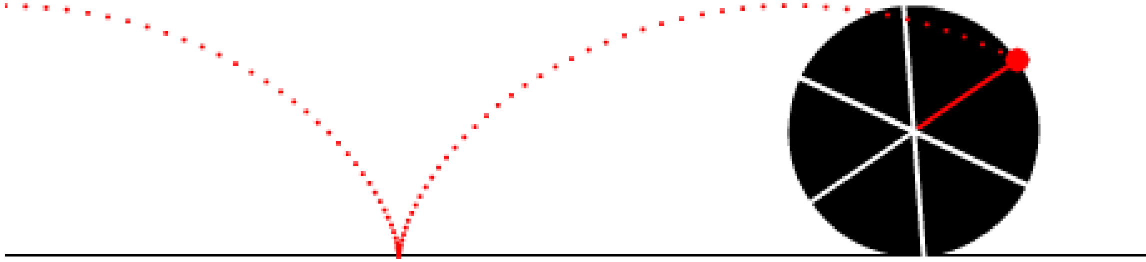
PHY131H1F - Class 17

Today:

Finishing Chapter 10:

Rolling Without Slipping

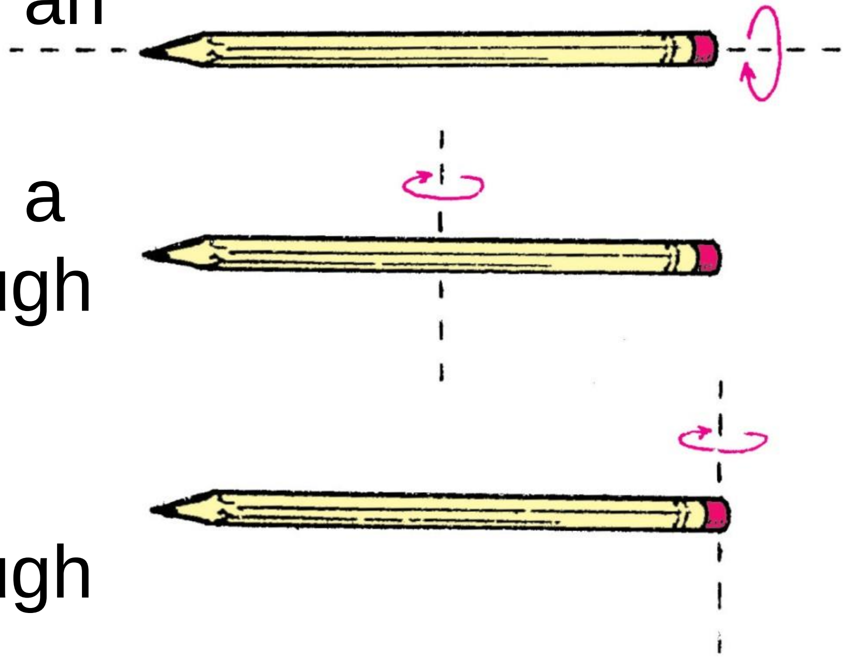
Rotational Energy



Learning Catalytics Question 1

Which pencil has the largest rotational inertia?

A. The pencil rotated around an axis passing through it.
B. The pencil rotated around a vertical axis passing through centre.
C. The pencil rotated around vertical axis passing through the end.



10.5: Rolling without slipping

- No matter what the speed, four points on this car are always ***at rest!***
- Which points? The bottoms of the four tires!

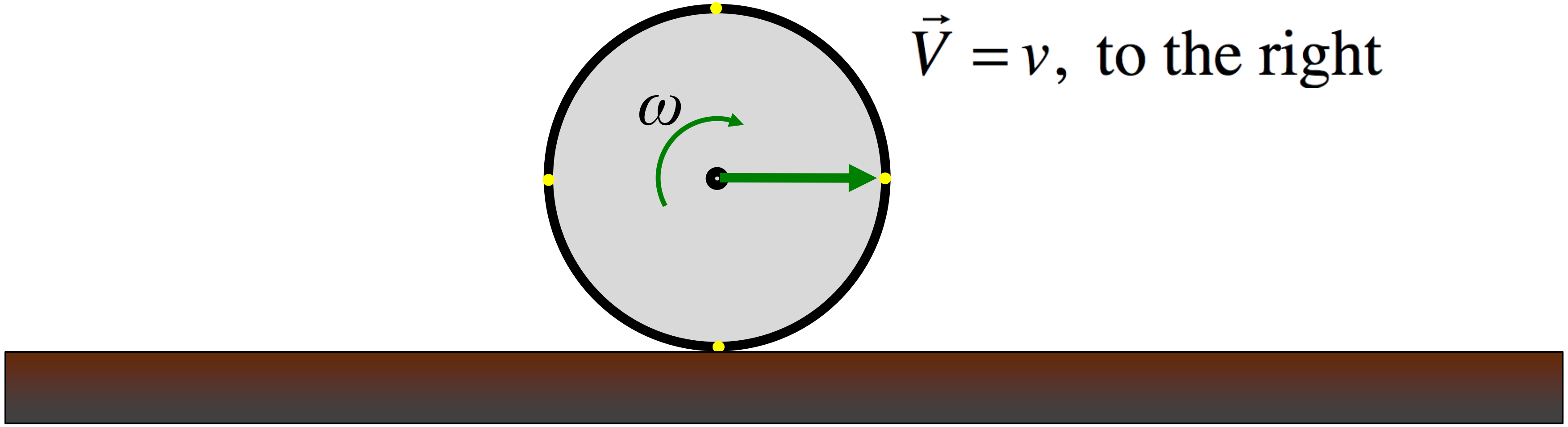
- A wheel rolls much like the treads of a tank.
- The bottom of the wheel is ***at rest*** relative to the ground as it rolls.

Rolling without slipping

S frame: the ground

S' frame: the axle

$\vec{V} = v,$ to the right



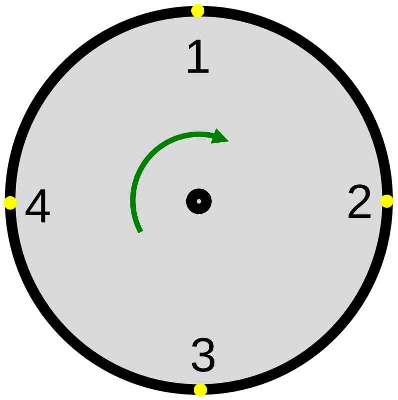
The wheel rotates with angular speed ω.

The tangential speed of a point on the rim is $v = \omega r$, relative to the axle.

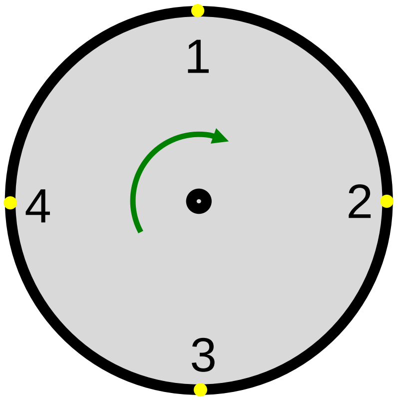
In "rolling without slipping", the axle moves at speed v. This is the *S'* frame.

Rolling without slipping

S' frame: the axle is at rest



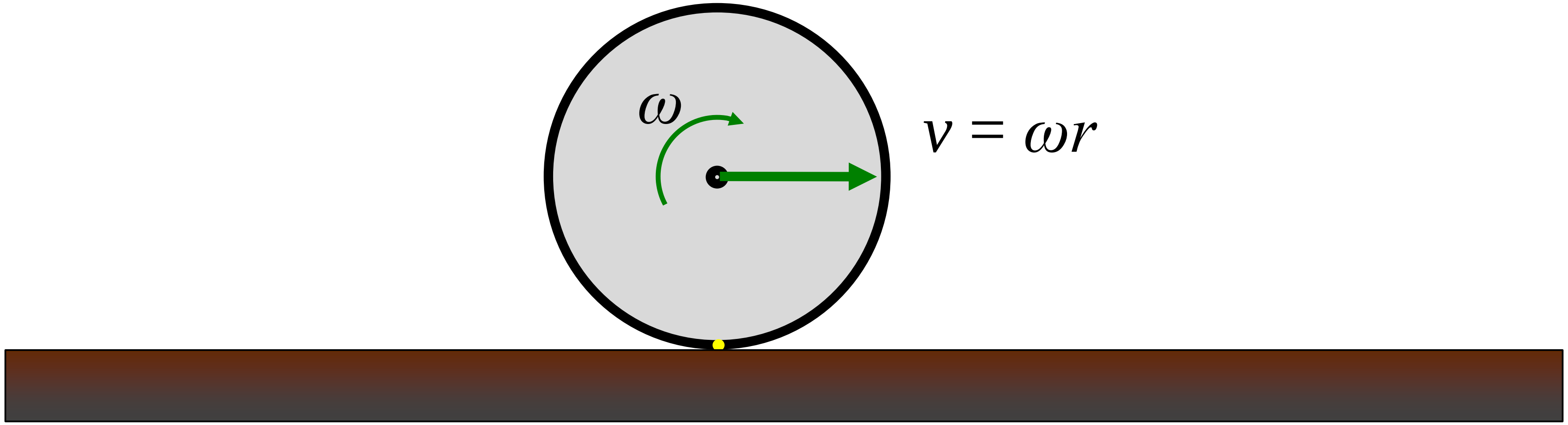
S frame: the ground is at rest



$\vec{V}$ is the velocity of the axle relative to the ground.

$$\vec{v} = \vec{v}\,' + \vec{V}$$

Rolling without slipping



The wheel rotates with angular speed ω.

The axle moves with linear speed $v = \omega r$.,
where r is the radius of the wheel.

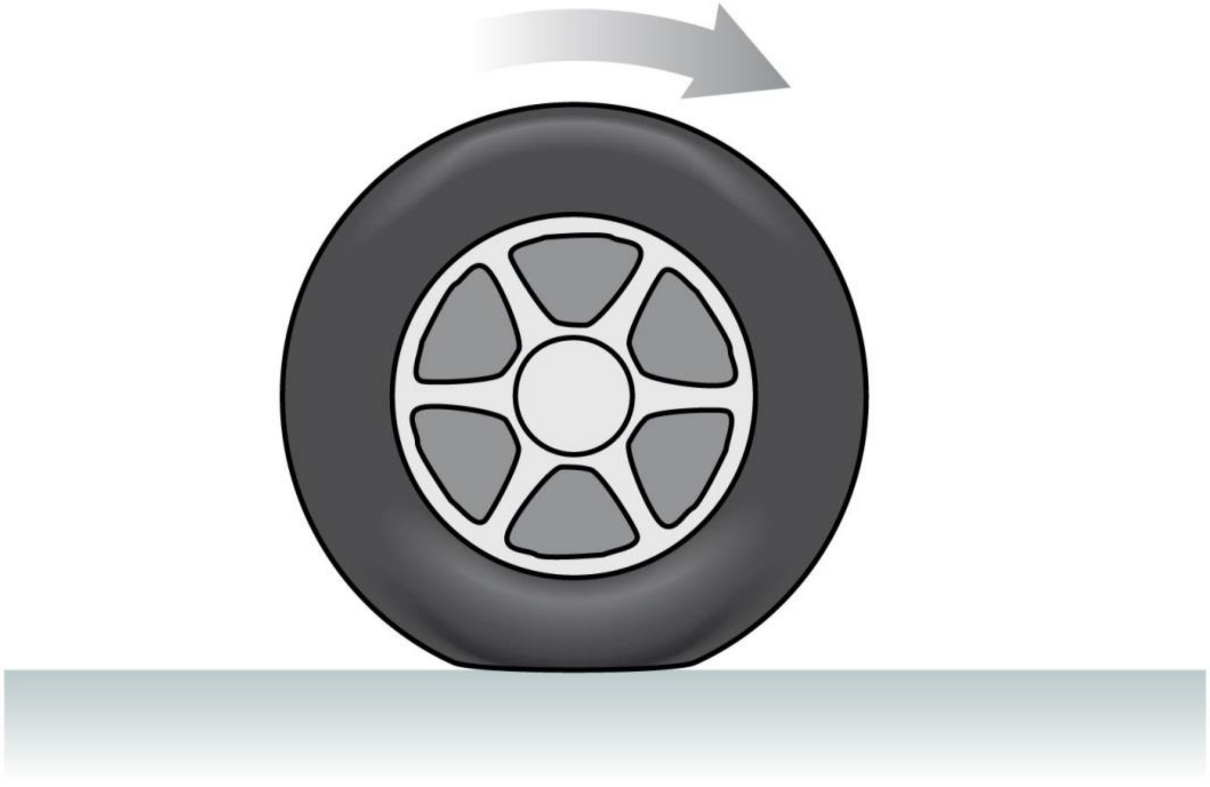
- If your car is accelerating or decelerating or turning, it is *static friction* of the road on the wheels that provides the net force which accelerates the car

Rolling without slipping Doc Cam Demo

Rolling Without Slipping

- Under normal driving conditions, the portion of the rolling wheel that contacts the surface is *stationary*, not sliding
- In this case the speed of the centre of the wheel is:

$$v = \frac{C}{T}$$



where C = circumference [m] and T = Period [s]

Example

- The circumference of the tires on your car is 0.9 m.
- The onboard computer in your car measures that your tires rotate 10 times per second.
- What is the speed as displayed on your speedometer?

The “Rolling Without Slipping” Constraints

When a round object rolls without slipping, the distance the axis, or centre of mass, travels is equal to the change in angular position times the radius of the object.

$$s = \theta R$$

The speed of the centre of mass is

$$v = \omega R$$

The acceleration of the centre of mass is

$$a = \alpha R$$

Rotational Kinetic Energy

A rotating rigid body has kinetic energy because all atoms in the object are in motion. The kinetic energy due to rotation is called **rotational kinetic energy**.

$$K_{\text{rot}} = \frac{1}{2} I \omega^2$$

Example: A 0.50 kg basketball rolls along the ground at 1.0 m/s. What is its *total* kinetic energy (linear plus rotational)? [Note that the rotational inertia of a hollow sphere is $I = 2/3\ MR^2$.]

Linear / Rotational Analogy

Linear	**Rotational Analogy**
$\vec{s},\ \vec{v},\ \vec{a}$	$\theta,\ \omega,\ \alpha$
Force: $\vec{F}$	Torque: τ
Mass: m	Rotational Inertia: I

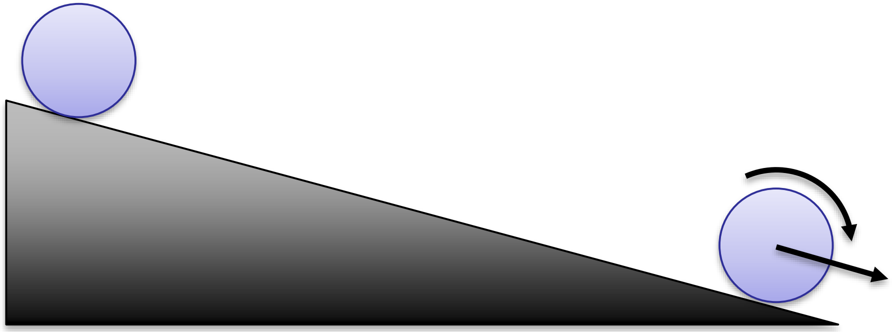
- Newton's 2nd law: $$\vec{a} = \frac{\vec{F}_{net}}{m} \qquad \alpha = \frac{\tau_{net}}{I}$$
- Kinetic energy: $$K_{\mathrm{cm}} = \frac{1}{2}mv^2 \qquad K_{\mathrm{rot}} = \frac{1}{2}I\omega^2$$

Summary of some Different Types of Energy:

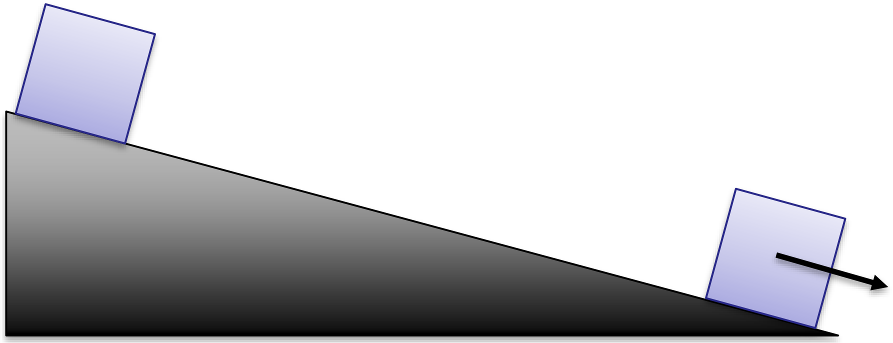
- Kinetic Energy due to linear motion of centre of mass: $K = ½\ mv^2$
- Gravitational Potential Energy $U_{\mathrm{g}} = mgh$
- Spring Potential Energy: $U_{\mathrm{s}} = ½\ kx^2$
- Rotational Kinetic Energy: $K_{\mathrm{rot}} = ½\ I\omega^2$
- Thermal Energy: ΔE_{th} (often created by kinetic friction)
  - An object can possess any or all of the above.
  - One way of transferring energy to or out of an object is work:
- Work done by a constant force: $W = Fr\cos\theta$

- Learning Catalytics Q2
- A hoop and a disk are both released from rest at the top of an incline. They both roll without slipping. Which reaches the bottom first? Shall we vote?
- A: hoop wins
- B: disk wins
- C: tie

Don’t forget: Nature is not a democracy!



- Learning Catalytics Q3. A solid disk is released from rest and rolls without slipping down an incline. A box is released from rest and slides down a frictionless incline of the same angle. Which reaches the bottom first?
- A: disk wins
- B: box wins
- C: tie



Compare and Contrast Soup Cans

- About same mass
- About same radius and shape
- Thick paste, so when this can is rolling, the contents rotate along with the can as one solid object, like a solid cylinder
- Low viscosity liquid, so the can itself rolls while the liquid may just "slide" along.

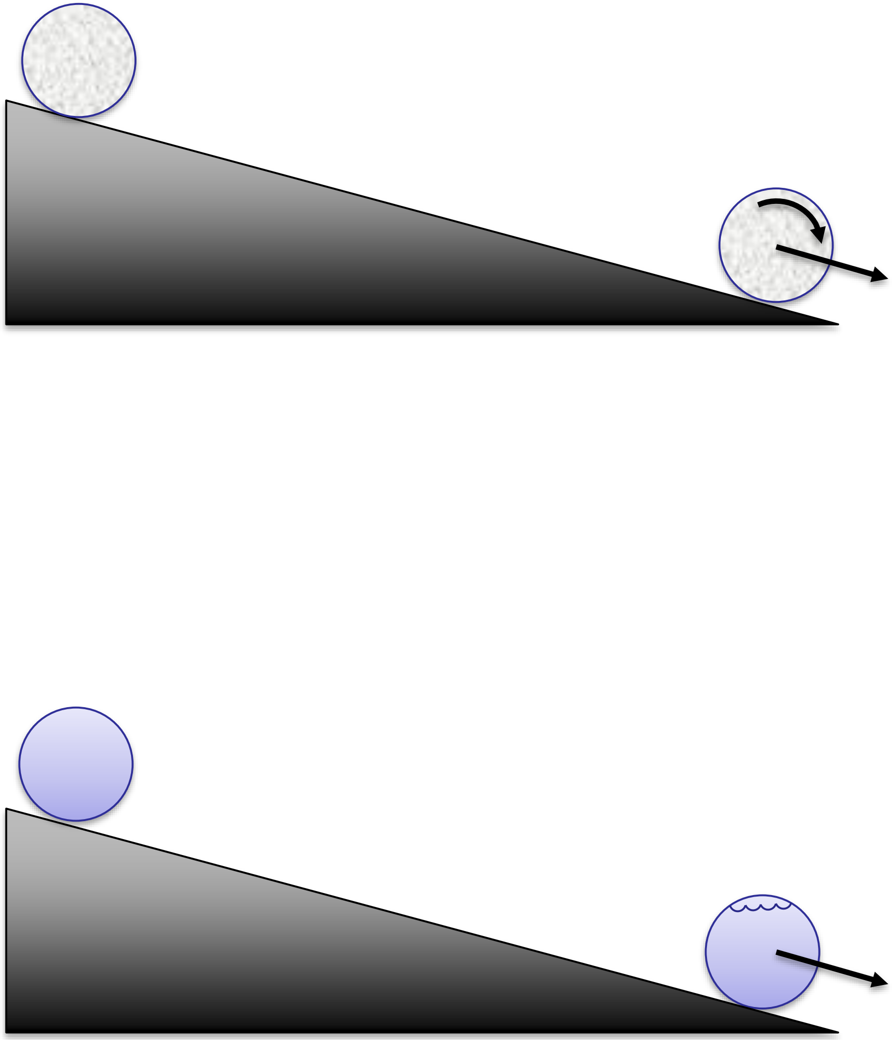
Learning Catalytics Q4

- Two soup cans begin at the top of an incline, are released from rest, and allowed to roll without slipping down to the bottom. Which will win?

Predict:

A. Cream of Mushroom will win
B. Chicken Broth will win
C. Both will reach the bottom at about the same time.

- Cream of Mushroom soup must rotate, like a solid disk.
- Chicken broth can slide down **without rotating** while the can rotates around it.



1. What is the acceleration of a slipping object down a ramp inclined at angle θ? [assume no friction]

2. What is the acceleration of a **solid disk** rolling down a ramp inclined at angle θ? [assume rolling without slipping]

3. What is the acceleration of a **hoop** rolling down a ramp inclined at angle θ? [assume rolling without slipping]

Before Class 18 on Monday

- The reading is all of Chapter 11 on Rotational Vectors and Angular Momentum.
- Please read the chapter and/or watch the Preclass 18 Video.
- Something to think about: When a figure-skater starts a spin and brings in her arms, she spins even faster. Why?

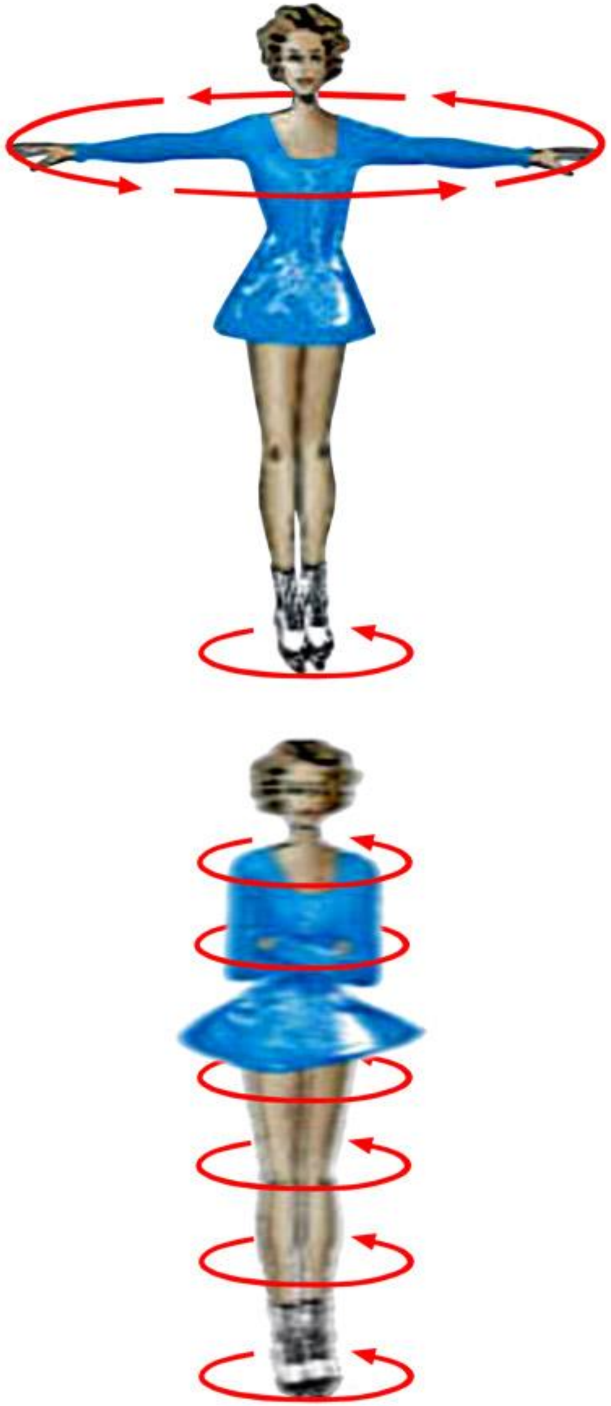


Image from http://www.daviddarling.info/encyclopedia/A/angular_momentum.html