

PHY131H1F - Class 16

Today I will talk about stuff that will **not** be on tomorrow's midterm, but **will** be on the final exam on Dec. 13:

10.1 Angular Velocity and Acceleration

10.2 Torque

10.3 Rotational Inertia and the Analog of Newton's Law



This is a torque wrench.



Term Test 2 Info

- The second term test is tomorrow, Tue., Nov. 14, 6:10pm - 7:30pm (80 minutes). The room is based on the first letter of your family name:
 - A - K: EX100
 - L - Ti: EX200
 - Tj - Y: EX300
 - Z: EX310
- To further help you understand what room you need to go to, you can look up under "My Grades" to find the room under the column Test 2 Room Assignments.
- Alternate sitting students have received a separate email letting you know the room and time.


Term Test 2 Info

- Testable material is:
 - Chapters 6 - 9 of Wolfson
 - Classes 9 - 15, from Oct.11-Nov.1, inclusive
 - Practicals 4 - 7 and Homeworks 4 - 7, from Oct.10-Nov.13, inclusive, including the “PHY131 Uncertainty Propagation Summary” which is used in all Practical activities in which you do measurements.


Term Test 2 Info

- The format will be the same as for test 1. There will be 8 multiple choice questions, worth 2 points each, plus two free-form problems worth 6 points each, for which you must write out your reasoning.
- Aids allowed: A calculator with no communication ability (programmable calculators and graphing calculators are okay). A single hand-written aid-sheet prepared by the student, no larger than 8.5”x11”, written on both sides. A hard-copy English translation dictionary. A ruler.
- Please be sure to bring your T-Card, as invigilators will be collecting signatures and checking your photo-ID.

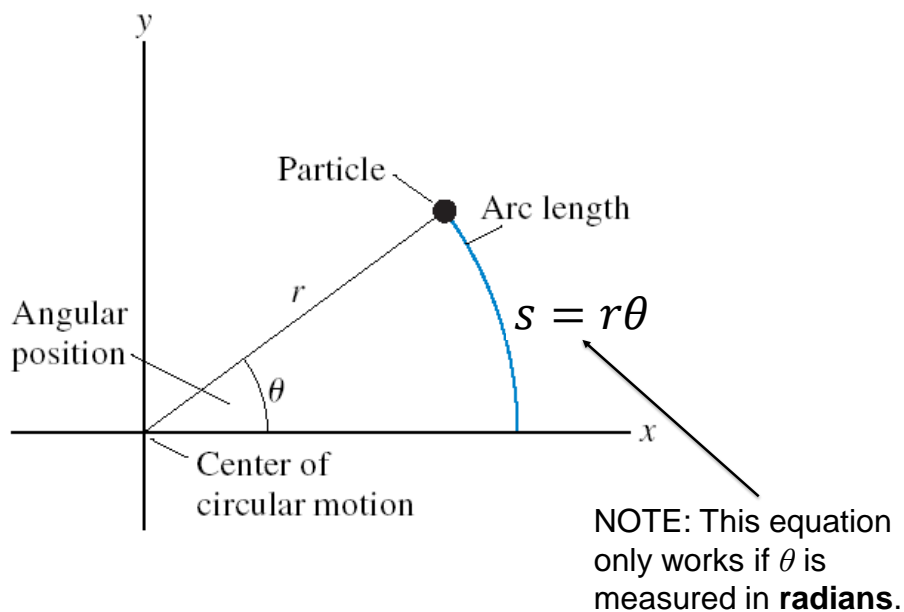
Midterm Test 2 - hints

- **Don't be late.** If you're very early, just wait outside the room.
- Spend the first 2 or 3 minutes skimming over the entire test from front to back before you begin. Look for the easy problems that you have confidence to solve first.
- Before you answer anything, read the question *very carefully*. The **most common mistake** is misreading the question!
- Manage your time; if you own a watch, bring it. 10 problems over 80 minutes means an average of about 8 minutes per problem.
- You CANNOT HAVE YOUR PHONE with you or in your pocket at a test or exam at U of T – you must store it in your backpack at the edge of the room, or in a special bag underneath your desk 

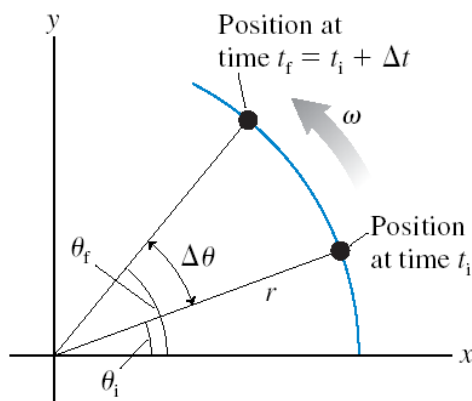
Midterm Test 1 – *more hints!*

- Some of the multiple choice are conceptual and can be answered in less than 2 minutes.. Maybe do these ones first?
- If you start a longer problem but can't finish it within about 10 minutes, leave it, make a mark on the edge of the paper beside it, and come back to it after you have solved all the easier problems.
- When you are in a hurry and your hand is not steady, you can make little mistakes; if there is time, do the calculation twice and obtain agreement.
- Bring a snack or drink. 
- Don't leave a test early! You might spend the first half getting 95% of the marks you're going to get, and the second half getting the other 5%, but it's still worth it.

Chapter 10 begins with.... Angular Position

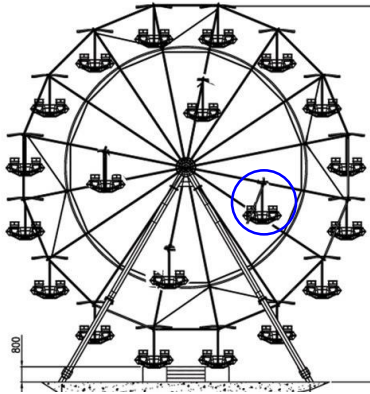


Angular Velocity



$$\omega \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad (\text{angular velocity})$$

$$\theta_f = \theta_i + \omega \Delta t \quad (\text{uniform circular motion})$$



Learning Catalytics Question :

A carnival has a Ferris wheel where some seats are located halfway between the center and the outside rim. Compared with the seats on the outside rim, the inner cars have

- A. Smaller angular speed and greater tangential speed
- B. Greater angular speed and smaller tangential speed
- C. The same angular speed and smaller tangential speed
- D. Smaller angular speed and the same tangential speed
- E. The same angular speed and the same tangential speed

Rigid Body Rotation

Angular velocity is

$$\omega = \frac{d\theta}{dt}$$

The units of ω are rad/s.

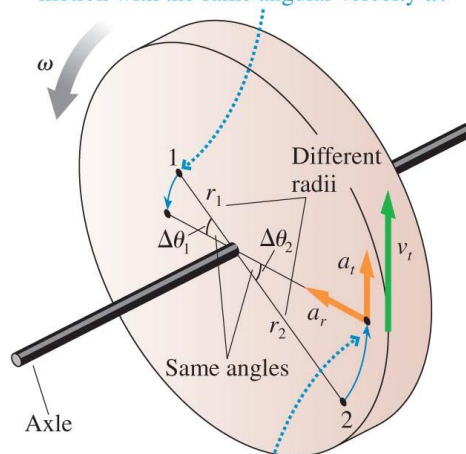
If the rotation is speeding up or slowing down, its angular acceleration is

$$\alpha = \frac{d\omega}{dt}$$

The units of α are rad/s².

All points on a rotating rigid body have the same ω and the same α .

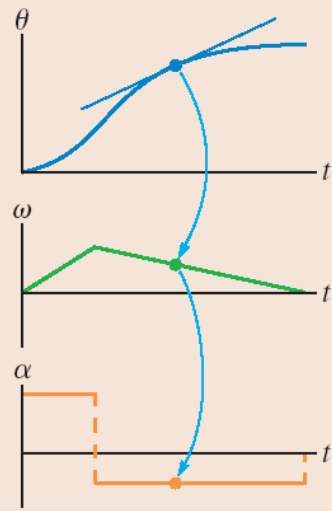
Every point on the wheel turns through the same angle and thus undergoes circular motion with the same angular velocity ω .



All points on the wheel have a tangential velocity and a radial (centripetal) acceleration. They also have a tangential acceleration if the wheel has angular acceleration.

Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.
- Arc length: $s = \theta r$
- Tangential velocity: $v_t = \omega r$



- Tangential acceleration: $a_t = \alpha r$

Rotational Kinematics

Linear

- s (or x or y) specifies position.

• Velocity:

$$v_x = \frac{d}{dt}(x) \quad v_y = \frac{d}{dt}(y)$$

• Acceleration:

$$a_x = \frac{d}{dt}(v_x) \quad a_y = \frac{d}{dt}(v_y)$$

Rotational Analogy

- θ is angular position. The S.I. Unit is radians, where 2π radians = 360° .

- Angular velocity:

$$\omega = \frac{d}{dt}(\theta)$$

- Angular acceleration:

$$\alpha = \frac{d}{dt}(\omega)$$

Radians are the Magical Unit!

- Radians appear and disappear as they please in your equations!!!
- They are the only unit that is allowed to do this!
- Example: $v_t = \omega r$



Rotational Kinematics

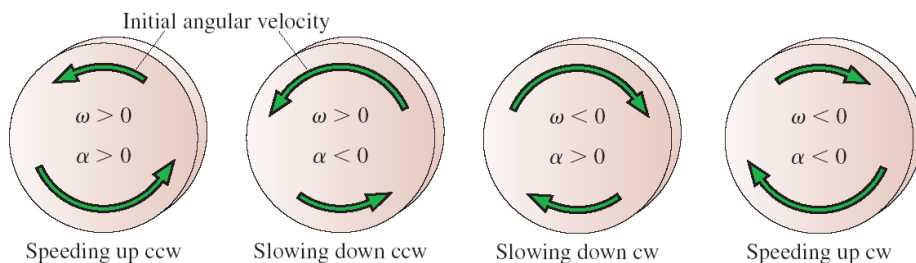
Rotational kinematics for constant angular acceleration

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

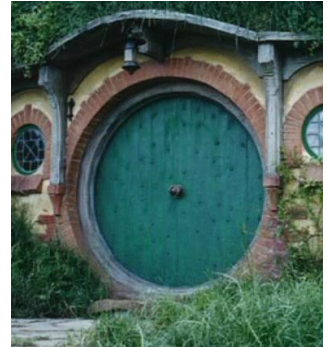
$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$$

The signs of angular velocity and angular acceleration.



Last day I asked at the end of class:

- Why is a door easier to open when the handle is far from the hinge, and more difficult to open when the handle is in the middle?



- ANSWER:
- Torque is the rotational analog of force:
- Force causes things to accelerate along a line.
- Torque causes things to have angular acceleration.
- Torque = Force \times Lever Arm
- Lever arm is the distance between where you apply the force and the hinge or pivot point.
- Putting the handle further from the hinge increases your lever arm, therefore it increases your torque for the same applied force.

Rotational Dynamics

Linear

- x
- v_x
- a_x

Rotational Analogy

- θ
- ω
- α

-
- Force: F_x
 - Mass: m

- Torque: τ
- Rotational Inertia: I

Newton's Second Law:

$$a_x = \frac{(F_{net})_x}{m}$$

$$\alpha = \frac{\tau_{net}}{I}$$

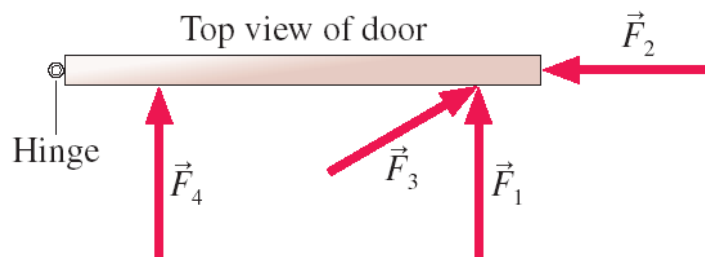
Example

The engine in a small airplane is specified to have a torque of 60.0 N m . This engine drives a propeller whose rotational inertia is 13.3 kg m^2 . On start-up, how long does it take the propeller to reach 200 rpm ?



Torque

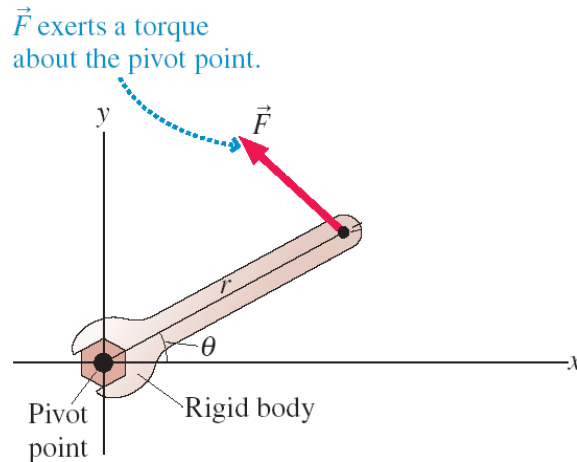
Consider the common experience of pushing open a door. Shown is a top view of a door hinged on the left. Four pushing forces are shown, all of equal strength. Which of these will be most effective at opening the door?



- A. F_1
- B. F_2
- C. F_3
- D. F_4

Torque

The effectiveness of a force at causing a rotation is called torque. Torque is the rotational equivalent of force. We say that a torque is exerted *about* the pivot point.

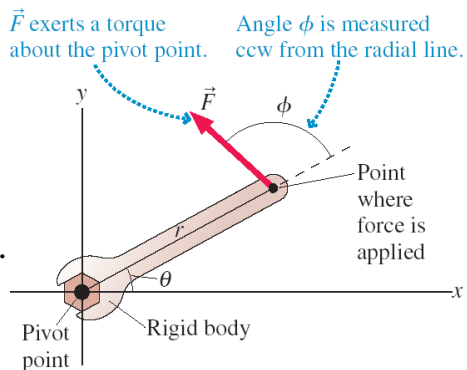


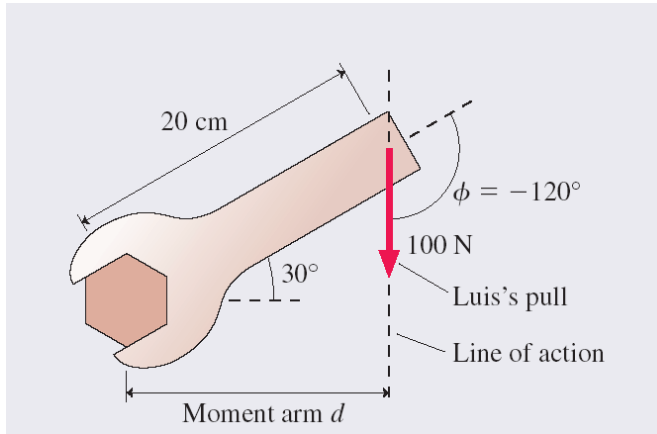
Torque

Mathematically, we define torque τ (Greek tau) as

$$\tau \equiv rF \sin \phi$$

SI units of torque are N m.
English units are foot-pounds.





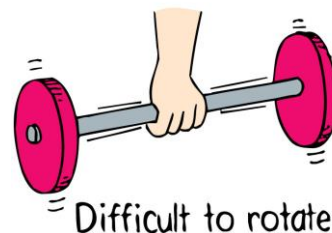
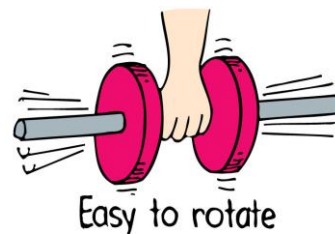
Example

Luis uses a 20 cm long wrench to turn a nut. The wrench handle is tilted 30° above the horizontal, and Luis pulls straight down on the end with a force of 100 N. How much torque does Luis exert on the nut?

Rotational Inertia

Depends upon:

- mass of object.
- distribution of mass around axis of rotation.
 - The greater the distance between an object's mass concentration and the axis, the greater the rotational inertia.



Rotational Inertia

Consider a body made of N particles, each of mass m_i , where $i = 1$ to N . Each particle is located a distance r_i from the axis of rotation. For this body made of a countable number of particles, the rotational inertia is:

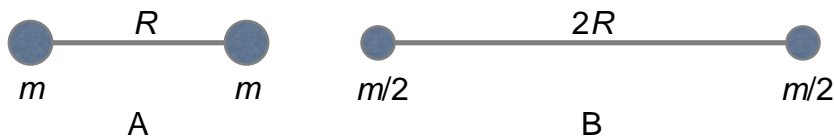
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2$$

The units of rotational inertia are kg m^2 . An object's rotational inertia depends on the axis of rotation.

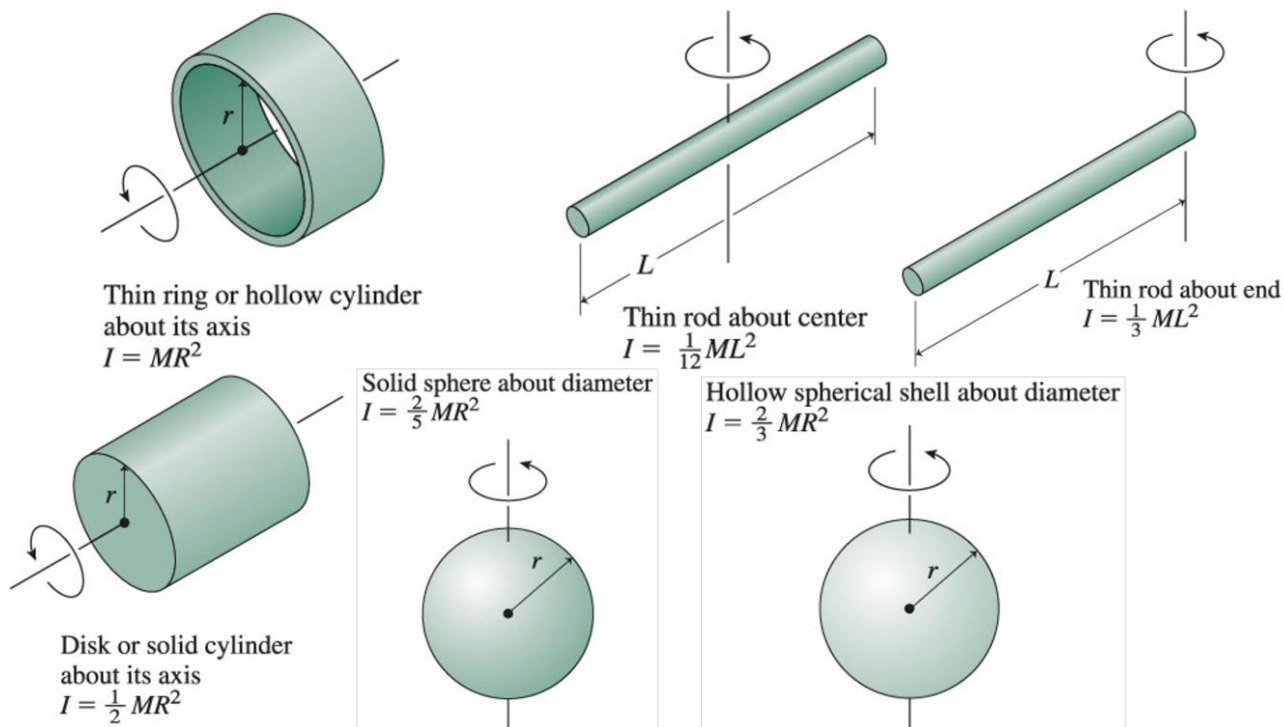
For a continuous distribution of mass (uncountably high number of particles), you must use an integral to compute rotational inertia:

$$I = \int r^2 dm$$

Which dumbbell has the larger rotational inertia about the midpoint of the rod? The connecting rod is massless.



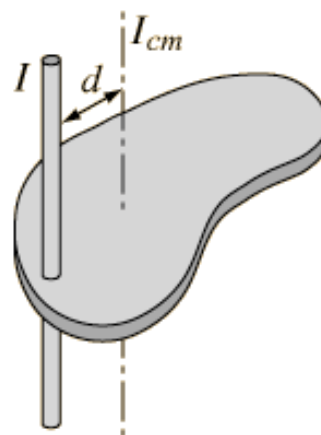
- A. Dumbbell A.
- B. Dumbbell B
- C. Their rotational inertias are the same.



The Parallel-Axis Theorem

- Suppose you know the rotational inertia of an object when it rotates about axis through center of mass: I_{cm}
- You can find the rotational inertia when it is rotating about another axis, which is a distance d away from the center of mass:

$$I = I_{cm} + Md^2$$



Before Class 17 on Wednesday



- Remember, there are Practicals this week!
- For Wednesday please finish reading Chapter 10, and/or watch the preclass 17 video.
- Yes there is a preclass quiz on Learning Catalytics due Wednesday at 8:00am sorry about that – you can do it any time tomorrow...
- Something to think about:
- A hoop and a disk are both released from rest at the top of an incline. They both roll without slipping. Which reaches the bottom first? Why?

