N s. $= J_x$

What is the velocity of the cart immediately after the collision?

Before: $v_{ix} = +1.2\ \frac{m}{s}$

$p_{ix} = 0.5(1.2)$

$p_{ix} = +0.6\ \frac{kg\,m}{s}$

$J_x = \Delta p_x = p_{fx} - p_{ix}$

$p_{fx} = p_{ix} + J$

$p_{fx} = 0.6 + (-1.0)$

$= -0.4\ \frac{kg \cdot m}{s}$

$p_{fx} = m\, v_{fx}$

$v_{fx} = \frac{p_{fx}}{m} = \frac{-0.4}{0.5}$

$\boxed{v_{fx} = -0.8\ \frac{m}{s}}$

$|v_{fx}| < |v_{ix}| \rightarrow$ Energy was lost in collision

Image from http://images.thecarconnection.com/med/25-years-of-the-airbag_100223157_m.jpg

$$\vec{J} = \Delta\vec{p} = \vec{F} \cdot t$$

- Consider a car accident in which a car, initially traveling at 50 km/hr, collides with a large, massive bridge support.
- The car comes to an abrupt stop.
- Why is it better to hit the airbag as opposed to the hard plastic steering wheel or dashboard?
- ANSWER:
- The people must reduce their momentum from mv to zero. This requires a force applied over some amount of time. If the time is very short, the force must be very large (ie hitting steering wheel).
- If the person hits the airbag, this squishes during impact, lengthening the time of the stop. If the stopping process **takes longer**, then the maximum force is **less**.

Learning Catalytics Question 3

A 100 g rubber ball and a 100 g damp cloth are dropped on the floor from the same height. They both are traveling at the same speed just before they hit the floor.

The rubber ball bounces, the damp cloth does not.
Which object receives a larger upward impulse from the floor?

A. They receive equal impulses.
B. The damp cloth receives a larger impulse.
C. The rubber ball receives a larger impulse.

Learning Catalytics Question 3

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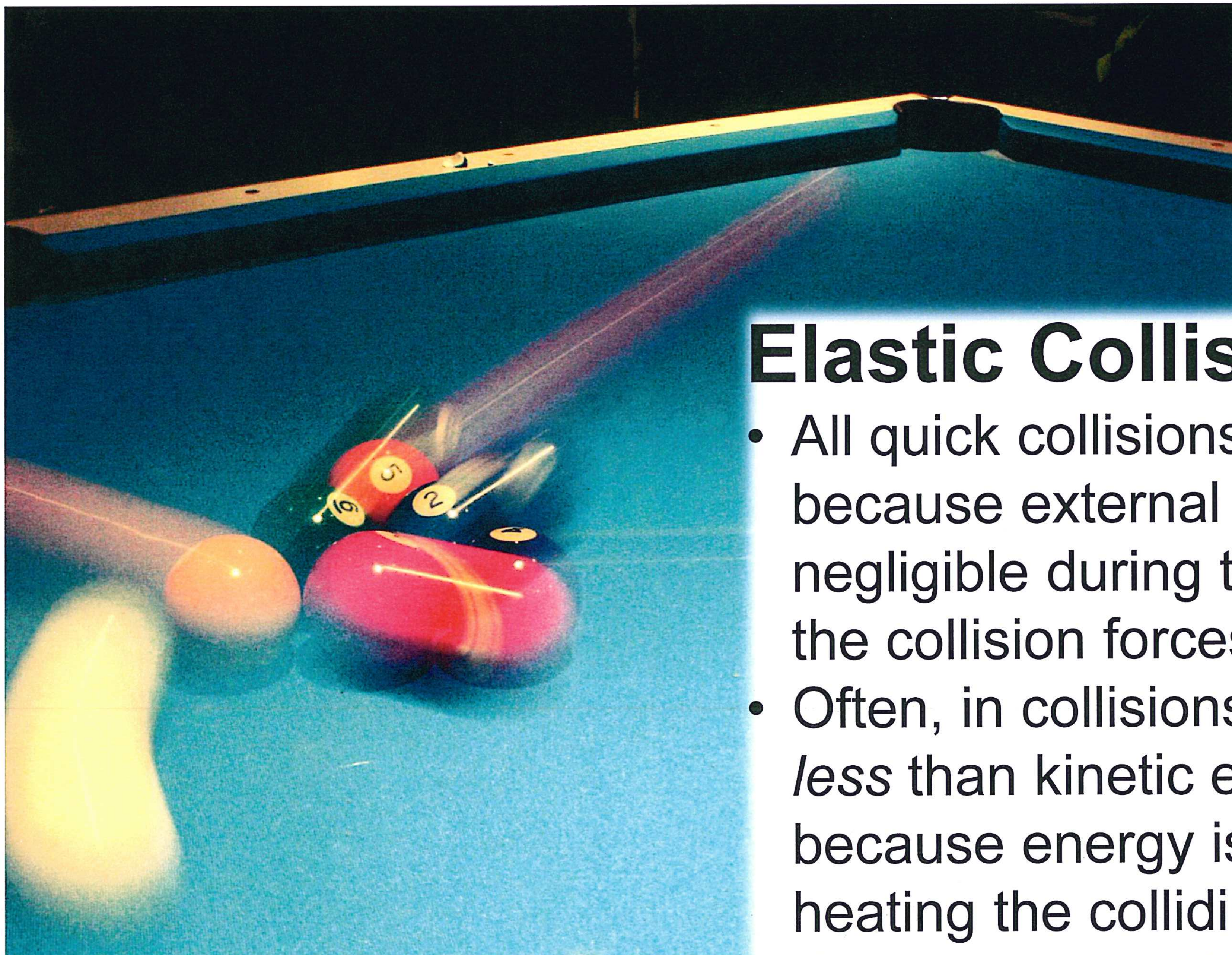
The rubber ball bounces, the damp cloth does not.
Which object receives a larger upward impulse from the floor?

A. They receive equal impulses.
B. The damp cloth receives a larger impulse.
C. The rubber ball receives a larger impulse.

Damp cloth: $p_f = 0$ so $\Delta p = 0 - p_i = -p_i$.

Rubber ball: $p_f \approx -p_i$ so $\Delta p \approx -p_i - p_i = -2p_i$.



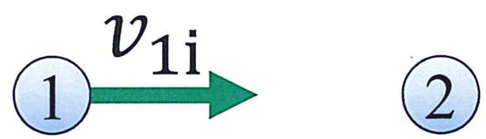
Elastic Collisions

- All quick collisions conserve momentum, because external forces are usually negligible during the collision compared to the collision forces themselves.
- Often, in collisions, kinetic energy after is *less* than kinetic energy before. That's because energy is used up in deforming or heating the colliding objects.
- If kinetic energy is conserved, the collision is said to be **elastic**.

Elastic Collision in 1 Dimension when ball 2 is initially at rest.

Consider a head-on, perfectly elastic collision of a ball of mass m_1 having initial velocity v_{1i}, with a ball of mass m_2 that is initially at rest.

Before:



K_i

During:

During the collision energy is stored as elastic potential energy.

After:

$K_f = K_i$

The balls' velocities after the collision are v_{1f} and v_{2f}.

Elastic Collision in 1 Dimension when ball 2 is initially at rest.

Momentum conservation: $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i}$

Kinetic energy conservation: $\frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2 = \frac{1}{2} m_1 {v_{1i}}^2$

There are two equations, and two unknowns: v_{1f} and v_{2f}.
Solving for the unknowns gives:

Eq. 9.15:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

(Elastic collision with ball 2 initially at rest.)

← If ball 2 is not at rest, switch to a reference frame where it is.

Elastic Collision in 1 Dimension when ball 2 is initially at rest.

Eq. 9.15

$$v_{1\mathrm{f}} = \frac{m_1 - m_2}{m_1 + m_2} v_{1\mathrm{i}}$$

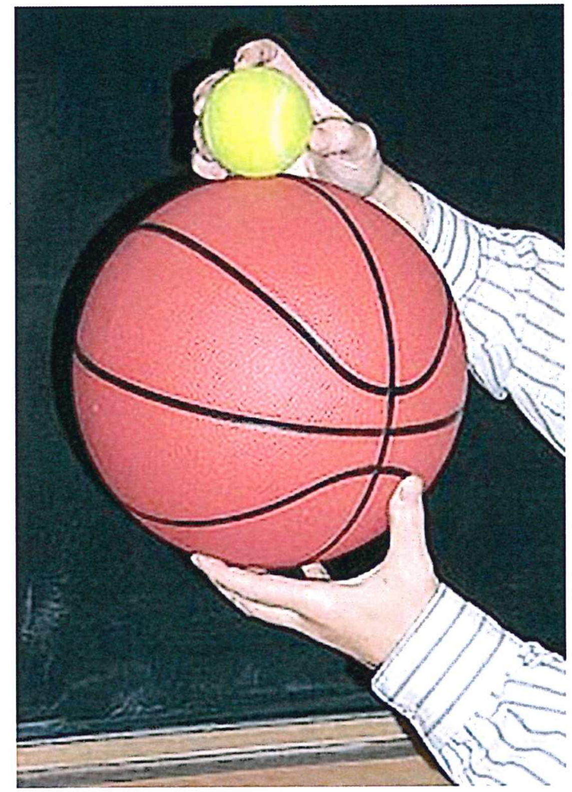
$$v_{2\mathrm{f}} = \frac{2m_1}{m_1 + m_2} v_{1\mathrm{i}}$$

(Elastic collision with ball 2 initially at rest.)

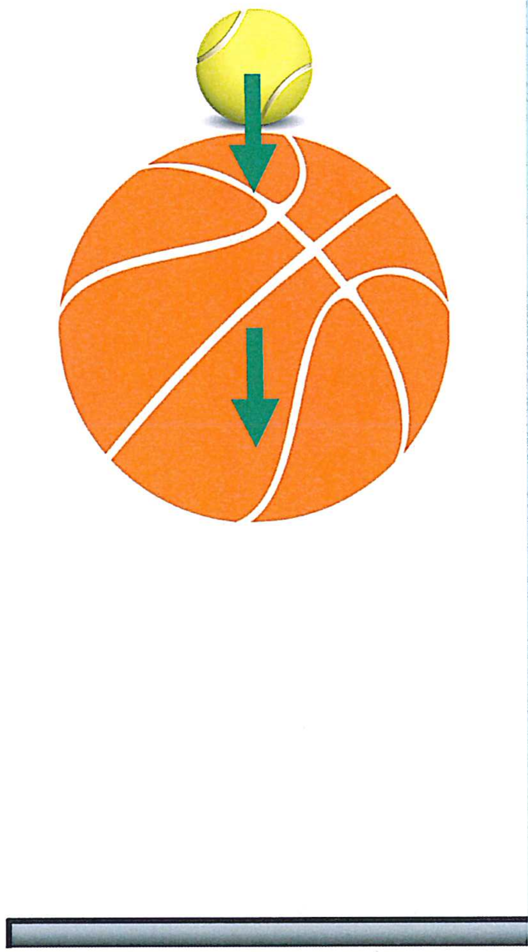
These equations come in especially handy, because you can always switch into an inertial reference frame in which ball 2 is initially at rest!

Demonstration and Example

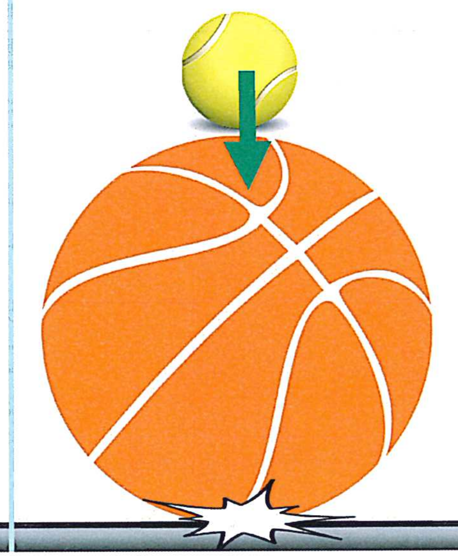
- A 0.50 kg basketball and a 0.05 kg tennis ball are stacked on top of each other, and then dropped from a height of 0.82 m above the floor.
- How high does the tennis ball bounce?
- Assume all perfectly elastic collisions.



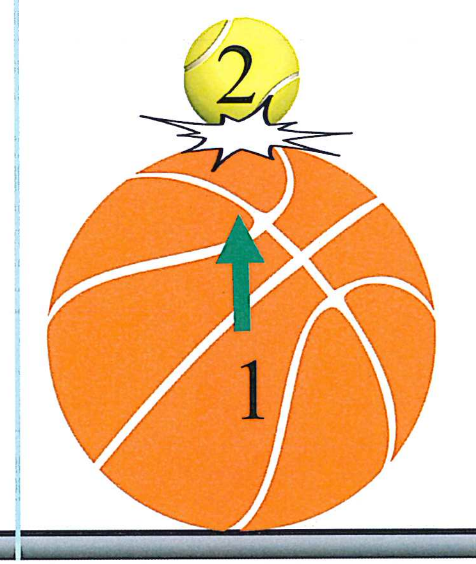
Segment 1: freefall of both balls as they fall, $v_i = 0$.



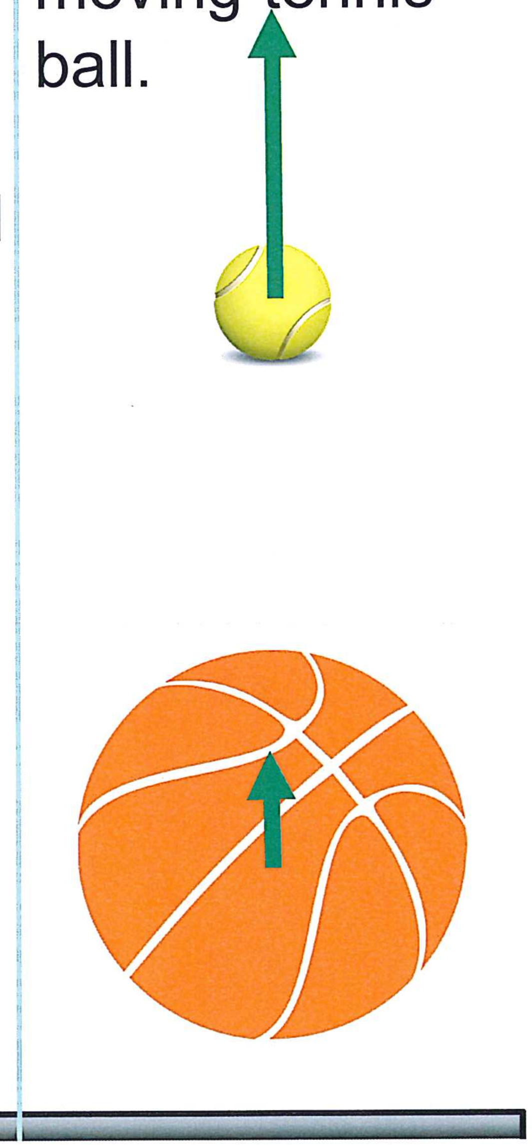
Segment 2: Elastic collision of basketball with **floor**. Tennis ball continues downward, unaffected.



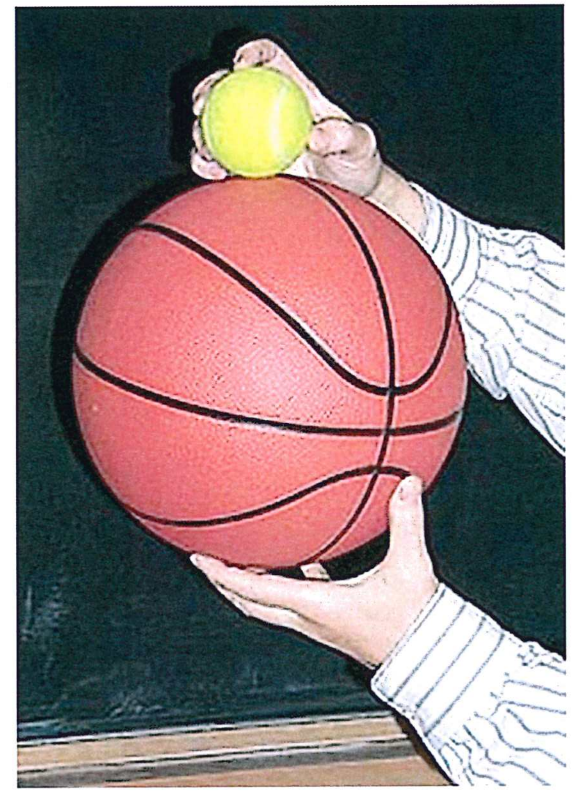
Segment 3: Elastic collision of upward moving basketball (1) with downward moving tennis ball (2).



Segment 4: freefall of upward moving tennis ball.



Demonstration and Example



- Divide motion into segments.
- **Segment 1:** free-fall of both balls from a height of h = 0.82 m. Use conservation of energy: $U_f + K_f = U_i + K_i$

$$0 + \tfrac{1}{2} m v_f^2 = mgh + 0$$

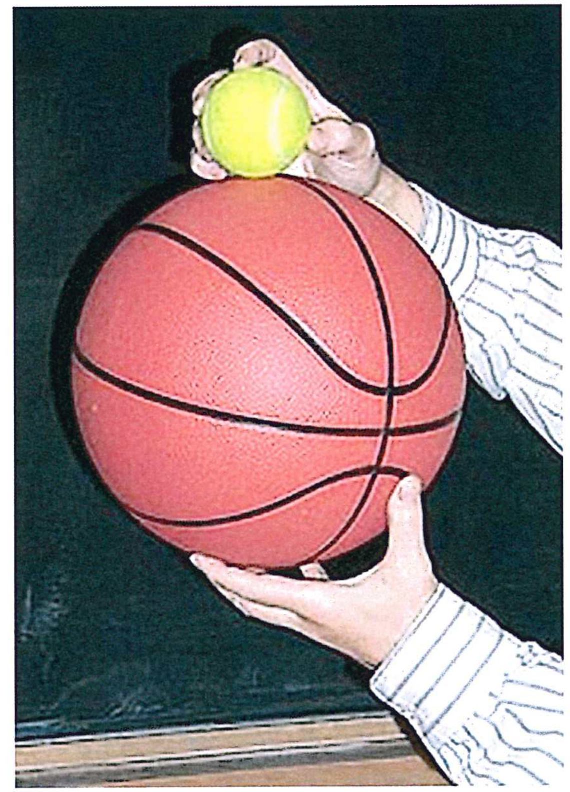
$$v_f = \pm[2gh]^{1/2} = -4.0 \text{ m/s, for both balls.}$$

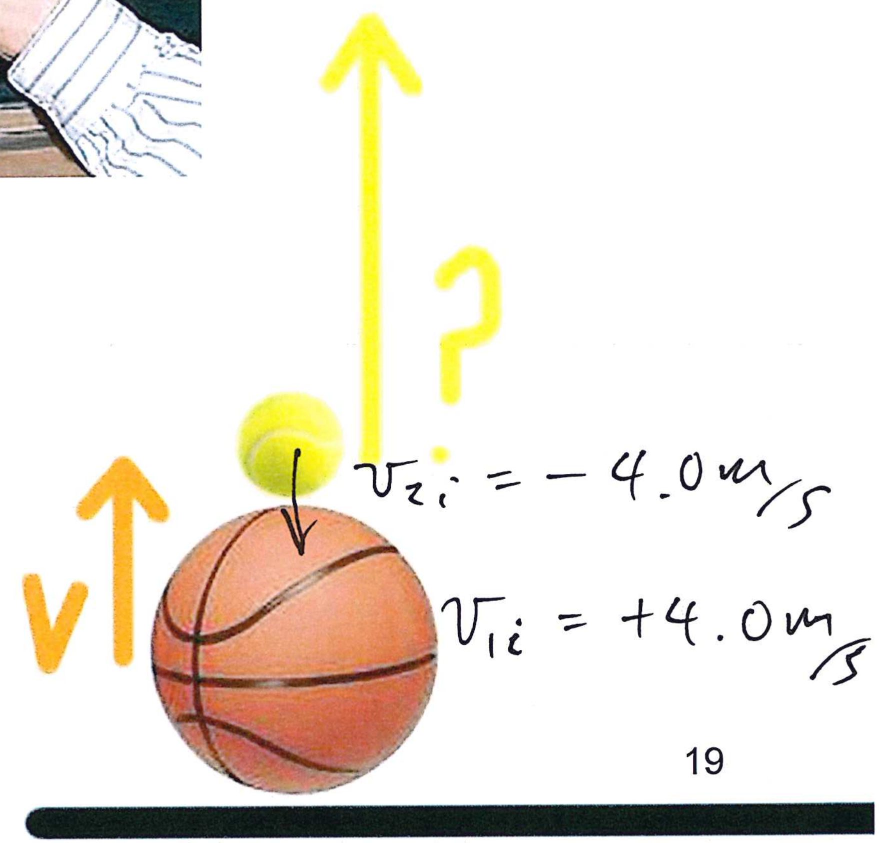
$$v_f = \pm\sqrt{2(9.8)(0.82)} = -4.0 \text{ m/s}$$

- **Segment 2:** basketball bounces elastically with the floor, so its new velocity is +4.0 m/s.

Demonstration and Example

- **Segment 3:** A 0.50 kg basketball moving upward at 4.0 m/s strikes a 0.05 kg tennis ball, initially moving downward at 4.0 m/s.
- Their collision is perfectly elastic.
- What is the speed of the tennis ball immediately after the collision?





- A 0.50 kg basketball moving upward at 4.0 m/s strikes a 0.05 kg tennis ball, initially moving downward at 4.0 m/s.
- Their collision is perfectly elastic. What is the speed of the tennis ball immediately after the collision?

$m_1 = 0.5$ kg

$m_2 = 0.05$ kg.

Ground frame:

Before:

② ↓ $v_{2i} = -4$

① ↑ $v_{1i} = +4$

Switch to reference frame in which $v_{2i} = 0$ by adding +4 to all velocity. (tennis ball frame)

Before:

② $v_{2i} = 0$

① ↑ $v_{1i} = +8$ m/s

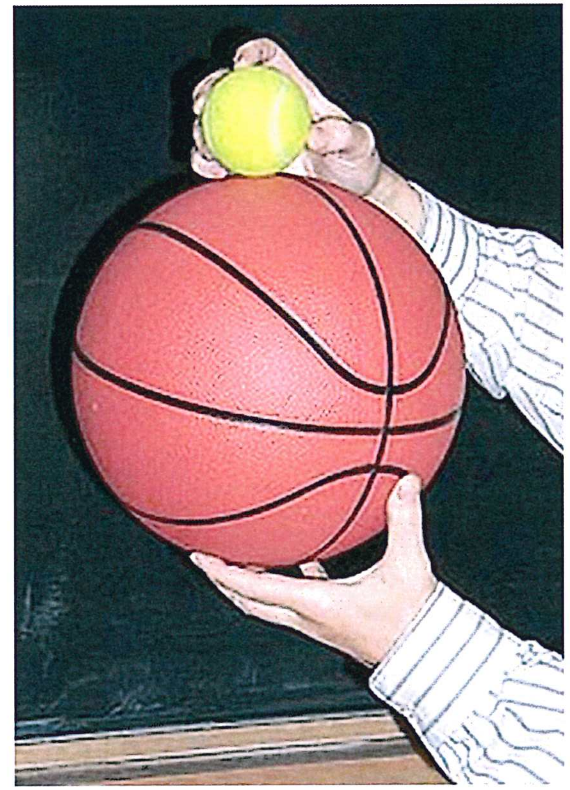
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2(0.5)}{0.5 + 0.05} 8$$

$$= +14.5 \text{ m/s}$$

Don't forget to convert back to Ground frame by subtracting 4:

$$\boxed{v_{2f} = +10.5 \text{ m/s}}$$

Demonstration and Example

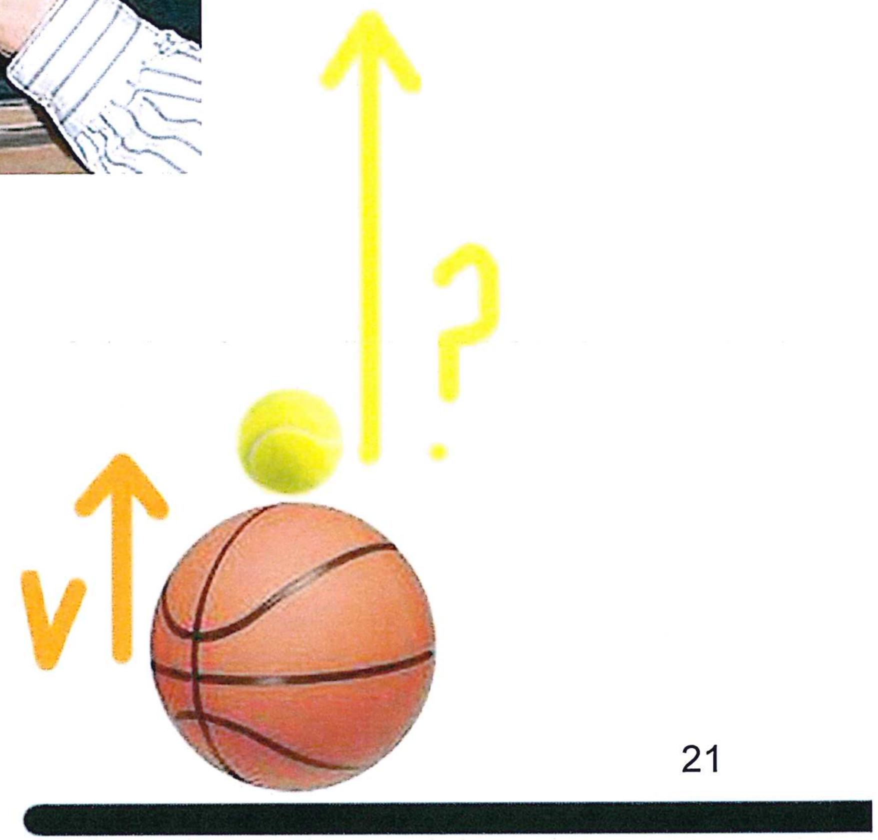


- **Segment 4:** freefall of tennis ball on the way up. $v_{i2} = +10.5$ m/s.
- Use conservation of energy: $U_f + K_f = U_i + K_i$

$$mgh + 0 = 0 + \tfrac{1}{2} m v_i^2$$

$$h = v_i^2/(2g) = 5.6 \text{ m.}$$

- So the balls were dropped from 0.82 m, but the tennis ball rebounds up to 5.6 m! (Assuming no energy losses.)



Before Class 16 next Monday Nov. 13

- Complete Problem Set 7 on Chapters 8 and 9 by Monday Nov.13 at 11:59pm.
- Read the first 3 sections of Chapter 10 and/or watch the Preclass 16 video.
- Have a great Reading Week!

Something to think about: Why is a door easier to open when the handle is far from the hinge, and more difficult to open when the handle is in the middle?

[image from http://s693.photobucket.com/user/rorimac/media/Shire_dg7.jpg.html]