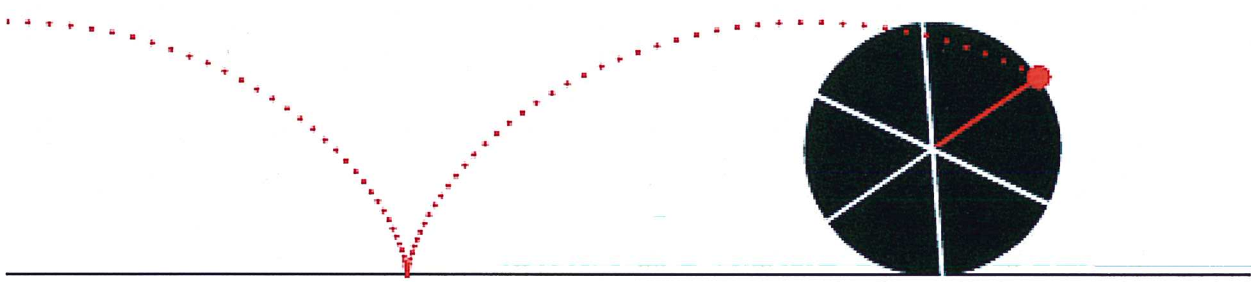
PHY131H1F - Class 17

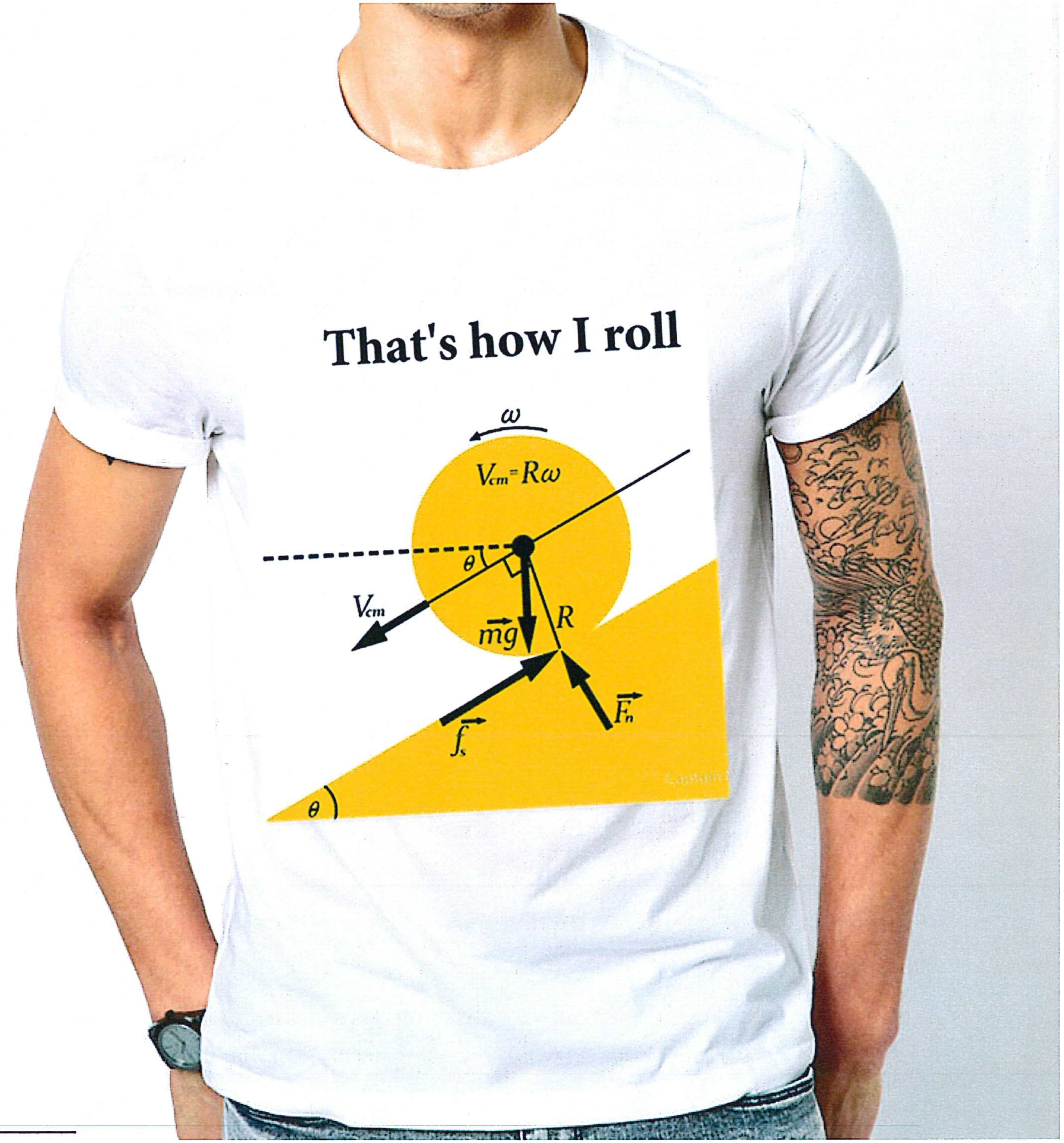
Today:

Finishing Chapter 10:

Rolling Without Slipping

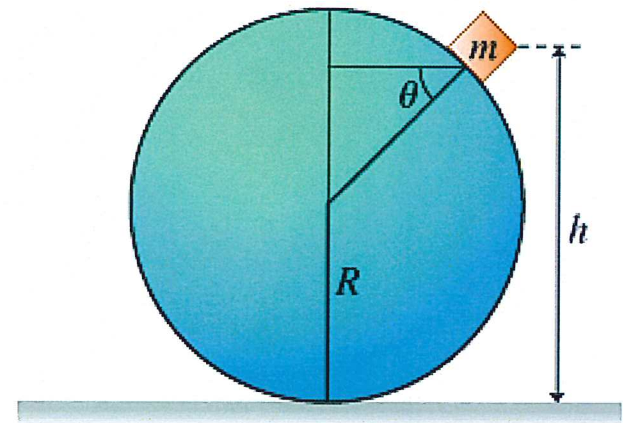
Rotational Energy





MasteringPhysics Homework 6 Question

A large globe, with a radius of about 5 m, was built in Italy between 1982 and 1987. Imagine that such a globe has a radius R and a frictionless surface. A small block of mass m starts to slide with a tiny (negligible) speed from the very top of the globe and slides along the surface of the globe. The block leaves the surface of the globe when it reaches a height h_{crit} above the ground. The geometry of the situation is shown in the figure for an arbitrary height h. (Figure 1)



Find h_{crit}, the height from the ground at which the block leaves the surface of the globe.

- This was due Oct. 30.
- The correct answer was $h_{\mathrm{crit}} = \frac{5R}{3}$

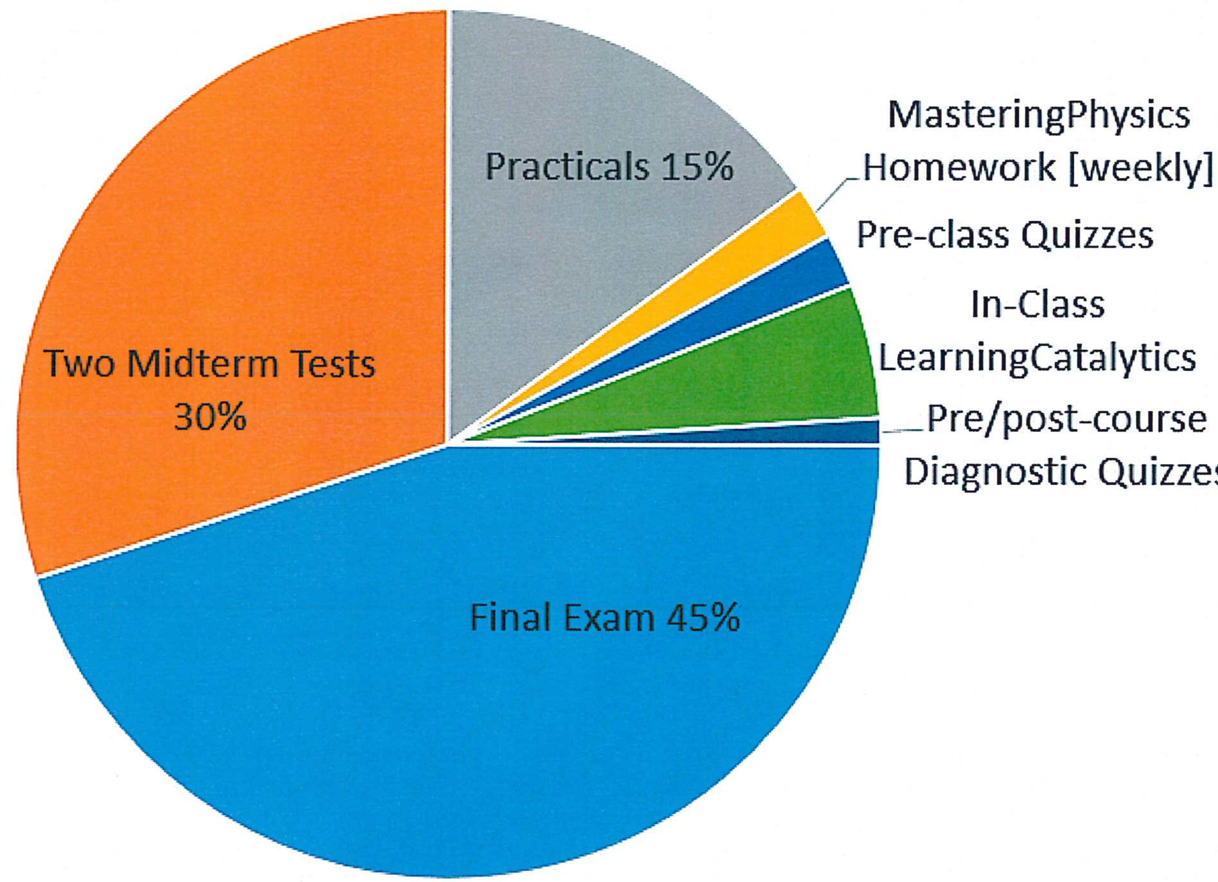
- 656 students (84% of the class) answered the question on time, and 96.3% of these students got the correct answer.

Preclass 17 comments

- Quote from a wise student:

  “That test is the reason why homework should be not only finished, but *understood*.”

PHY131 Mark



Pre-class 17 results: Q.1

1. multiple choice

At any angular speed, a certain uniform solid sphere of diameter *D* has half as much rotational kinetic energy as a certain uniform thin-walled hollow sphere of the same diameter when both are spinning about an axis through their centers. If the mass of the solid sphere is *M*, the mass of the hollow sphere is

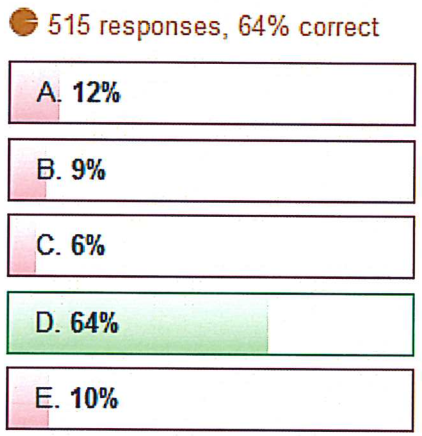
A. 3/5 M.

B. 5/3 M.

C. 5/6 M.

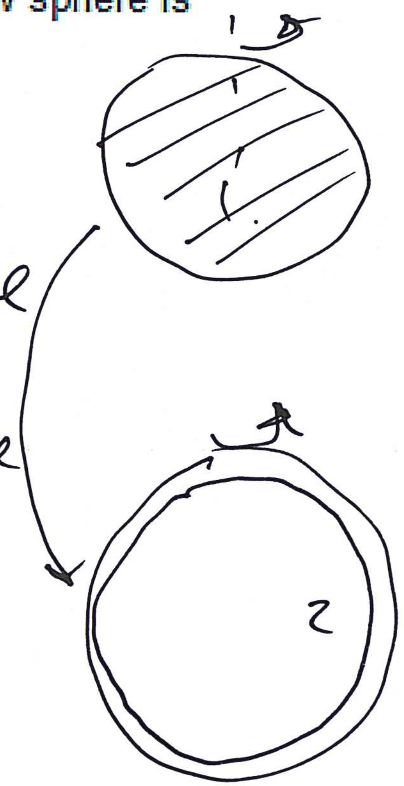
D. 6/5 M.

E. 2 M.



same ω

same R



solid: $I_1 = \frac{2}{5} MR^2$

$K_{rot} = \frac{1}{2} I \omega^2$

hollow: $I_2 = \frac{2}{3} M_2 R^2$

↑ find M_2

$K_1 = \frac{K_2}{2}$

$\cancel{\frac{1}{2}} I_1 \cancel{\omega^2} = \frac{\cancel{\frac{1}{2}} I_2 \cancel{\omega^2}}{2}$

$\frac{2}{5} M \cancel{R^2} = \frac{1}{2}\left(\frac{2}{3} M_2 \cancel{R^2}\right)$

$\frac{2M}{5} = \frac{1}{3} M_2$

$M_2 = \frac{6}{5} M$

10.5: Rolling without slipping

- No matter what the speed, four points on this car are always ***at rest!***
- Which points? The bottoms of the four tires!

- A wheel rolls much like the treads of a tank.
- The bottom of the wheel is ***at rest*** relative to the ground as it rolls.

Learning Catalytics Question (part 1 of 2)

You are sitting in your car, and you step on the gas pedal. The car accelerates forward.

Since the car is accelerating, there must be an external net force on the car, $\vec{F}_{1\ on\ 2}$.

Here, object 2 is the car.

What is object 1?

A. The Earth

B. ~~The engine~~

C. The air

D. The ~~gas pedal~~

E. An ~~invisible string attached~~ to the front of the car

Learning Catalytics Question (part 2 of 2)

You are sitting in your car, and you step on the gas pedal. The car accelerates forward.

Since the car is accelerating, there must be an external net force on the car, $\vec{F}_{1\ on\ 2}$.

Here, object 2 is the car, and object 1 is the Earth.

What kind of force is $\vec{F}_{1\ on\ 2}$?

A. air resistance

B. applied force

C. electric

D. kinetic friction

E. magnetic

F. normal

G. spring force

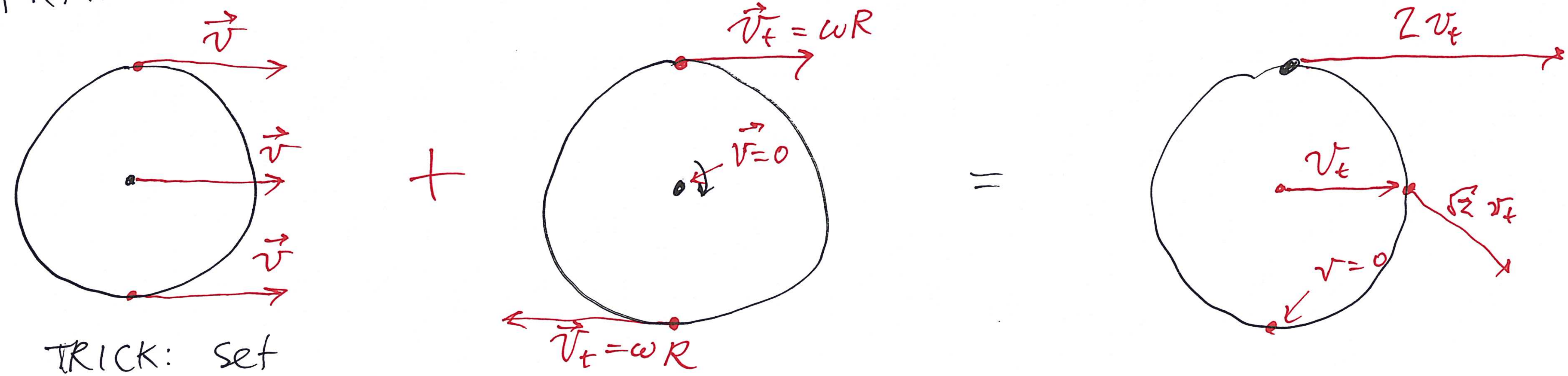
H. static friction

I. tension

J. thrust

Rolling without slipping Doc Cam Demo

TRANSLATION: ROTATION:



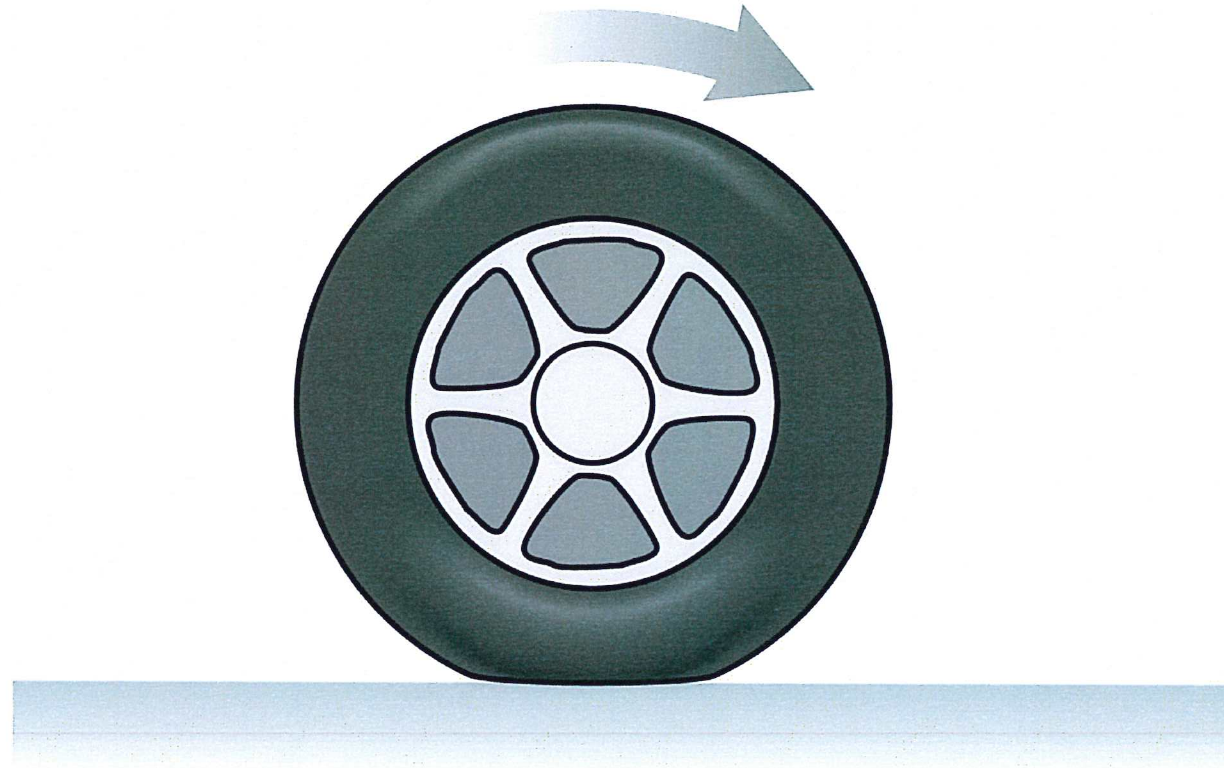
TRICK: set
$v = v_t = \omega R$

$$v_{rolling} = \frac{\text{Circ}}{\text{period}} = \frac{2\pi R}{T} = \underbrace{\left(\frac{2\pi}{T}\right)}_{\omega} R = \omega R$$

Rolling Without Slipping

- Under normal driving conditions, the portion of the rolling wheel that contacts the surface is *stationary*, not sliding
- In this case the speed of the centre of the wheel is:

$$v = \frac{C}{T}$$



where C = circumference [m] and T = Period [s]

Example

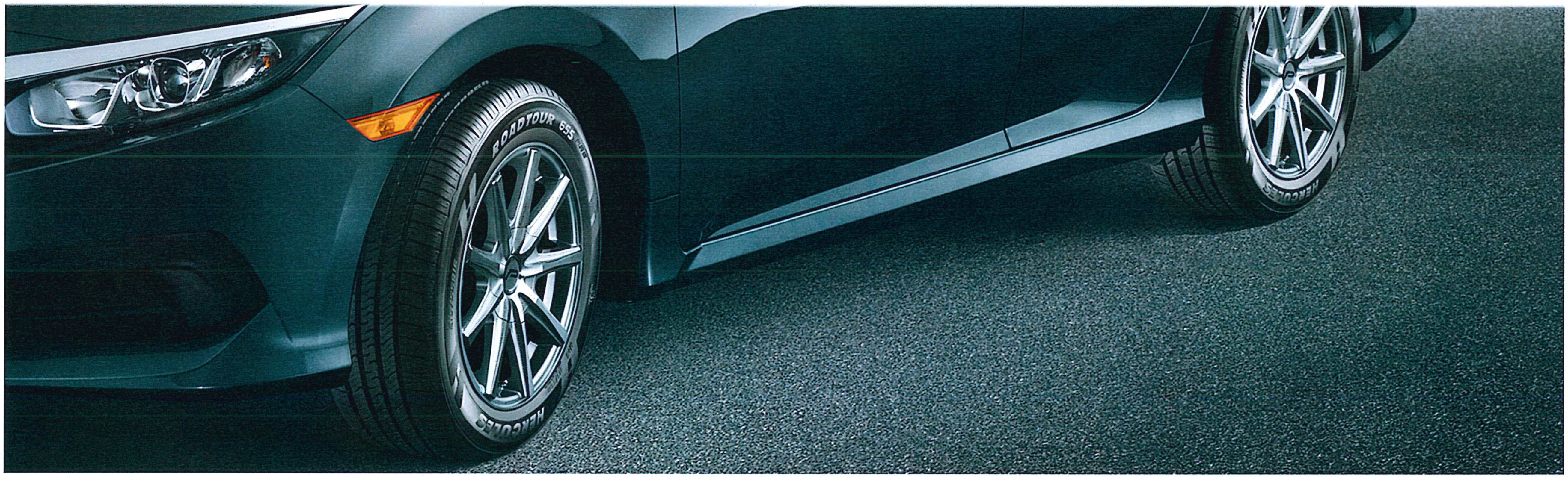
- The circumference of the tires on your car is 0.9 m.
- The onboard computer in your car measures that your tires rotate 10 times per second.
- What is the speed as displayed on your speedometer?

0.9m 0.9m

$$f = \frac{1}{T} = 10 \frac{rot}{s}$$

$$T = 0.1 s$$

$$v = \frac{c}{T} = \frac{0.9 m}{0.1 s} = \boxed{9 \ m/s}$$



The "Rolling Without Slipping" Constraints

When a round object rolls without slipping, the distance the axis, or centre of mass, travels is equal to the change in angular position times the radius of the object.

$$\boxed{s = \theta R}$$

"constraints of rolling without slipping"

The speed of the centre of mass is

$$\boxed{v = \omega R}$$

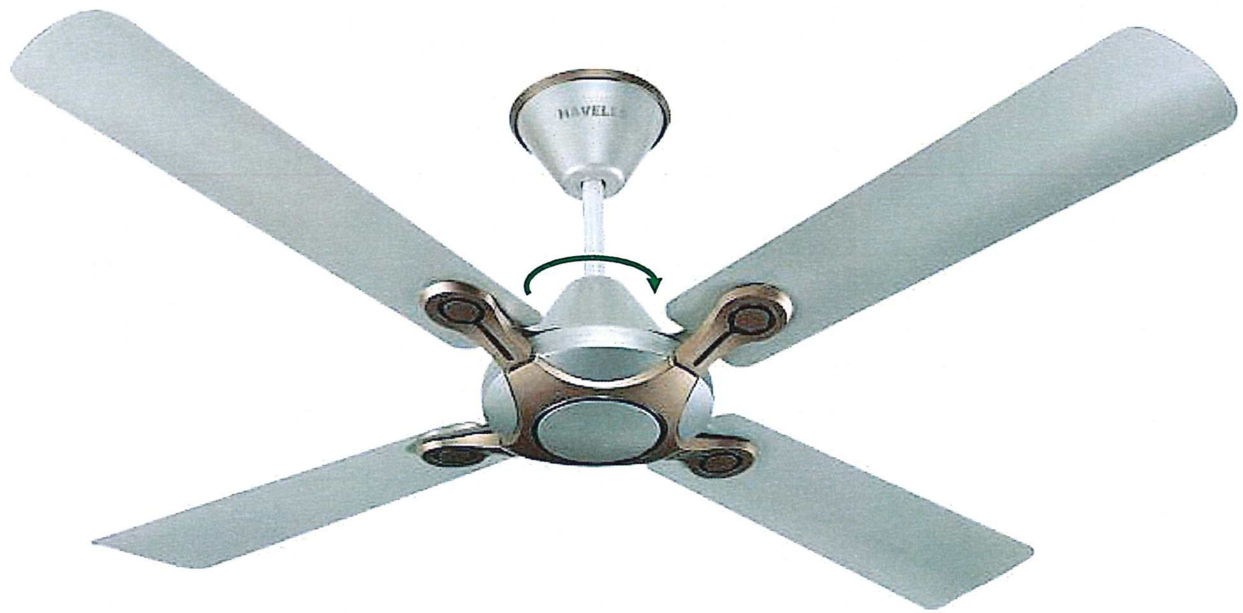
The acceleration of the centre of mass is

$$\boxed{a = \alpha R}$$

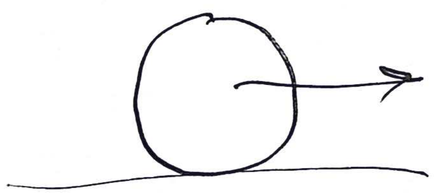
Rotational Kinetic Energy

A rotating rigid body has kinetic energy because all atoms in the object are in motion. The kinetic energy due to rotation is called **rotational kinetic energy**.

$$K_{\text{rot}} = \frac{1}{2} I \omega^2$$



Example: A 0.50 kg basketball rolls along the ground at 1.0 m/s. What is its *total* kinetic energy (linear plus rotational)? [Note that the rotational inertia of a hollow sphere is $I = 2/3\ MR^2$.]



$v = 1.0\ m/s$

$M = 0.5\ kg.$

Find: $K = K_{trans.} + K_{rot}$

Use: $\boxed{I = \frac{2}{3} MR^2}$ $\quad R = ?\ \ldots$

Use Rolling Without slipping constraint

$v = \omega R \rightarrow \boxed{\omega = \frac{v}{R}}$

$K = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$

$$K = \frac{1}{2} M v^2 + \frac{1}{2}\left(\frac{2}{3} M R^2\right)\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2} M v^2 + \frac{2}{6} \frac{M \cancel{R^2} v^2}{\cancel{R^2}}$$

$$= \frac{1}{2} M v^2 + \frac{2}{6} M v^2$$

$$= \left(\frac{3}{6} + \frac{2}{6}\right) M v^2$$

$$K = \frac{5}{6} M v^2 = \frac{5}{6}(0.5)(1)^2$$

$$\boxed{K = 0.42\ J}$$

Pre-class 17 results: Q.3

3. multiple choice

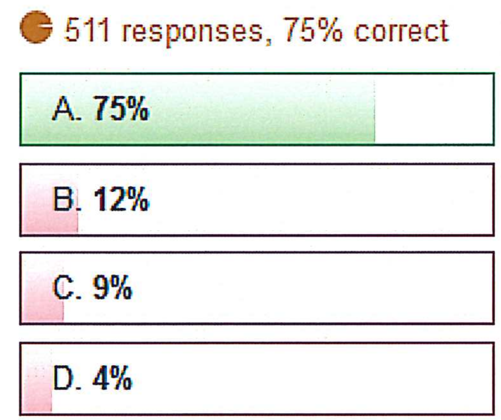
Consider a uniform solid sphere of radius *R* and mass *M* rolling without slipping. Which form of its kinetic energy is larger, translational or rotational?

A. Its translational kinetic energy is larger than its rotational kinetic energy.

B. Its rotational kinetic energy is larger than its translational kinetic energy.

C. Both forms of energy are equal.

D. You need to know the speed of the sphere to tell.



solid sphere : $I = \frac{2}{3} MR^2$

$K_{trans} = \frac{1}{2} M v^2$

$K_{Rot} = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(\frac{2}{3} MR^2\right)\left(\frac{v}{R}\right)^2$

$>$ $\quad K_{rot} = \frac{2}{6} M v^2$

Linear / Rotational Analogy

Linear	Rotational Analogy
• $\vec{s},\ \vec{v},\ \vec{a}$	• $\theta,\ \omega,\ \alpha$
• Force: $\vec{F}$	• Torque: τ
• Mass: m	• Rotational Inertia: I

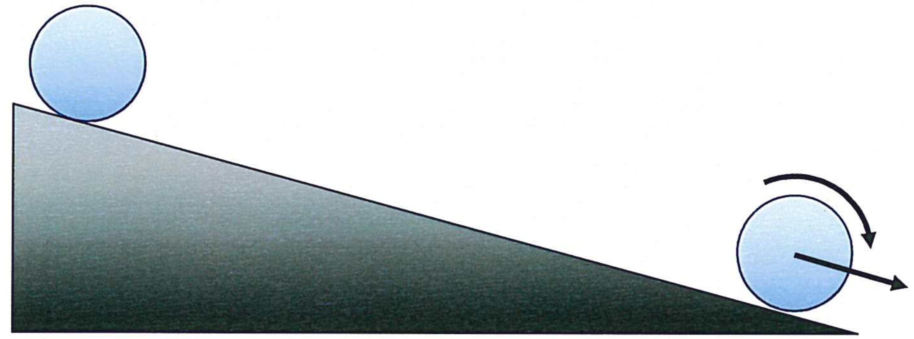
- Newton's 2nd law: $\vec{a} = \dfrac{\vec{F}_{net}}{m}$ $\qquad \alpha = \dfrac{\tau_{net}}{I}$
- Kinetic energy: $K_{\text{cm}} = \dfrac{1}{2} m v^2$ $\qquad K_{\text{rot}} = \dfrac{1}{2} I \omega^2$

Summary of some Different Types of Energy:

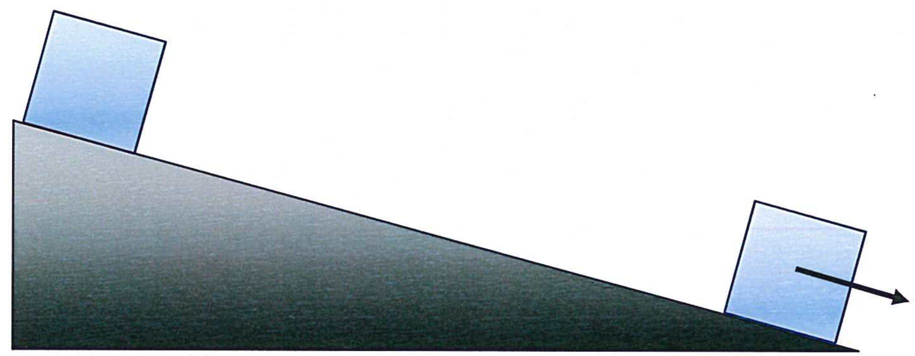
- Kinetic Energy due to linear motion of centre of mass: $K = \frac{1}{2} mv^2$
- Gravitational Potential Energy $U_g = mgh$
- Spring Potential Energy: $U_s = \frac{1}{2} kx^2$
- Rotational Kinetic Energy: $K_{rot} = \frac{1}{2} I\omega^2$
- Thermal Energy: ΔE_{th} (often created by kinetic friction)
  - An object can possess any or all of the above.
  - One way of transferring energy to or out of an object is work:
- Work done by a constant force: $W = Fr\cos\theta$

- Learning Catalytics Q2
- A hoop and a disk are both released from rest at the top of an incline. They both roll without slipping. Which reaches the bottom first? Shall we vote?
- A: hoop wins
- B: disk ~~wins~~ won!!
- C: tie

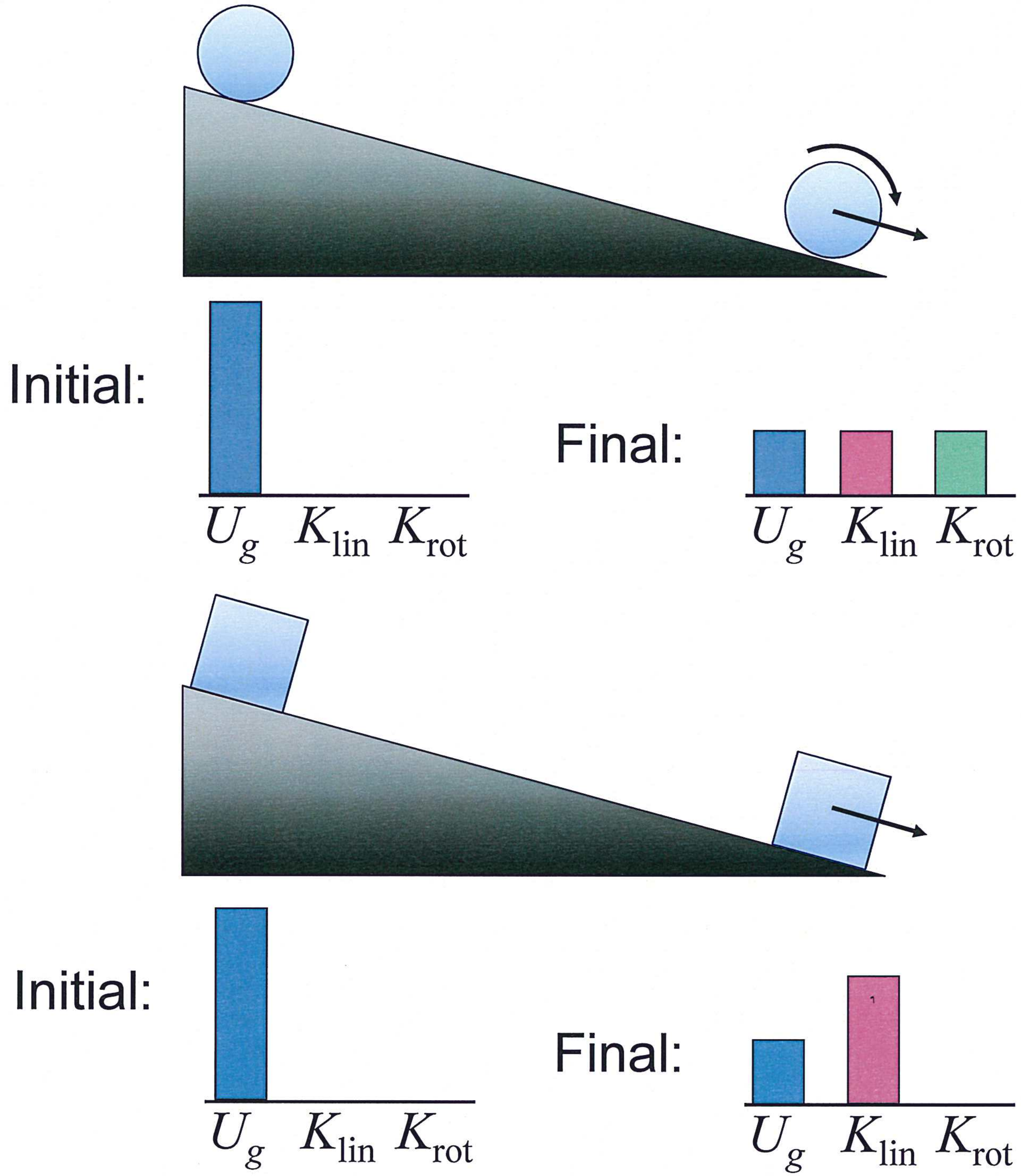
Don't forget: Nature is not a democracy!



- Learning Catalytics Q3. A solid disk is released from rest and rolls without slipping down an incline. A box is released from rest and slides down a frictionless incline of the same angle. Which reaches the bottom first?
- A: disk wins
- B: box wins ← thought experiment... (block of ice?)
- C: tie



- A: disk wins
- **B: box wins**
- C: tie
- Think about conservation of energy.
- A rolling object has two forms of kinetic energy which must be **shared**



Compare and Contrast Soup Cans

- About same mass
- About same radius and shape

- Thick paste, so when this can is rolling, the contents rotate along with the can as one solid object, like a solid cylinder

- Low viscosity liquid, so the can itself rolls while the liquid may just “slide” along.

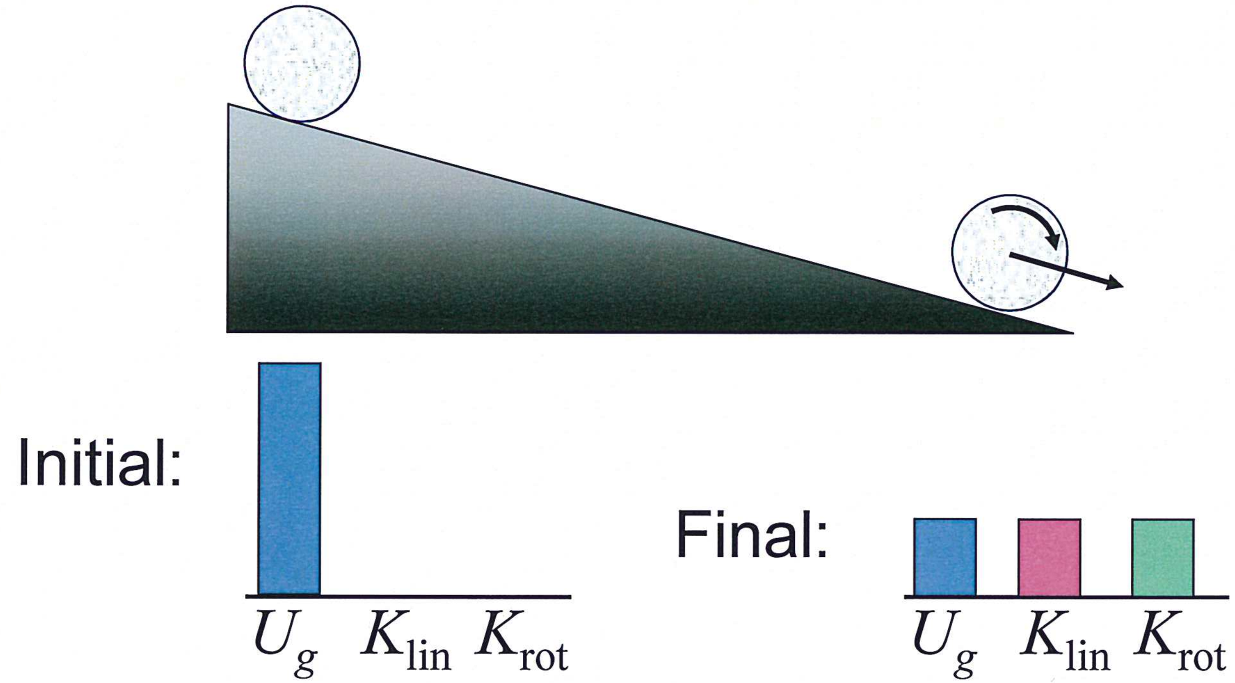
Learning Catalytics Q4

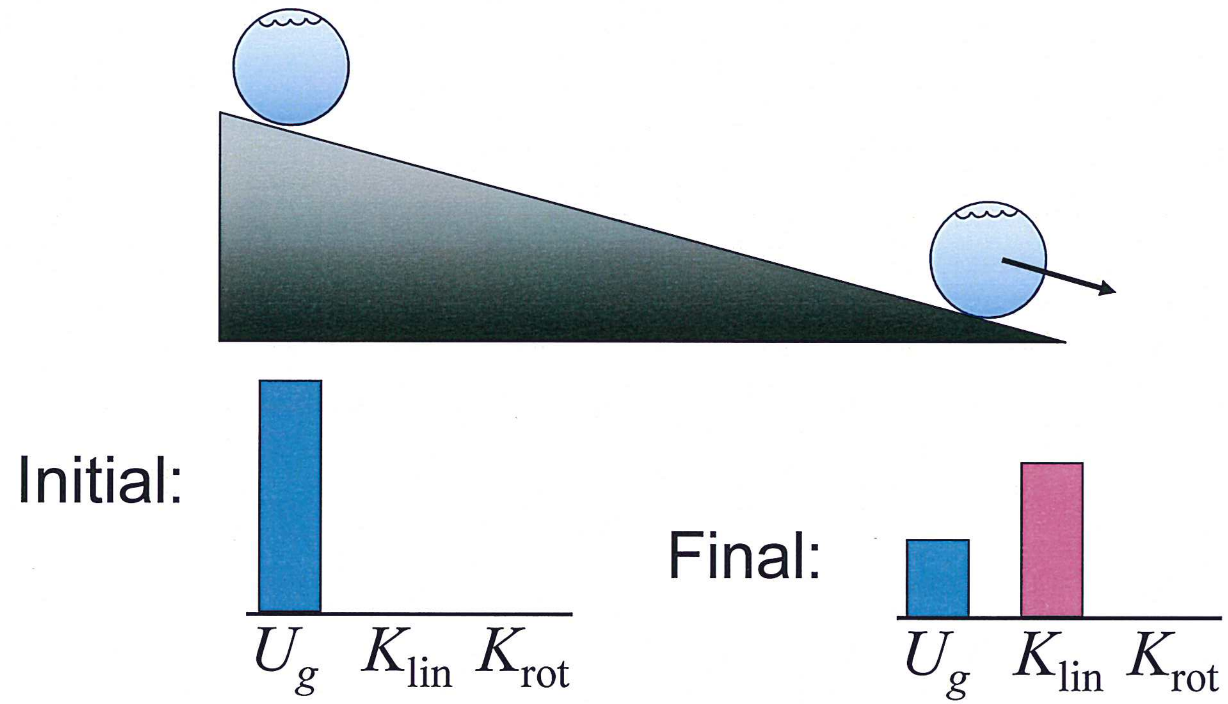
- Two soup cans begin at the top of an incline, are released from rest, and allowed to roll without slipping down to the bottom. Which will win?

Predict:

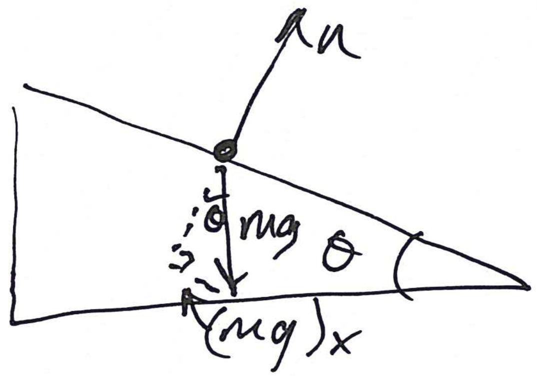
A. Cream of Mushroom will win
B. Chicken Broth will win — won !!
C. Both will reach the bottom at about the same time.

- Cream of Mushroom soup must rotate, like a solid disk.
- Chicken broth can slide down **without rotating** while the can rotates around it.



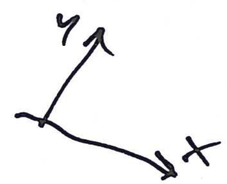


1. What is the acceleration of a slipping object down a ramp inclined at angle θ? [assume no friction]



$f = 0$

Define:



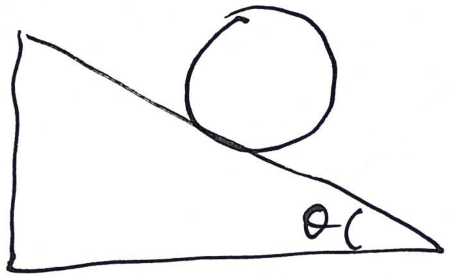
$$\sin\theta = \frac{(mg)_x}{mg}.$$

$$(F_{net})_x = (mg)_x = mg\sin\theta$$

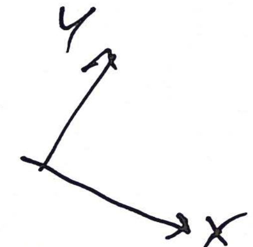
$$\cancel{m}a = \cancel{m}g\sin\theta$$

$$\boxed{a = g\sin\theta}$$

2. What is the acceleration of a **solid disk** rolling down a ramp inclined at angle θ? [assume rolling without slipping]

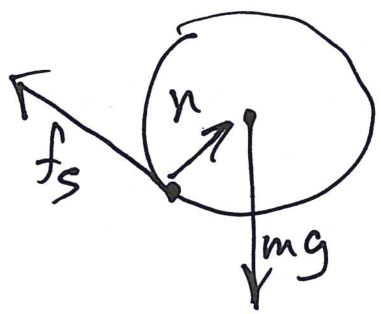


Define



↺ = positive

Extended f.b.d.



n, f_s are unknowns

Define rotation axis to be centre of disk.

$\tau_n = 0 \rightarrow$ n acts toward centre of disk

$\tau_{mg} = 0 \rightarrow$ mg acts at centre

$$\tau_{Net} = I\alpha = -f_s R$$

Rolling without slipping constraint: $\boxed{\alpha = \frac{a}{R}}$

$$-f_s R = I\left(\frac{a}{R}\right)$$

$$f_s = -\frac{I a}{R^2}$$

$$(F_{net})_x = mg\sin\theta - |f_s| = ma$$

$$a = g\sin\theta - \frac{I a}{mR^2}$$

solve for a:

$$a\left(1 + \frac{I}{mR^2}\right) = g\sin\theta$$

$$a = \frac{g\sin\theta}{1 + I/mR^2}$$

Solid Disk: $I = \frac{1}{2}mR^2$

$$\Rightarrow a = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}mR^2}{mR^2}} = \frac{g\sin\theta}{1 + \frac{1}{2}}$$

$$\boxed{a = \frac{2}{3}g\sin\theta} < g\sin\theta$$

Before Class 18 on Monday

- The reading is all of Chapter 11 on Rotational Vectors and Angular Momentum.
- Please read the chapter and/or watch the Preclass 18 Video.

- Something to think about: When a figure-skater starts a spin and brings in her arms, she spins even faster. Why?

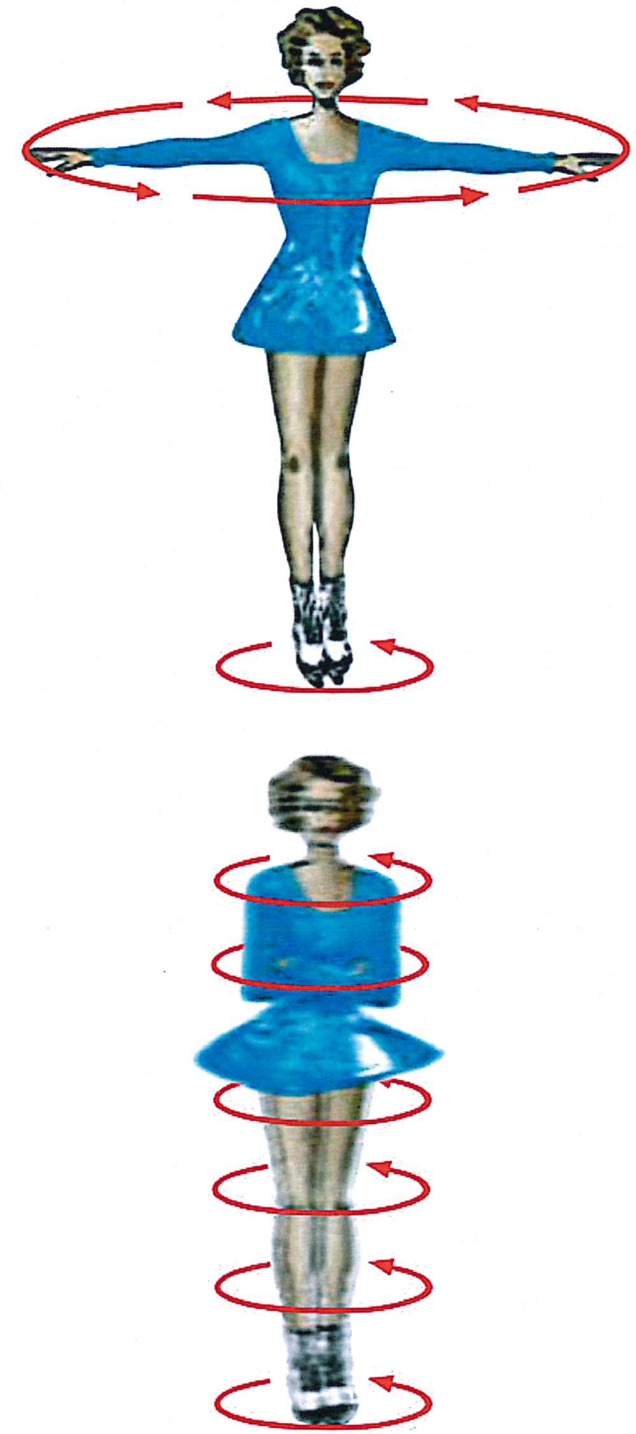


Image from http://www.daviddarling.info/encyclopedia/A/angular_momentum.html