PHY131H1F - Class 16

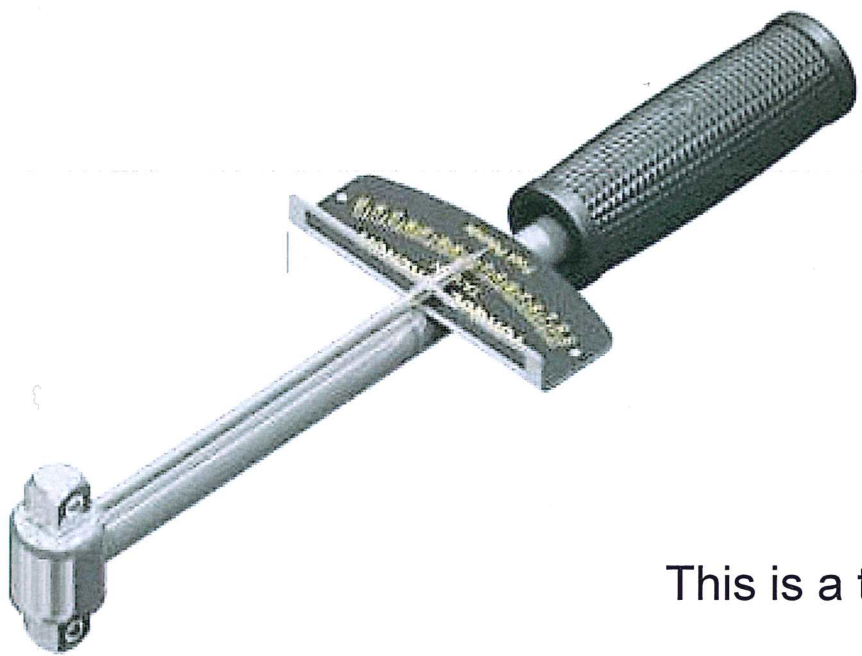
Today I will talk about stuff that will **not** be on tomorrow's midterm, but **will** be on the final exam on Dec. 13:

Welcome Back!! ☺

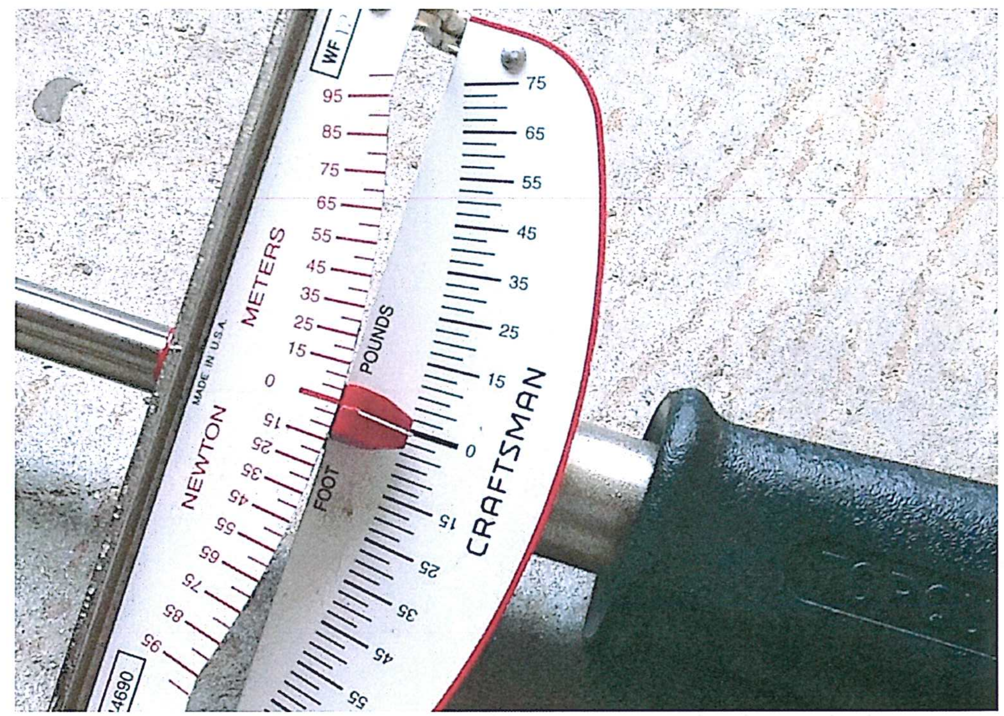
10.1 Angular Velocity and Acceleration

10.2 Torque

10.3 Rotational Inertia and the Analog of Newton's Law



This is a torque wrench.



Term Test 2 Info

- The second term test is tomorrow, Tue., Nov. 14, 6:10pm - 7:30pm (80 minutes). The room is based on the first letter of your family name:
  - A - K: EX100
  - L - Ti: EX200
  - Tj - Y: EX300
  - Z: EX310
- To further help you understand what room you need to go to, you can look up under "My Grades" to find the room under the column Test 2 Room Assignments.
- Alternate sitting students have received a separate email letting you know the room and time.

Term Test 2 Info

- Testable material is:
  - Chapters 6 - 9 of Wolfson
  - Classes 9 - 15, from Oct.11-Nov.1, inclusive
  - Practicals 4 - 7 and Homeworks 4 - 7, from Oct.10-Nov.13, inclusive, including the “PHY131 Uncertainty Propagation Summary” which is used in all Practical activities in which you do measurements.

Term Test 2 Info

- The format will be the same as for test 1. There will be 8 multiple choice questions, worth 2 points each, plus two free-form problems worth 6 points each, for which you must write out your reasoning.
- Aids allowed: A calculator with no communication ability (programmable calculators and graphing calculators are okay). A single hand-written aid-sheet prepared by the student, no larger than 8.5”x11”, written on both sides. A hard-copy English translation dictionary. A ruler.
- Please be sure to bring your T-Card, as invigilators will be collecting signatures and checking your photo-ID.

FAMILY NAME as on student card	Given Name(s) as on student card	Student Number	Practical Group Code (ie F3B)

PHY131H1F
Term Test 2 —version A
Tuesday, November 14, 2017
Duration: 80 minutes

Aids allowed: A calculator with no communication ability (programmable calculators and graphing calculators are okay). A single hand-written aid-sheet prepared by the student, no larger than 8.5"x11", written on both sides. A hard-copy English translation dictionary. A ruler.

- **Completely turn off** any communication device you may have and leave it with your belongings at the front of the room.
- **DO NOT separate the sheets of your question paper.** You can, however, *carefully* tear off the blank page at the end, as it does not have to be handed in.
- Before starting, please **PRINT IN BLOCK LETTERS your name, student number, and practical group code** at the top of this page **and** on the answer sheet.

Locate your test version letter (A or B) in the header at the top of the page and fill in the circle with the corresponding version code on your answer sheet in the "Form Code" box. Mark in your student number by shading the circles at the top-right of the sheet, starting with a 0 if the first digit is a 9. It is not required to bubble in your surname on the lower half of the sheet.

5

Scanned Area of the Answer Sheet.

1. **Use a dark-black, soft-lead pencil or a black pen.**
2. Indicate your answer to a multiple-choice question by thoroughly filling the appropriate circle on the answer sheet and also by recording your answer on the test paper.
3. If you wish to modify an answer, erase your pencil mark thoroughly.
4. **Do not write anything else on the answer sheet.** Use the blank sheets at the end or the back of the question sheets for rough work.

The first part of the test consists of **8** multiple-choice questions, worth 2 points each, or altogether 16 points. Each multiple-choice question has one best answer, and up to four answers that are not the best. Blank, incorrect or multiple answers are awarded zero points. Note that only the bubbled answer sheet is marked for the multiple choice part; anything you write in this booklet is ignored.

The second part of the test is a set of free-form questions, worth a total of 12 points. To be awarded maximum credit, you must write out fully worked solutions to all parts of the free-form questions in the space provided in this question booklet. In addition to showing your work, please put your answer(s) for each part in the boxes provided. You can use the back-side of the sheets and the blank pages at the end for your rough work which will not be graded or taken into account.

The total number of points available for the test is 28.

Possibly helpful information for this test:
$\pi = 3.14159$ is the ratio of the circumference to the diameter of a circle
$g = 9.80$ m/s^2 is the acceleration due to gravity near the Earth's surface.
Air resistance may be neglected in all questions, unless otherwise stated.

Gravitational energy of a system comprising a mass m located at a distance r from the centre of another mass M is: $U = -\frac{GMm}{r}$.

The equation of a circle of radius, R, centred at the origin is: $x^2 + y^2 = R^2$

When a ball of mass m_1,originally moving at speed v_{1i}, collides elastically head-on with a ball of mass m_2, originally at rest, the final velocities of the two balls, in the direction of the original velocity of m_1, are:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

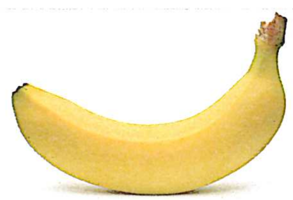
Some definite integrals, where a, b and c are constants:

- $\int_a^b x\sqrt{c^2 - x^2}\,dx = -\frac{1}{3}(c^2 - b^2)^{\frac{3}{2}} + \frac{1}{3}(c^2 - a^2)^{\frac{3}{2}}$
- $\int_a^b x^n dx = \frac{1}{n+1} b^{n+1} - \frac{1}{n+1} a^{n+1}$
- $\int_a^b \cos(x)\,dx = \sin(b) - \sin(a)$
- $\int_a^b \sin(x)\,dx = -\cos(b) + \cos(a)$

Midterm Test 2 - hints

- **Don't be late.** If you're very early, just wait outside the room.
- Spend the first 2 or 3 minutes skimming over the entire test from front to back before you begin. Look for the easy problems that you have confidence to solve first.
- Before you answer anything, read the question *very carefully*. The **most common mistake** is misreading the question!
- Manage your time; if you own a watch, bring it. 10 problems over 80 minutes means an average of about 8 minutes per problem.
- You CANNOT HAVE YOUR PHONE with you or in your pocket at a test or exam at U of T – you must store it in your backpack at the edge of the room, or in a special bag underneath your desk

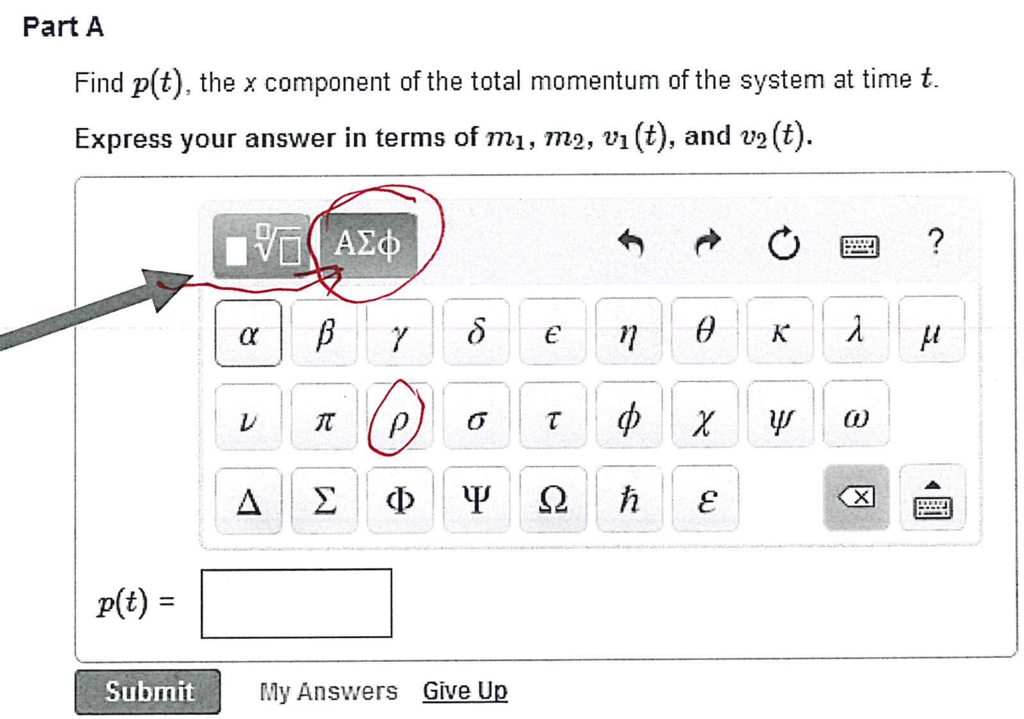
Midterm Test 1 – *more hints!*

- Some of the multiple choice are conceptual and can be answered in less than 2 minutes.. Maybe do these ones first?
- If you start a longer problem but can't finish it within about 10 minutes, leave it, make a mark on the edge of the paper beside it, and come back to it after you have solved all the easier problems.
- When you are in a hurry and your hand is not steady, you can make little mistakes; if there is time, do the calculation twice and obtain agreement.
- Bring a snack or drink. 
- Don't leave a test early! You might spend the first half getting 95% of the marks you're going to get, and the second half getting the other 5%, but it's still worth it.

Pre-class 16 results: Q.4

- In this week's mastering physics homework, one of the questions involved the Greek letter Rho which very much ressembles the lowercase letter p and after numerous tries and opening all the hints, did it occur to me that it was actually the Greek letter Rho.

Harlow answer: Be ***careful*** about this! $\rho \neq p$ and also $\omega \neq w$!!! You have to click on this symbol and use the drop-down symbol keyboard.

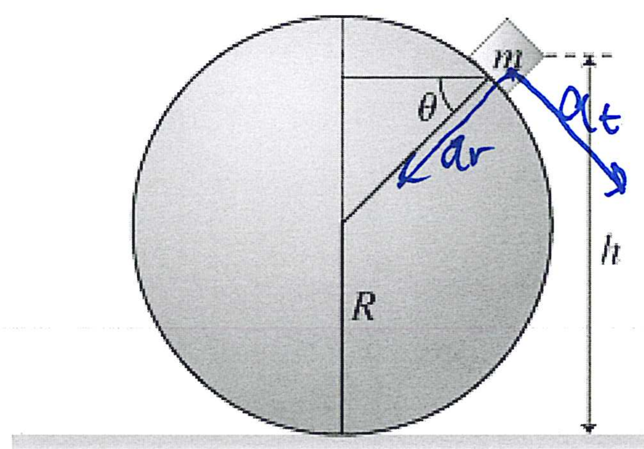


Pre-class 16 results: Q.4

- For the object moving in a circle, if the actual acceleration is not pointed towards the centre, how is the object still moving in a circle?

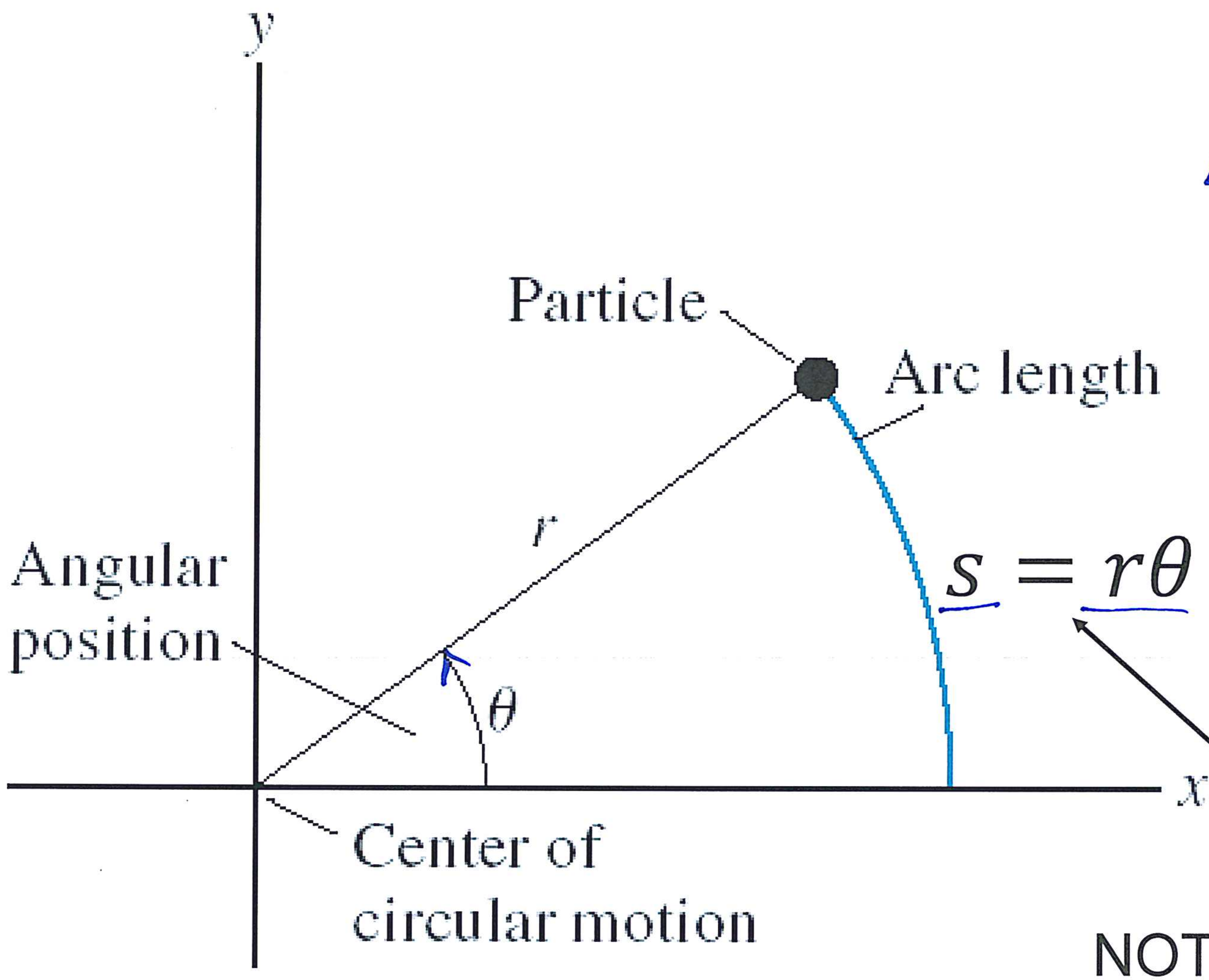
Harlow answer:
Remember: In non-uniform circular motion, the acceleration is not toward the centre! It has two components: a_r (toward the centre) and a_t (tangential).

$$a = \sqrt{{a_r}^2 + {a_t}^2}$$



$a_r = \frac{v^2}{R}$

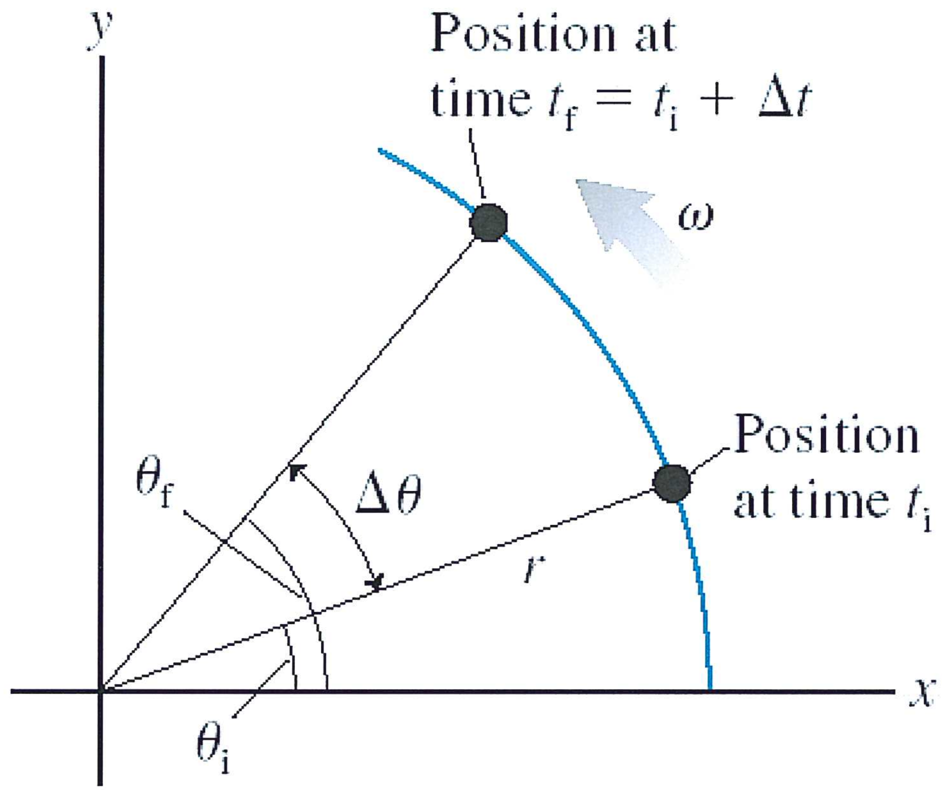
Chapter 10 begins with…. Angular Position



$$s = r\theta$$

NOTE: This equation only works if θ is measured in **radians**.

Angular Velocity

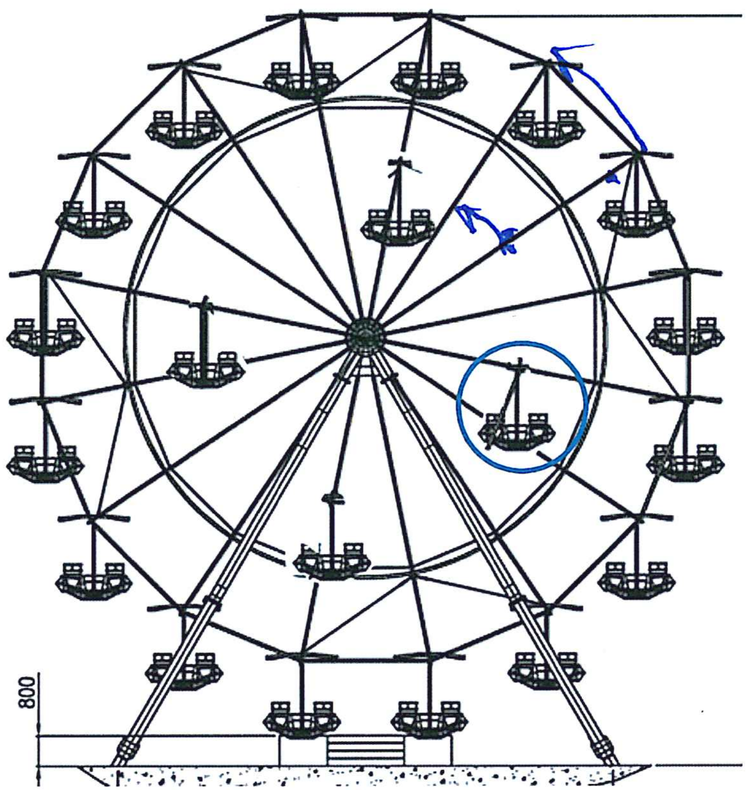


"omega"

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad \text{(angular velocity)}$$

$$\theta_f = \theta_i + \omega \Delta t \quad \text{(uniform circular motion)}$$

ω same for all points on rigid body rotation.



Learning Catalytics Question :

A carnival has a Ferris wheel where some seats are located halfway between the center and the outside rim. Compared with the seats on the outside rim, the inner cars have

$v_t = r\omega$

A. Smaller angular speed and greater tangential speed

B. Greater angular speed and smaller tangential speed

C. The same angular speed and smaller tangential speed

D. Smaller angular speed and the same tangential speed

E. The same angular speed and the same tangential speed

Rigid Body Rotation

Angular velocity is

$$\omega = \frac{d\theta}{dt}$$

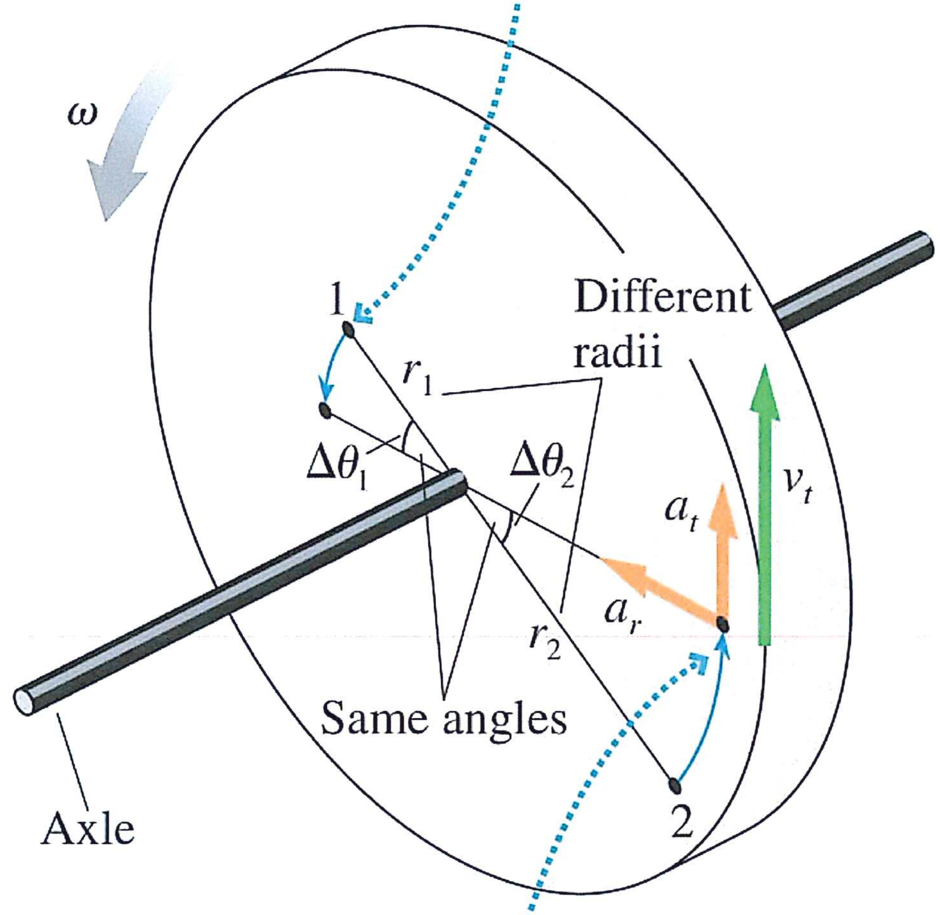
The units of ω are rad/s.
If the rotation is speeding up or slowing down, its angular acceleration is

$$\alpha = \frac{d\omega}{dt}$$

The units of α are rad/s^2.

All points on a rotating rigid body have the same ω and the same α.

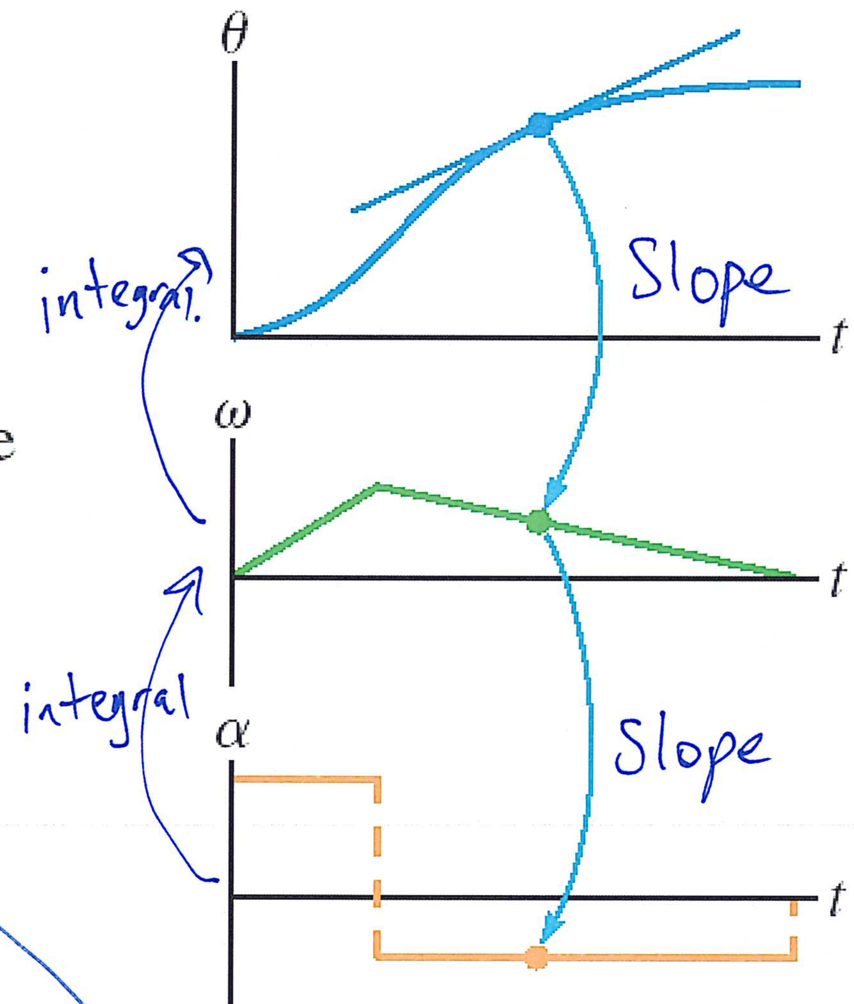
Every point on the wheel turns through the same angle and thus undergoes circular motion with the same angular velocity ω.



All points on the wheel have a tangential velocity and a radial (centripetal) acceleration. They also have a tangential acceleration if the wheel has angular acceleration.

Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.



- Arc length: $s = \theta r$
- Tangential velocity: $v_t = \omega r$
- Tangential acceleration: $a_t = \alpha r$

good eq's for aid sheet.

Rotational Kinematics

Linear	Rotational Analogy
• s (or x or y) specifies position.	• θ is angular position. The S.I. Unit is radians, where 2π radians = 360°.
• Velocity: $v_x = \frac{d}{dt}(x)$ $v_y = \frac{d}{dt}(y)$	• Angular velocity: $\omega = \frac{d}{dt}(\theta)$
• Acceleration: $a_x = \frac{d}{dt}(v_x)$ $a_y = \frac{d}{dt}(v_y)$	• Angular acceleration: $\alpha = \frac{d}{dt}(\omega)$

Radians are the Magical Unit!

- Radians appear and disappear as they please in your equations!!!
- They are the only unit that is allowed to do this!
- Example: $v_t = \omega r$

$v_t = \omega \cdot r$

Units:

$$\left[\frac{m}{s}\right] = \left[\frac{rad}{s}\right][m] = \left[\frac{rad \cdot m}{s}\right] = \left[\frac{m}{s}\right] \checkmark$$

Rotational Kinematics

Rotational kinematics for constant angular acceleration

$$\omega_f = \omega_i + \alpha \, \Delta t$$

$$\theta_f = \theta_i + \omega_i \, \Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \, \Delta\theta$$

Remember:

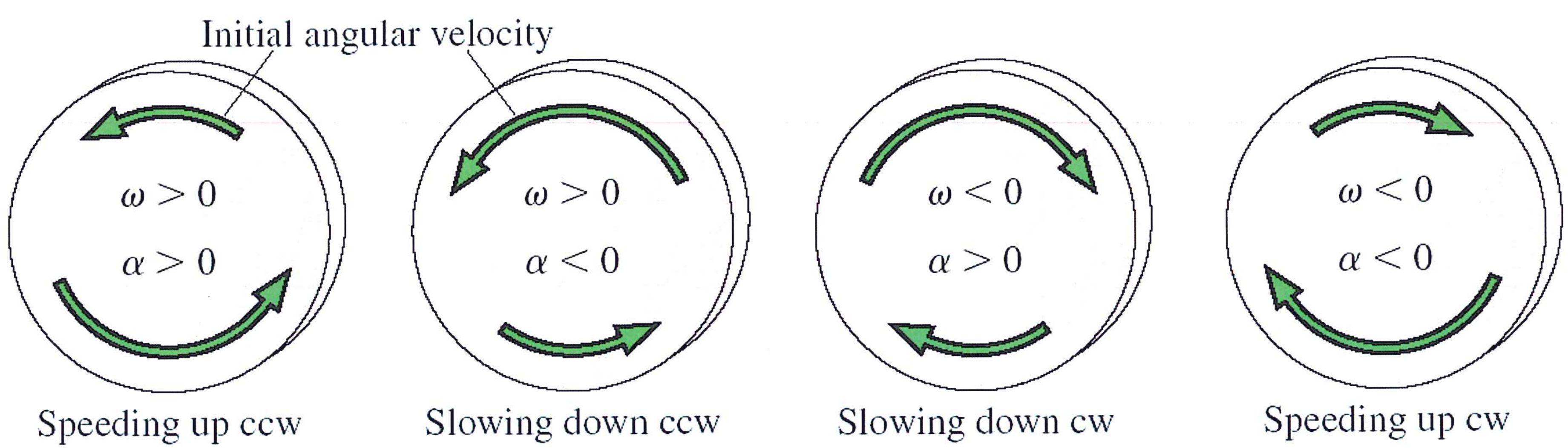
~~$x_f = x_i + a_x \Delta t$~~

$v_f = v_i + a_x \Delta t$

$x_f = x_i + v_i \Delta t + \frac{1}{2}a(\Delta t)^2$

$v_f^2 = v_i^2 + 2a_x(\Delta x)$

The signs of angular velocity and angular acceleration.



Pre-class 16: Q.2

539 responses, 55% correct

2. numerical

When a fan is turned off, its angular speed decreases uniformly from 10.0 rad/s to 6.30 rad/s in 5.00 s. What is the magnitude of the angular acceleration of the fan?

[Enter the number only of your answer, using the units rad/s^2.]

$\omega_i = 10 \frac{rad}{s}$

$\omega_f = 6.3 \frac{rad}{s}$

$\Delta t = 5\ s$.

find $|\alpha|$

Use:

$\omega_f = \omega_i + \alpha \Delta t$

solve for α:

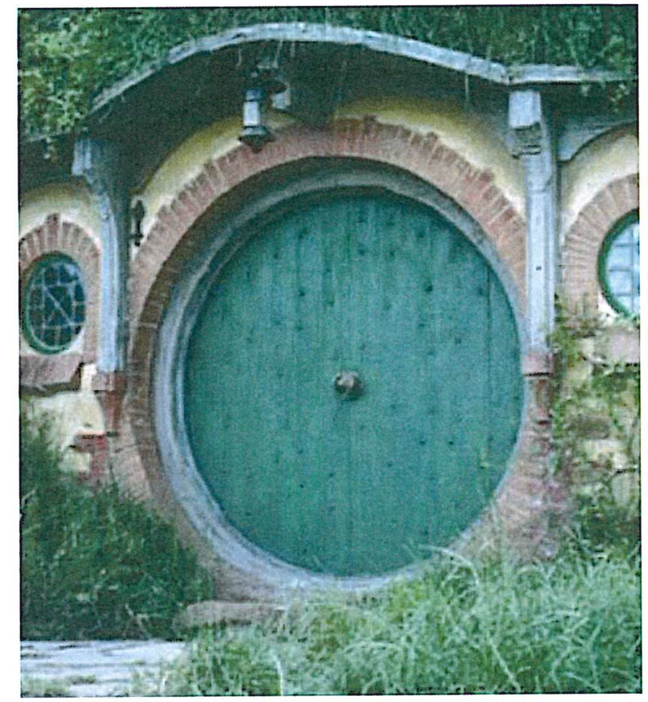
$\omega_f - \omega_i = \alpha \Delta t$

$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(10 - 6.3)}{5}$

$= -0.74 \frac{rad}{s^2}$

$|\alpha| = 0.74$

Last day I asked at the end of class:



- Why is a door easier to open when the handle is far from the hinge, and more difficult to open when the handle is in the middle?
  - ANSWER:
  - Torque is the rotational analog of force:
  - Force causes things to accelerate along a line.
  - Torque causes things to have angular acceleration.
  - Torque = Force × Lever Arm
  - Lever arm is the distance between where you apply the force and the hinge or pivot point.
  - Putting the handle further from the hinge increases your lever arm, therefore it increases your torque for the same applied force.

Rotational Dynamics

Linear	Rotational Analogy
x	θ
v_x	ω
a_x	α

- Force: F_x
- Mass: m

- Torque: τ ← "tau"
- Rotational Inertia: I

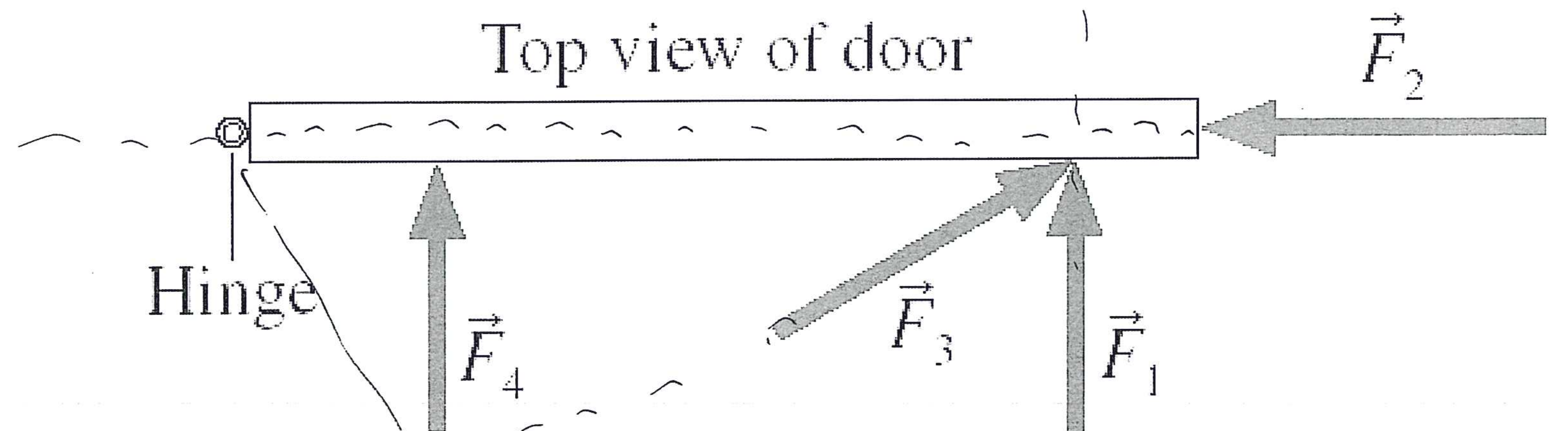
Newton's Second Law:

$$a_x = \frac{(F_{net})_x}{m}$$

$$\alpha = \frac{\tau_{net}}{I}$$

Torque

Consider the common experience of pushing open a door. Shown is a top view of a door hinged on the left. Four pushing forces are shown, all of equal strength. Which of these will be most effective at opening the door?



A. F_1

B. F_2

C. F_3

D. F_4

The ability of a force to cause a rotation depends on three factors:

1. the magnitude F of the force.
2. the distance r from the point of application to the pivot.
3. the angle at which the force is applied.

Pre-class 16 results: Q.1

1. multiple choice

If two forces of equal magnitude act on an object that is hinged at a pivot, the force acting farther from the pivot must produce the greater torque about the pivot.

A. true

B. false

C. unable to decide without knowing the shape of the object

angle could cause the farther force to have less torque.

550 responses, 43% correct

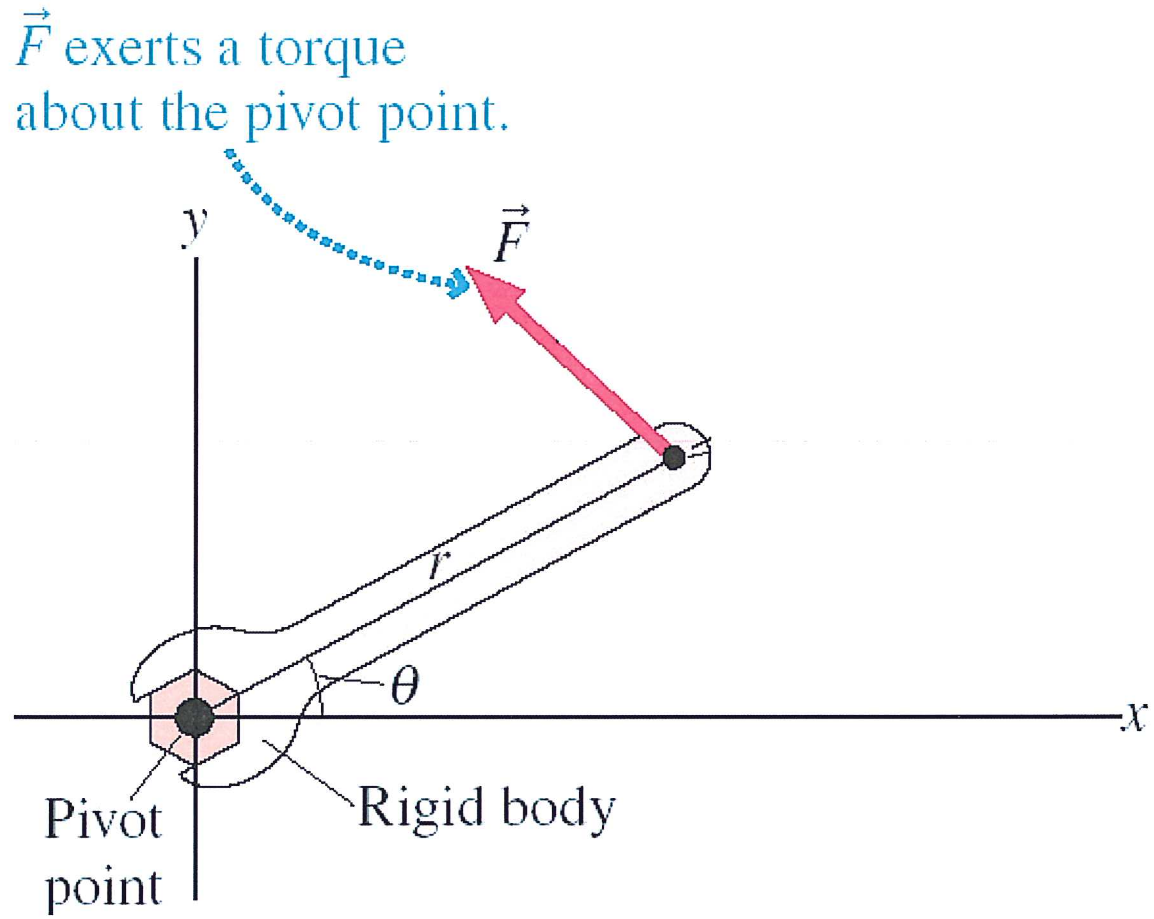
A. **54%**

B. **43%**

C. **3%**

Torque

The effectiveness of a force at causing a rotation is called torque. Torque is the rotational equivalent of force. We say that a torque is exerted *about* the pivot point.



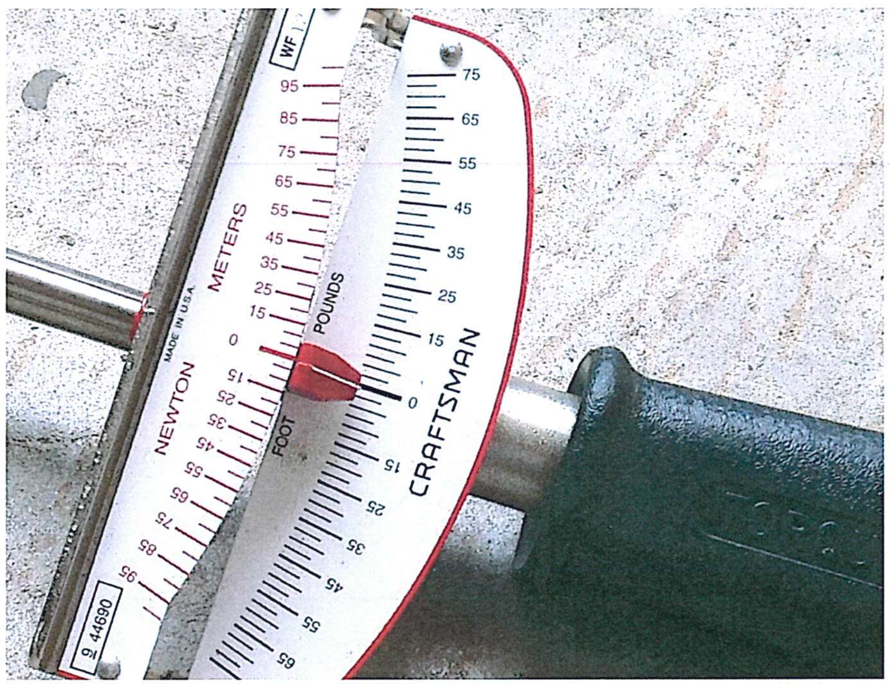
Torque

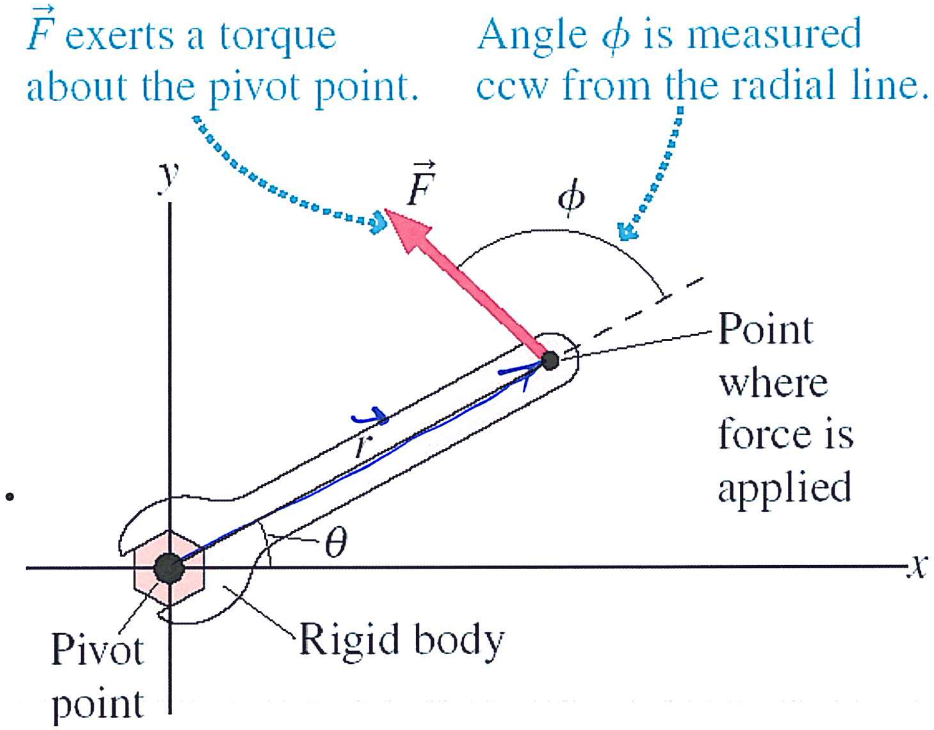
Mathematically, we define torque τ (Greek tau) as

$$\tau \equiv rF\sin\phi$$

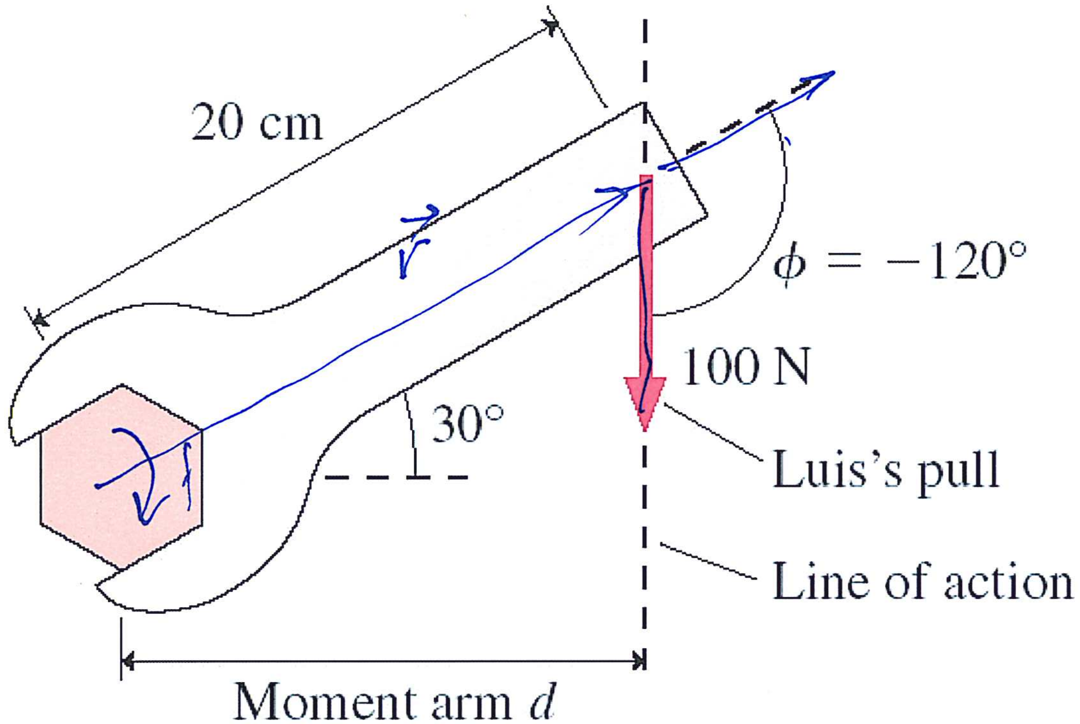
m·N · dimensionless

SI units of torque are N m.
English units are foot-pounds.





units of torque are [N · m]



Example

Luis uses a 20 cm long wrench to turn a nut. The wrench handle is tilted 30° above the horizontal, and Luis pulls straight down on the end with a force of 100 N. How much torque does Luis exert on the nut?

$$\tau = rF\sin\phi$$

$$= (0.2)(100)\sin(-120°)$$

$$\tau = -17.\ N\cdot m$$

clockwise

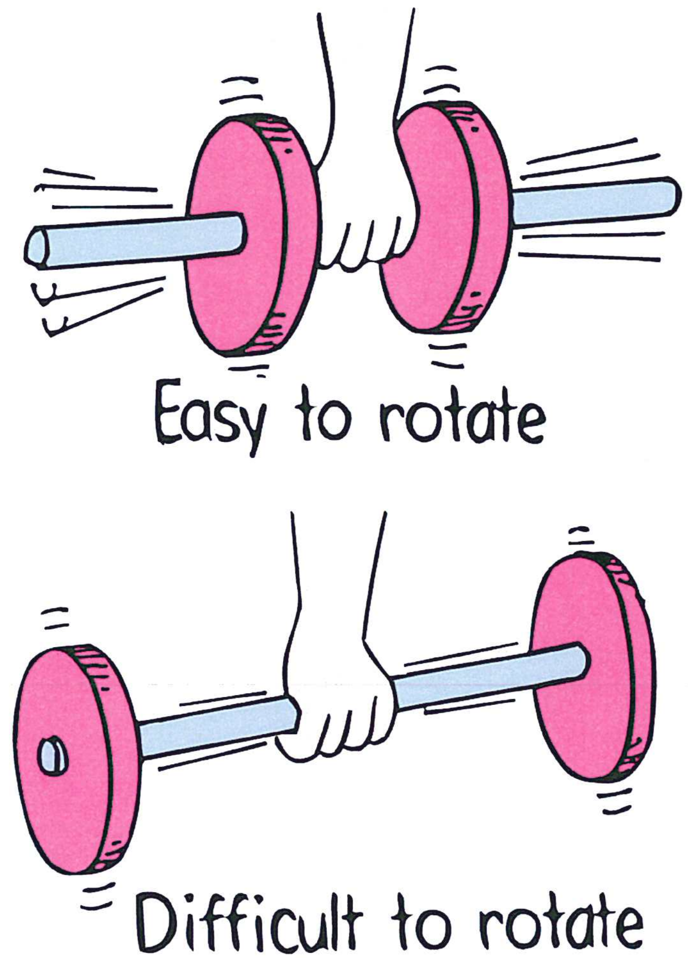
Magnitude of τ

$$= 17\ N\cdot m$$

Rotational Inertia

Depends upon:

- mass of object.
- distribution of mass around axis of rotation.
  - The greater the distance between an object's mass concentration and the axis, the greater the rotational inertia.



Rotational Inertia

Consider a body made of N particles, each of mass m_i, where $i = 1$ to N. Each particle is located a distance r_i from the axis of rotation. For this body made of a countable number of particles, the rotational inertia is:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2$$

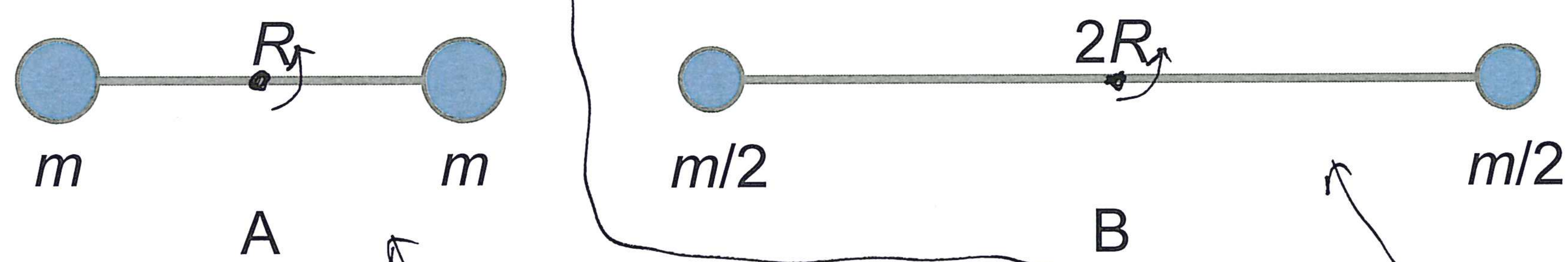
The units of rotational inertia are kg m^2. An object's rotational inertia depends on the axis of rotation.

For a continuous distribution of mass (uncountably high number of particles), you must use an integral to compute rotational inertia:

$$I = \int r^2 dm$$

2.9

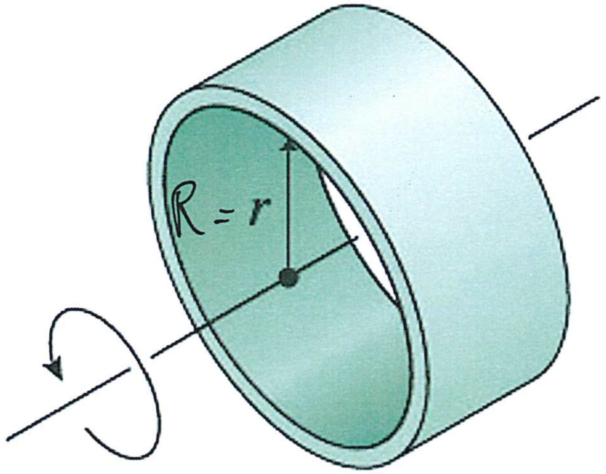
Which dumbbell has the larger rotational inertia about the midpoint of the rod? The connecting rod is massless.



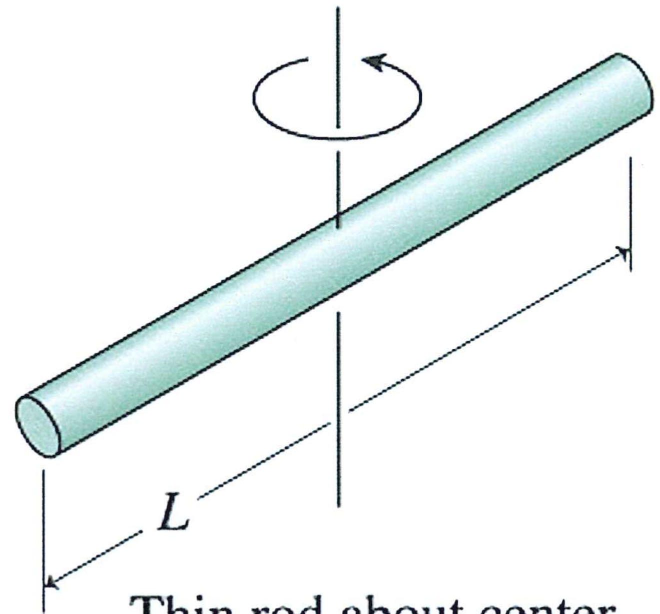
$I_A = m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2 = \frac{2mR^2}{4} = \frac{mR^2}{2}$

$I = \frac{m}{2}R^2 + \frac{m}{2}R^2 = mR^2$

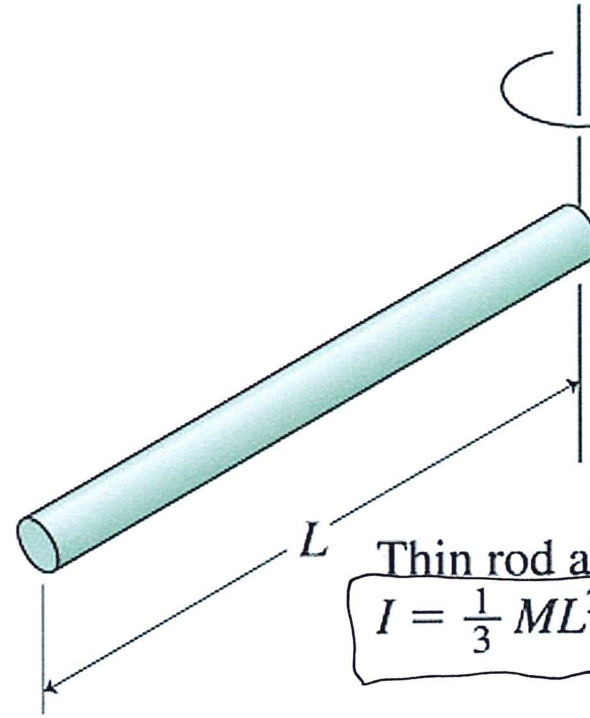
A. Dumbbell A.
(B.) Dumbbell B
C. Their rotational inertias are the same.



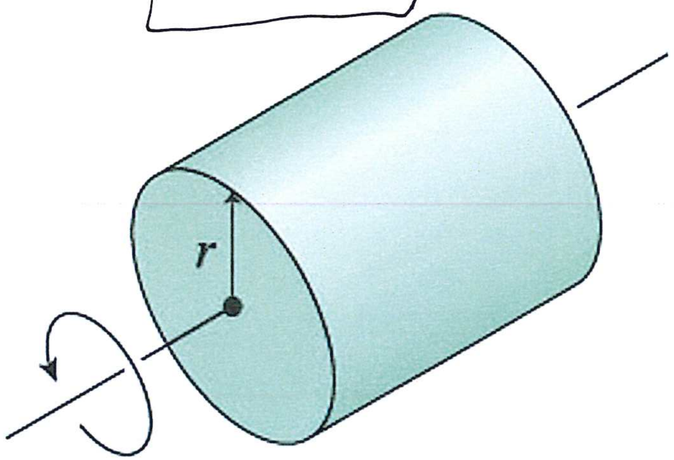
Thin ring or hollow cylinder about its axis
$I = MR^2$



Thin rod about center
$I = \frac{1}{12}ML^2$

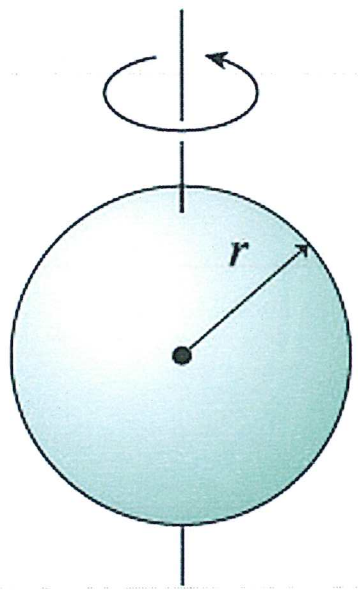


Thin rod about end
$I = \frac{1}{3}ML^2$

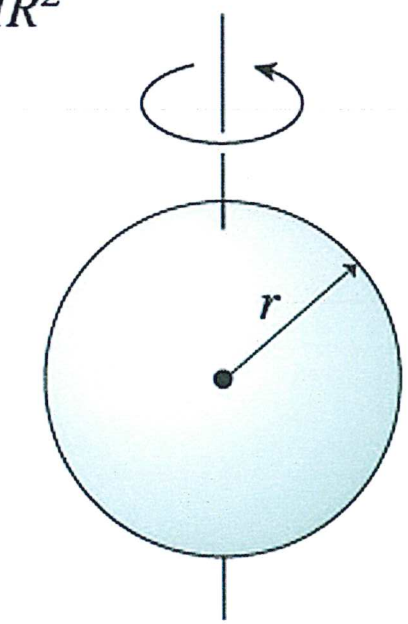


Disk or solid cylinder about its axis
$I = \frac{1}{2}MR^2$

Solid sphere about diameter
$I = \frac{2}{5}MR^2$



Hollow spherical shell about diameter
$I = \frac{2}{3}MR^2$



Pre-class 16 results: Q.4

· Will the rotational inertia of simple objects' table be given on the test??
Harlow answer: Yes! (on final exam).

· Why does a hallow sphere (which is lighter than a filled sphere) have a higher rotational inertia?
Harlow answer: It doesn't!!

Solid sphere about diameter
$I = \frac{2}{5} MR^2$

Hollow spherical shell about diameter
$I = \frac{2}{3} MR^2$

- If you take the same material and just cut out the middle, the rotational inertia goes way down!! Remember, rotational inertia is just the sum of all the mr^2 of every particle in the object.
- But if the mass is the **same**, then the hollow shell must be much more dense, and it has a higher rotational inertia, since this mass has a higher average value of r.

Before Class 17 on Wednesday

- Remember, there are Practicals this week!
- For Wednesday please finish reading Chapter 10, and/or watch the preclass 17 video.

- Yes there is a preclass quiz on Learning Catalytics due Wednesday at 8:00am sorry about that – you can do it any time tomorrow…
- Something to think about:
- A hoop and a disk are both released from rest at the top of an incline. They both roll without slipping. Which reaches the bottom first? Why?