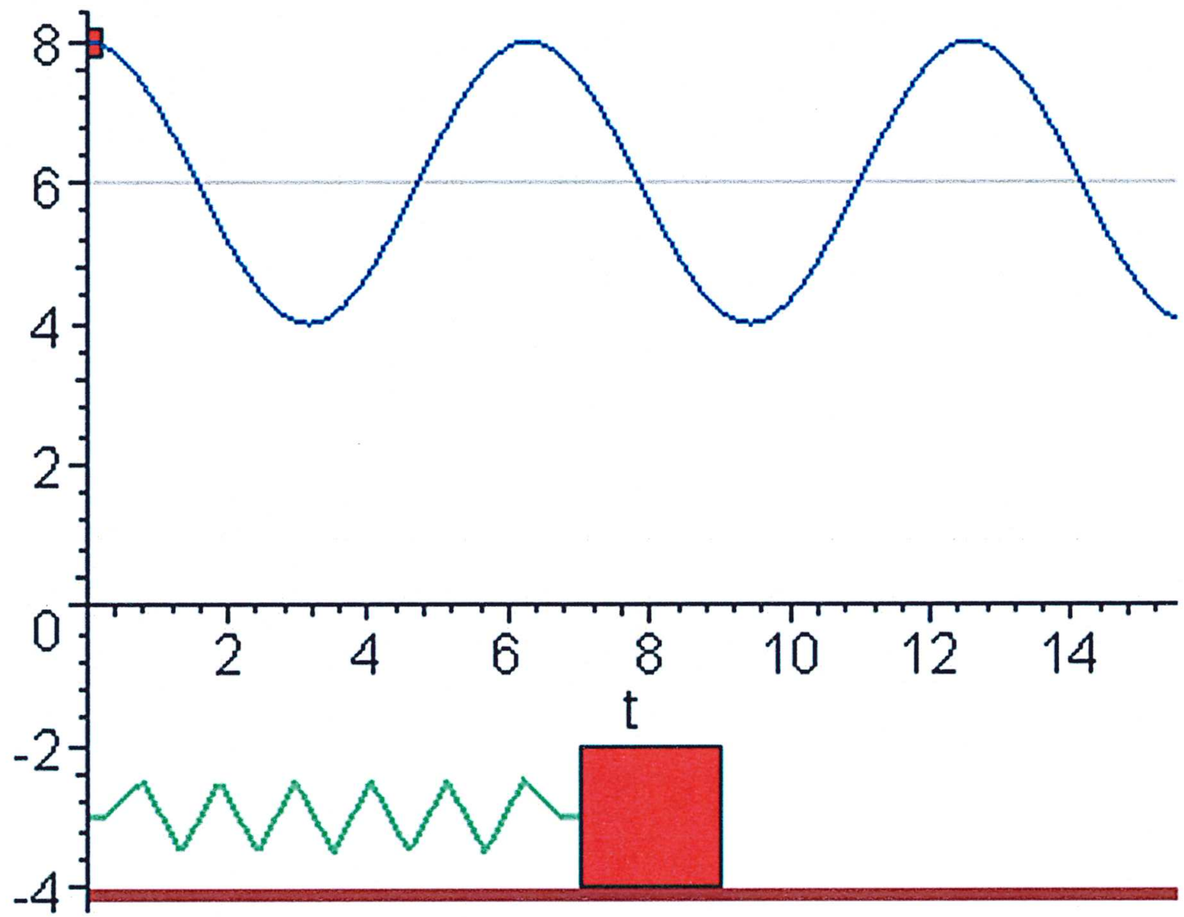
PHY131H1F - Class 20

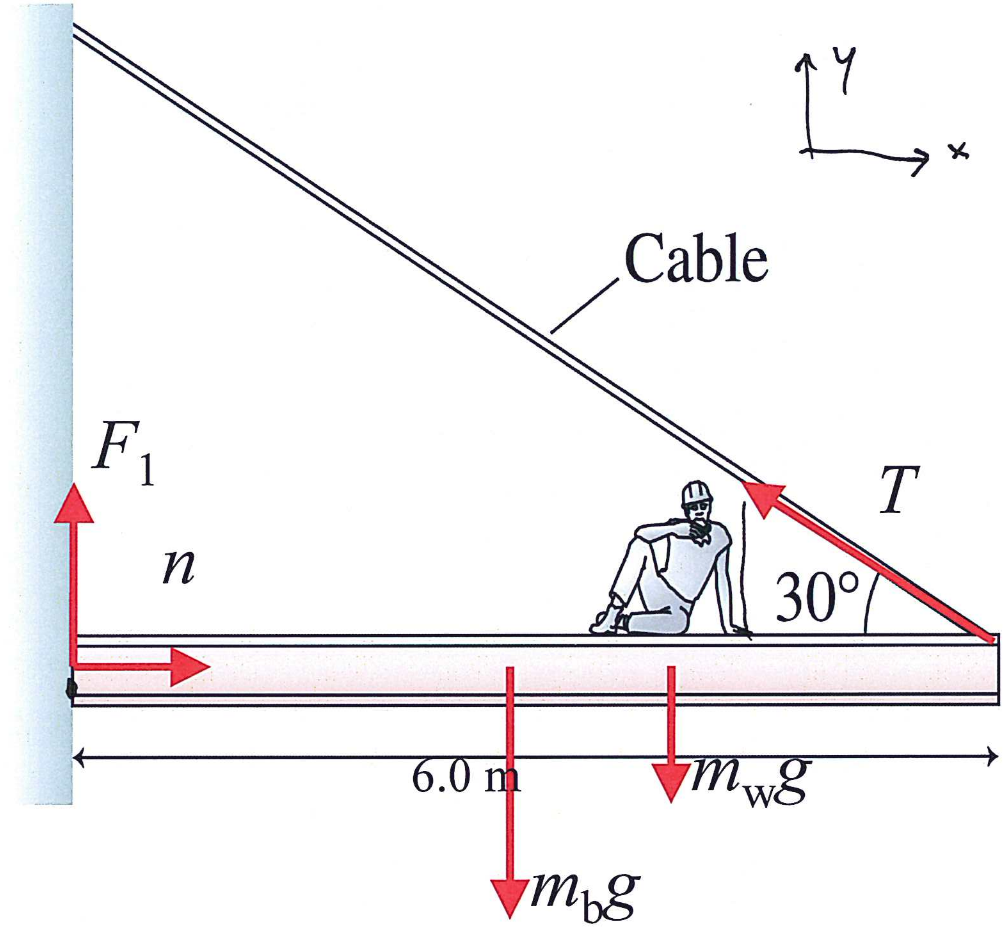
Today:

- **Today, Chapter 13:**
- General Periodic Oscillations
- Special case: Simple Harmonic Motion



Animation from http://www.uni-saarland.de/fak7/knorr/homepages/patrick/theorex/MapleScripts/oscillations/forced_oscillations1.html

Learning Catalytics Question 1



- A construction worker of mass m_w sits 2.0 m from the end of a steel beam of mass m_b, as shown.
- The tension in the Cable is T
- The wall exerts a normal force, n on the beam, and an upward force, F_1.
- Define $+x$ = to the right, $+y$ = up, and the pivot is the point where the beam touches the wall.
- What is the normal force, n?

$\sum \tau = 0$

$\sum F_x = 0 = n - T\cos(30°) = 0$

$\sum F_y = 0$

A. $(m_b + m_w)g$

B. $(m_b + m_w)g - T\cos(30°)$

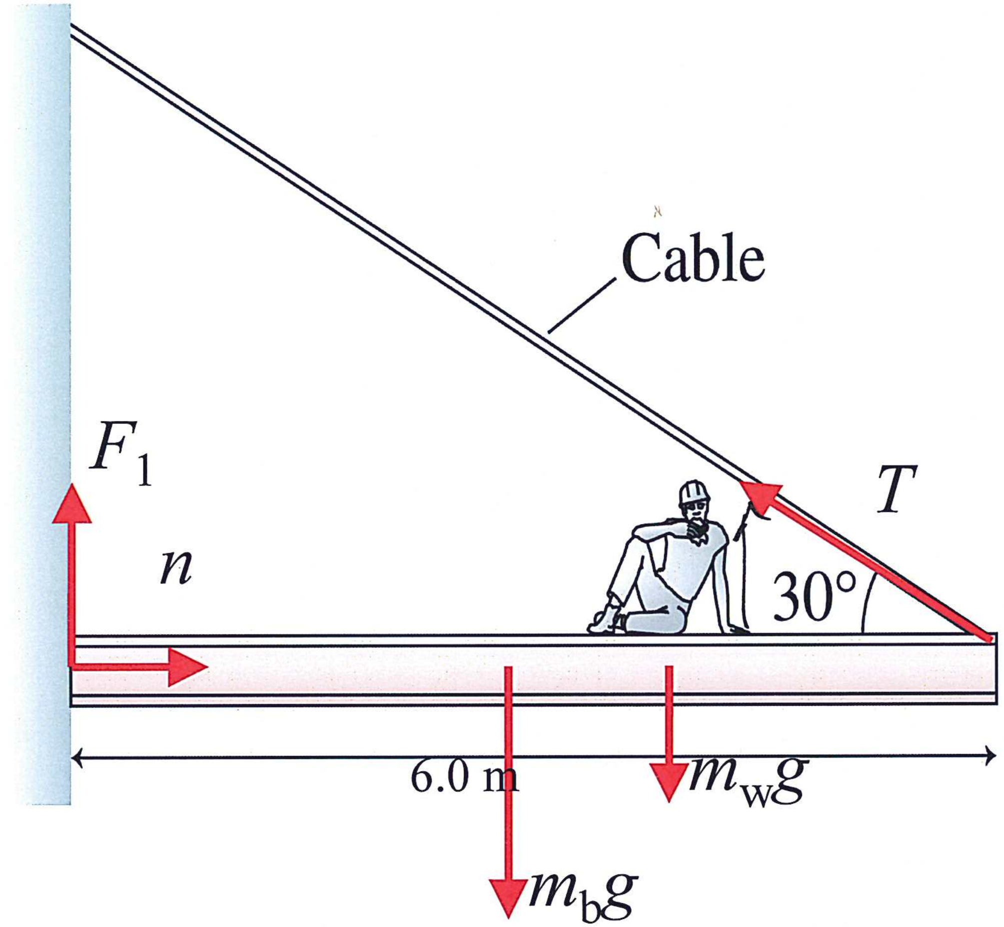
C. $(m_b + m_w)g - T\sin(30°)$

D. $T\sin(30°)$

E. $T\cos(30°)$

Learning Catalytics Question 2

- A construction worker of mass m_w sits 2.0 m from the end of a steel beam of mass m_b, as shown.
- The tension in the Cable is T
- The wall exerts a normal force, n on the beam, and an upward force, F_1.
- Define $+x$ = to the right, $+y$ = up, and the pivot is the point where the beam touches the wall.
- What is the force, F_1?



A. $(m_b + m_w)g$

B. $(m_b + m_w)g - T\cos(30°)$

C. $(m_b + m_w)g - T\sin(30°)$

D. $T\sin(30°)$

E. $T\cos(30°)$

$\sum \tau = 0$

$\sum F_x = 0$

$\sum F_y = 0 = F_1 - m_b g - m_w g + T\sin(30)$

$F_1 =$

Bonus Point for Over 65% Course Evaluation Response Rate

- An essential component of our commitment to teaching excellence is the regular evaluation of courses by students.
- On Nov. 24 you were sent an email by course.evaluations@utoronto.ca to evaluate PHY131H1F.
- It only takes 10 or 15 minutes to answer the questions and enter your typed thoughts about the course.
- Your answers and thoughts are **anonymous**, but are very important to me and Brian.
- I promise you that when the results become available to us in January, Brian and I will **read** every comment and scrutinize the responses to see if it can help us improve the course or my teaching in the future.

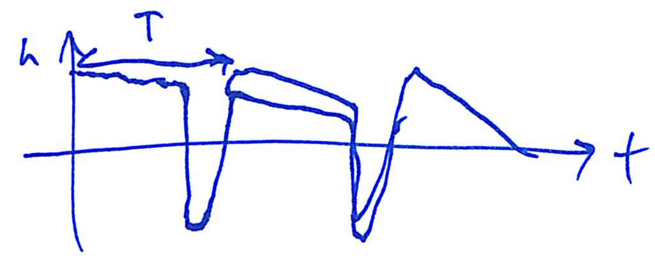
Bonus Point for Over 65% Course Evaluation Response Rate

- The end of the evaluation period for this semester is Thursday December 7 at 11:59PM.
- If, by the end of the course evaluation period, at least 65.00% of the students enrolled in this course have completed the course evaluations, then **every student in the course will have 1% bonus added to their final course mark.**
- If fewer than 65.00% of students complete the course evaluations by the deadline, then **no bonus point will be added for any student.**
- Results so far (as of 10:25 today):
- **Invited:** ___ students
- **Responded:** ___ students
- **Response Rate:** ___%

11:00am: 36%
5:00pm: 37%

Period, frequency, angular frequency

• The time to complete one full cycle, or one oscillation, is called the period, T.
• The frequency, f, is the number of cycles per second. Frequency and period are related by



$$f = \frac{1}{T} \qquad \text{or} \qquad T = \frac{1}{f}$$

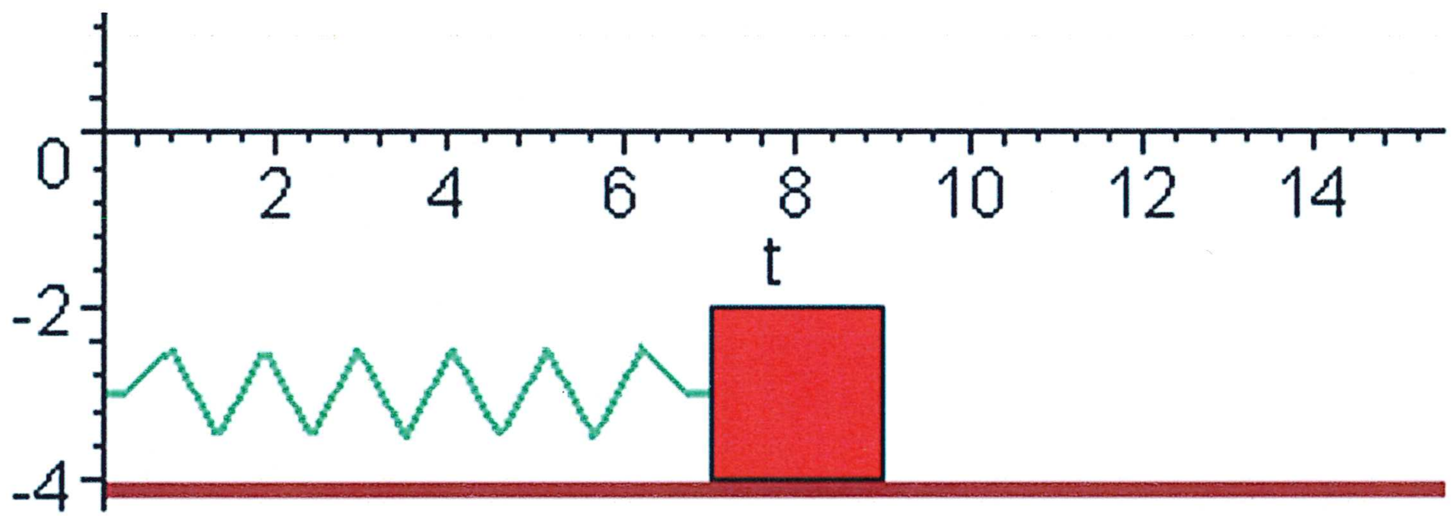
• The oscillation frequency f is measured in cycles per second, or Hertz.
• We may also define an angular frequency ω in radians per second, to describe the oscillation.

"angular" frequency of oscillation →

$$\omega \text{ (in rad/s)} = \frac{2\pi}{T} = 2\pi f \text{ (in Hz)}$$

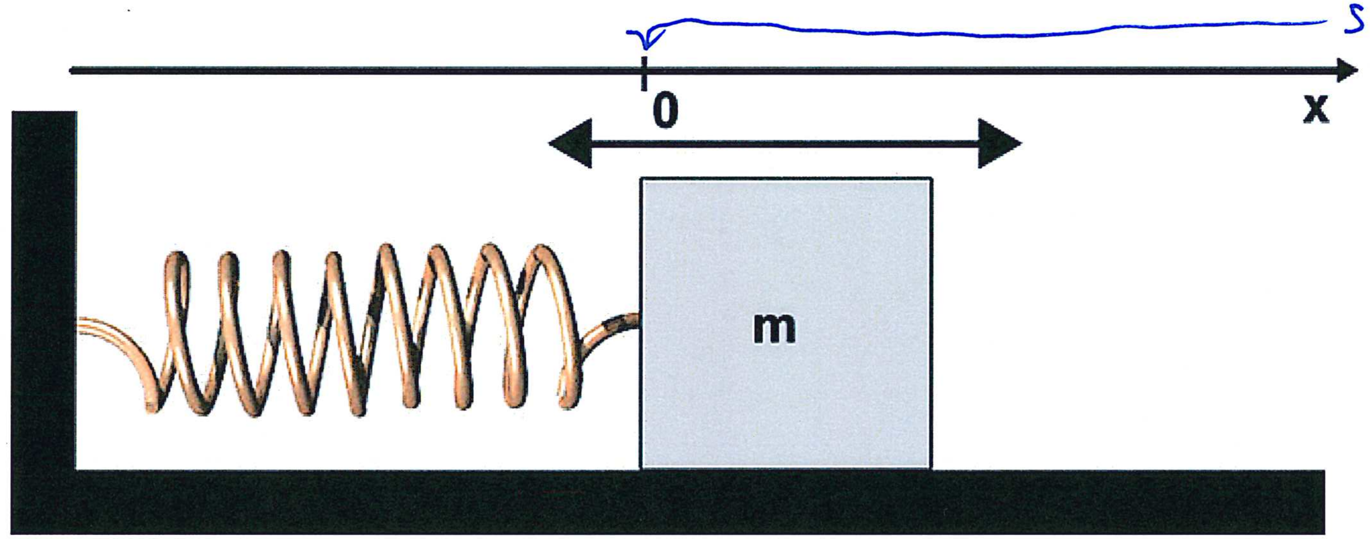
Last day I asked

- A spring with a mass attached to it is stretched and released. When the spring returns to equilibrium, is the mass moving?
- Answer: Yes! It is not **accelerating**, but it is **moving** at that moment. Inertia then carries it past equilibrium to the other side. After passing equilibrium, the acceleration is opposite the velocity, so it slows down, eventually turning around.



S HM

The Spring-Mass System



The force exerted on the mass by the spring:

$F = -k\,x$ (Hooke's Law)

$F_{net} = m\,a$ (Newton's Second Law)

$F_{net} = -kx = ma_x$

Combine to form a differential equation:

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Solving S.H.M.

$$a = \frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

Trial Solution:

$$x = A\cos(\omega t)$$

where A, ω are constants.

Does it work?

$$\frac{d}{dt}x = \frac{d}{dt}\left(A\cos(\omega t)\right)$$

$$= \omega\left[-A\sin(\omega t)\right]$$

$$v_x = -A\omega\sin(\omega t)$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left[-A\omega\sin(\omega t)\right]$$

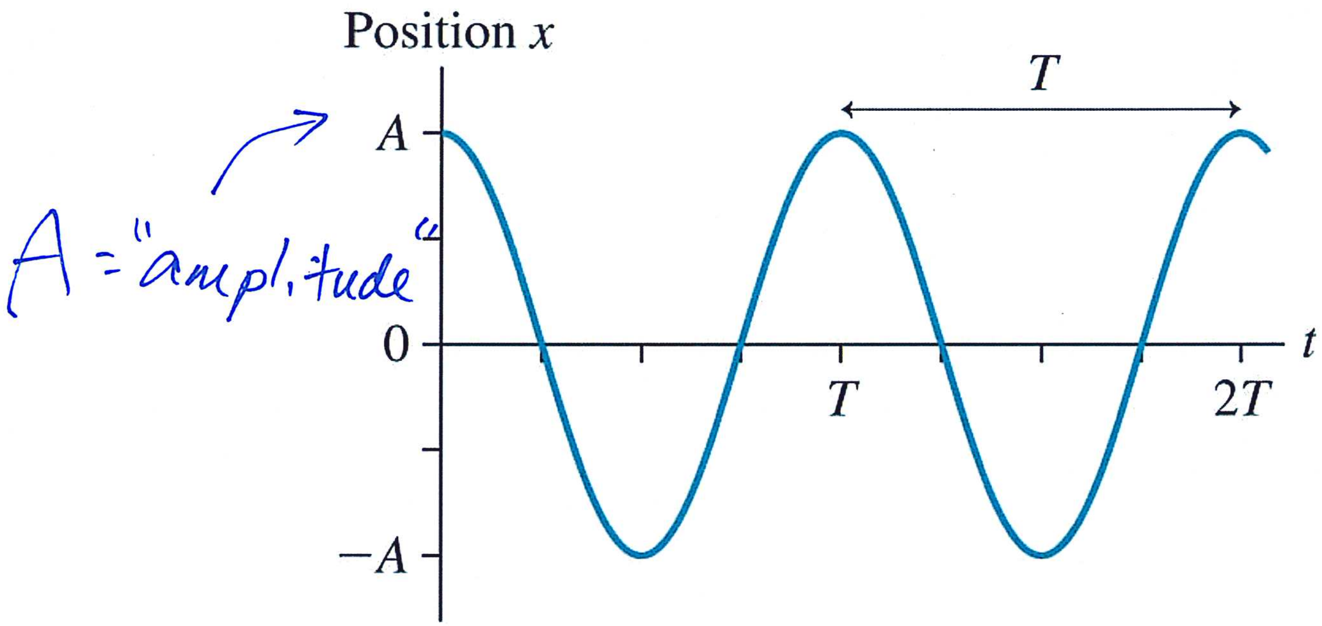
$$= -A\omega^2\cos(\omega t)$$

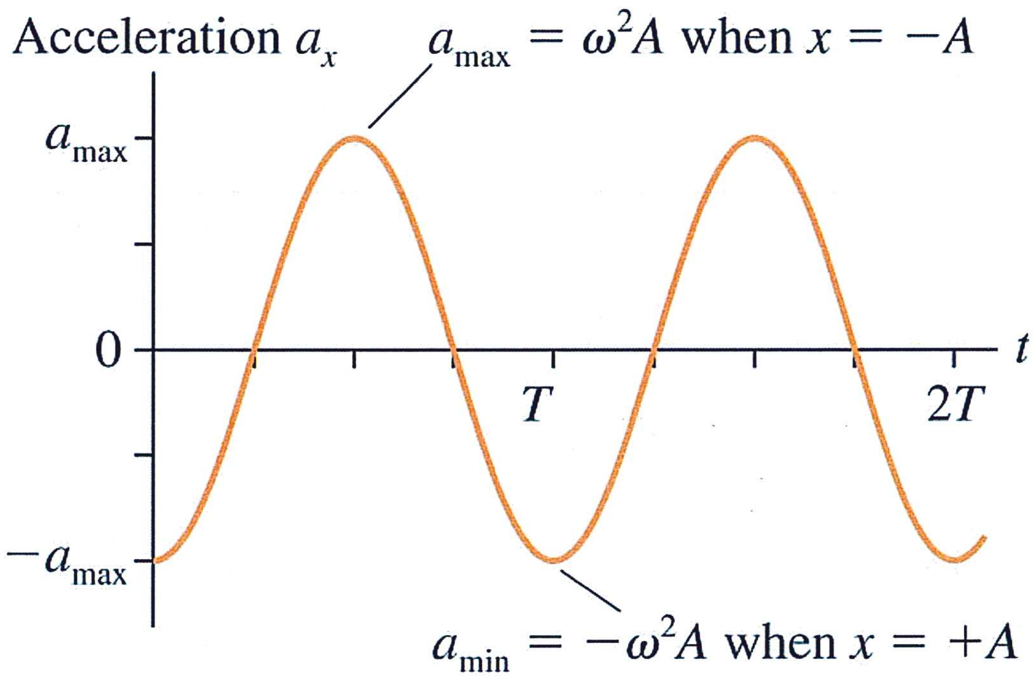
$$a_x = -\omega^2\left[A\cos(\omega t)\right]$$

$$a_x = -\omega^2 x$$

We solved it iff $\omega^2 = \frac{k}{m}$

A is arbitrary.





$$F_s = -kx$$

$$a = \frac{F_{Net}}{m}$$

$$a_x = -\frac{k}{m}x$$

Simple Harmonic Motion

If the initial position of an object in SHM is not A, then we may still use the cosine function, with a phase constant ϕ_0 measured in radians.

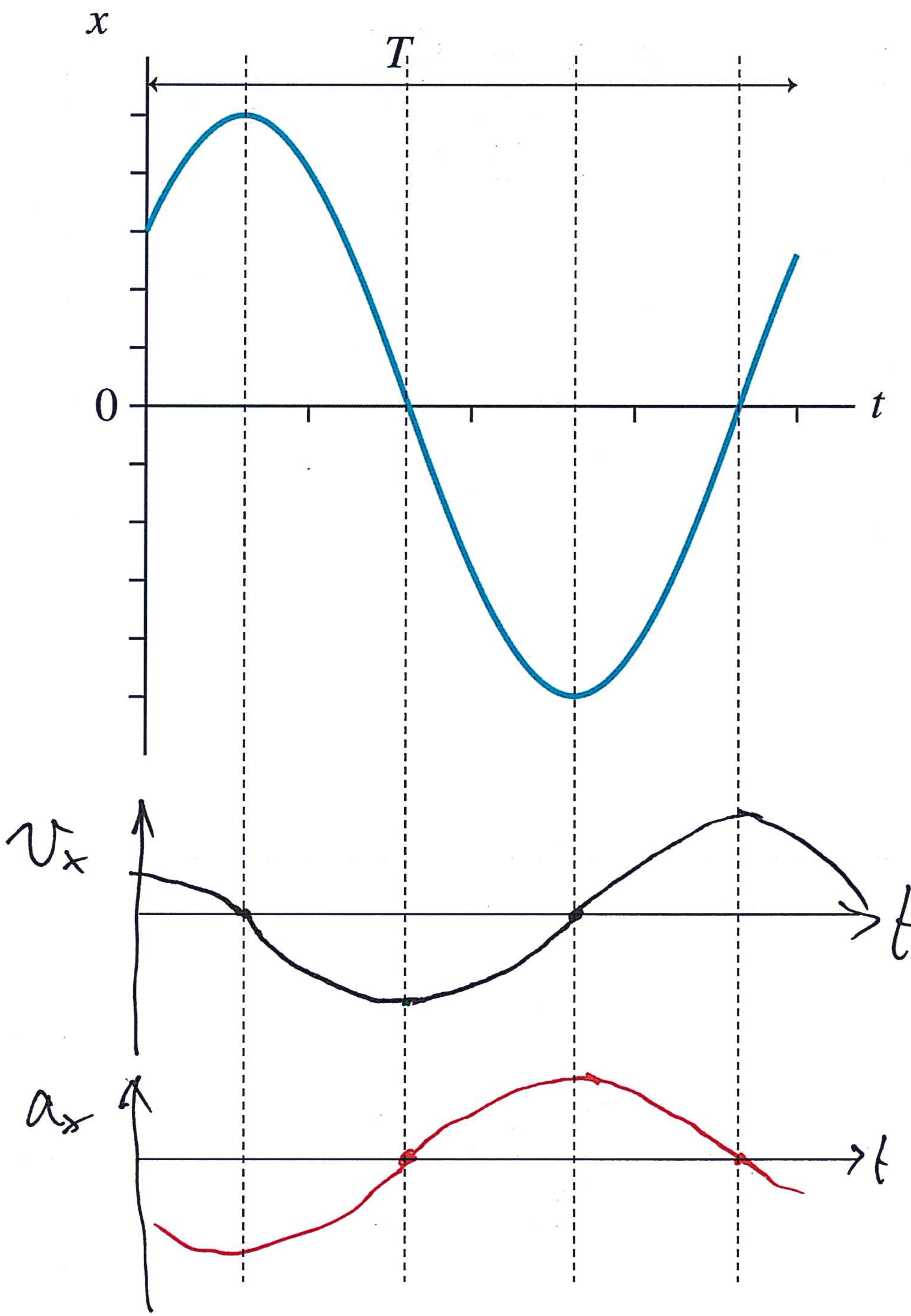
$$x = A\cos(\omega t + \phi_0)$$

← also a solution.
A, ϕ_0 arbitrary.
$\omega = \sqrt{k/m}$

$$v_x = -A\omega \sin(\omega t + \phi_0)$$

$$a_x = -A\omega^2 \cos(\omega t + \phi_0)$$

Simple Harmonic Motion (SHM)

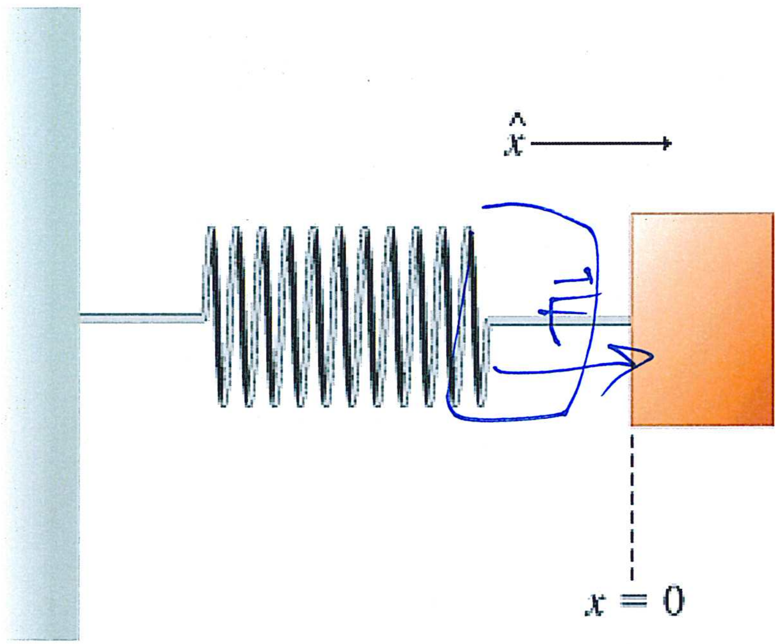


$$x = A\cos(\omega t + \phi_0)$$

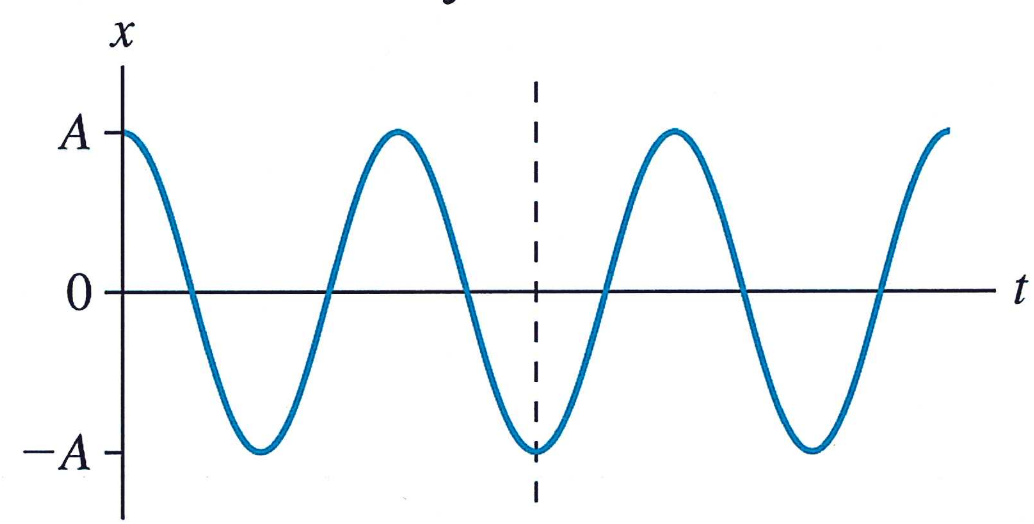
$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi_0)$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi_0)$$

Learning Catalytics Question 3

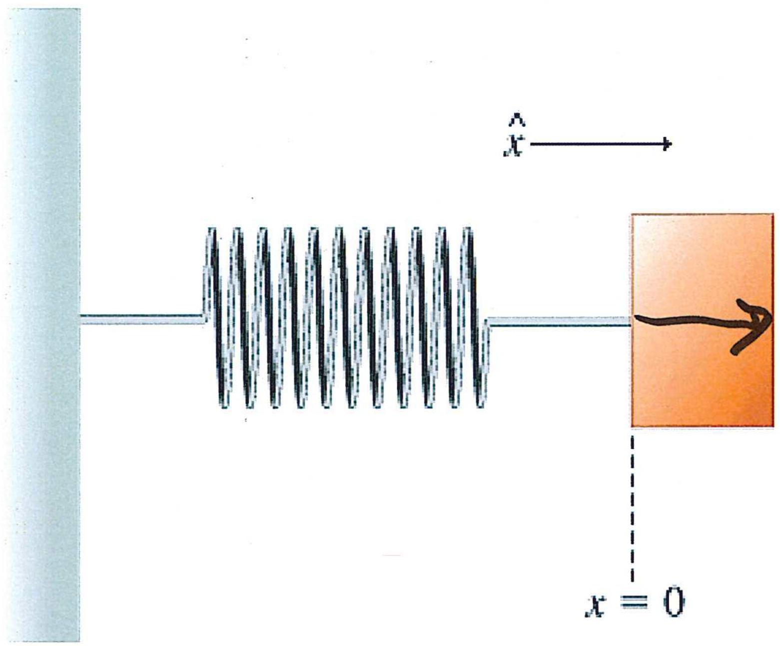


This is the position graph of a mass on a spring. What can you say about the velocity and the force at the instant indicated by the dotted line?

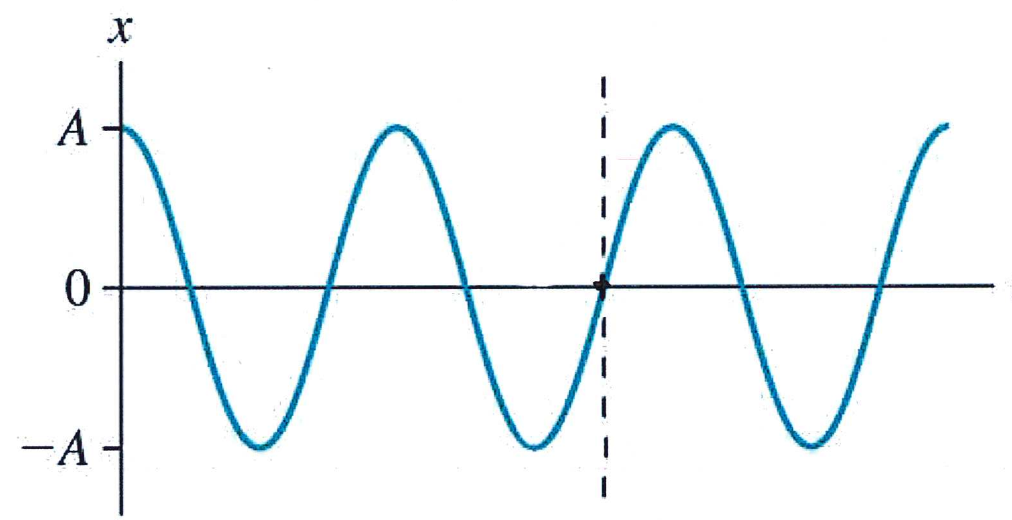


A. Velocity is positive; force is zero.
B. Velocity is negative; force is zero.
C. Velocity is negative; force is to the right.
D. Velocity is zero; force is to the right.
E. Velocity is zero; force is to the left.

Learning Catalytics Question 4



This is the position graph of a mass on a spring. What can you say about the velocity and the force at the instant indicated by the dotted line?

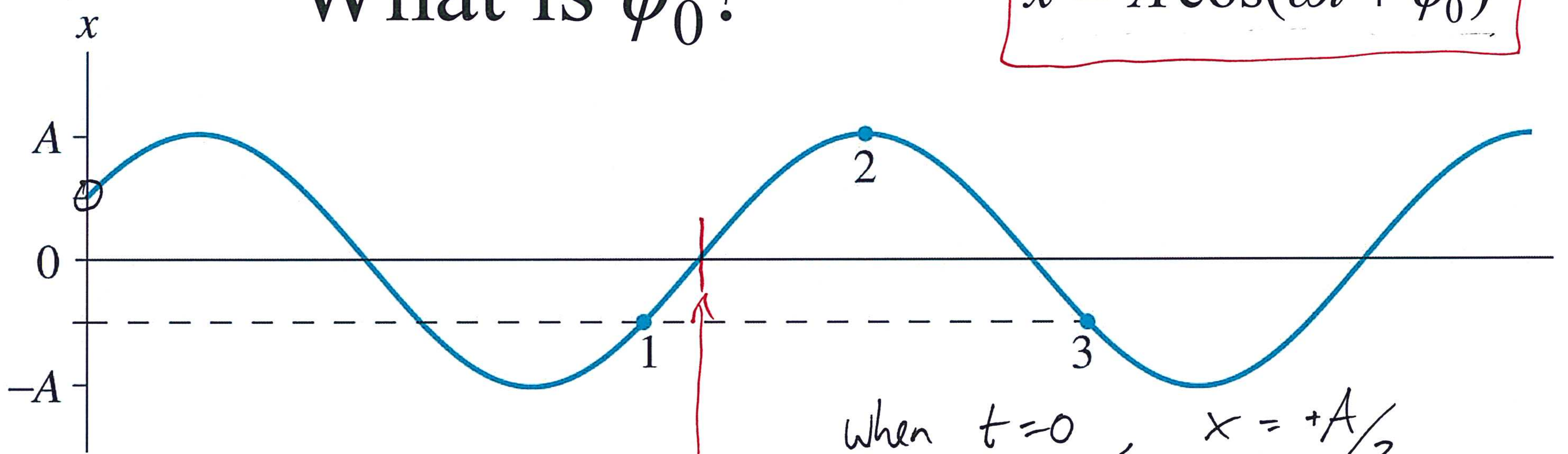


A. Velocity is positive; force is zero.
B. Velocity is negative; force is zero.
C. Velocity is negative; force is to the right.
D. Velocity is zero; force is to the right.
E. Velocity is zero; force is to the left.

Learning Catalytics Question 5

What is ϕ_0?

$$x = A\cos(\omega t + \phi_0)$$



A. $-\pi/3$

B. $+\pi/3$

C. $-2\pi/3$

D. $+2\pi/3$

E. 0

when $t=0$, $x = +A/2$

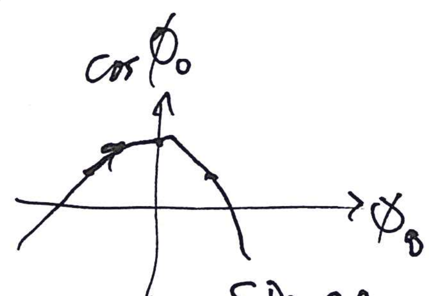
$x = A\cos(\omega t + \phi_0)$

$\frac{A}{2} = A\cos\phi_0$

$\cos\phi_0 = \frac{1}{2}$

$\phi_0 = \cos^{-1}(0.5) = 60°$

$\leftarrow \phi_0 = \pm 60°\left(\frac{\pi}{180}\right) = \pm\frac{\pi}{3}$ rad.



since x is increasing ⟶ choose − solution.

Pre-class 20 Results

1. many choice

An object is executing simple harmonic motion. What is true about the acceleration of this object? (There may be more than one correct choice.)

573 responses, 62% correct

A. The acceleration is a maximum when the displacement of the object is a maximum. — A. 79%

B. The acceleration is a maximum when the speed of the object is a maximum. — B. 7%

C. The acceleration is a maximum when the displacement of the object is zero. — C. 9%

D. The acceleration is zero when the speed of the object is a maximum. — D. 83%

E. The acceleration is a maximum when the object is instantaneously at rest. — E. 82%

Pre-class 20 Results

2. multiple choice

In simple harmonic motion, the speed is greatest at that point in the cycle when

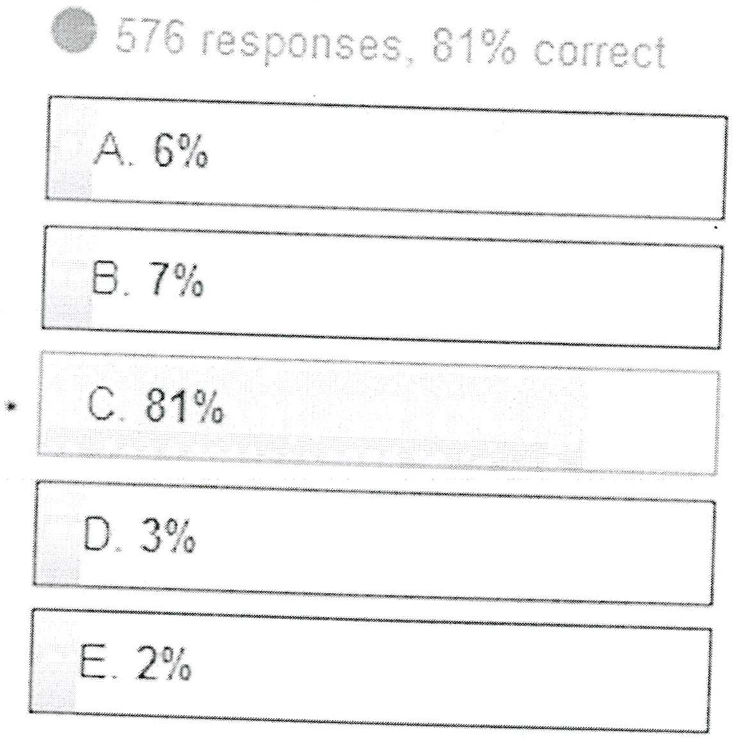
A. the magnitude of the acceleration is a maximum.

B. the displacement is a maximum.

C. the magnitude of the acceleration is a minimum.

D. the potential energy is a maximum.

E. the kinetic energy is a minimum.



Pre-class 20 Results

3. numerical

A sewing machine needle moves up and down in simple harmonic motion with an amplitude of 1.27 cm and a frequency of 2.55 Hz. What is the maximum speed of the needle?

[Enter your answer as a number only, using the units of m/s.]

568 responses, 52% correct

0.203

SHM. $A = 1.27\ cm$

$A = 0.0127\ m$

$f = 2.55\ Hz$, $\omega = 2\pi f = 2(3.14159)(2.55)$

$\omega = 16.02211\ \frac{rad}{s}$

$v_{max} = A\omega = (0.0127\ m)(16.02211\ \frac{rad}{s})$

$= 0.203\ m/s$

Pre-class 20 Comments

- *"yay more calculus..."*
- **Harlow comment:** Isn't it beautiful?
- *"Do we need to know how to get the formula x = Acos(wt + phi)?"*
- **Harlow answer:** No. Even I don't actually know how to get this one. I just know it works!
- *"What is the difference between angular frequency and angular velocity if they are both represented by the lowercase omega?"*
- **Harlow answer:** Context. Angular velocity refers to rotation, angular frequency refers to oscillation.
- *"How did we get v-max and a-max from equations 13.9 and 13.10?"*

13.9: $v(t) = -\omega A \sin(\omega t)$
13.10: $a(t) = -\omega^2 A \cos(\omega t)$

- **Harlow answer:** just find the minimum values of sin and cos, both of which are -1.

S.H.M. notes.

- The frequency, f, is set by the properties of the system. In the case of a mass m attached to a spring of spring-constant k, the frequency is always

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

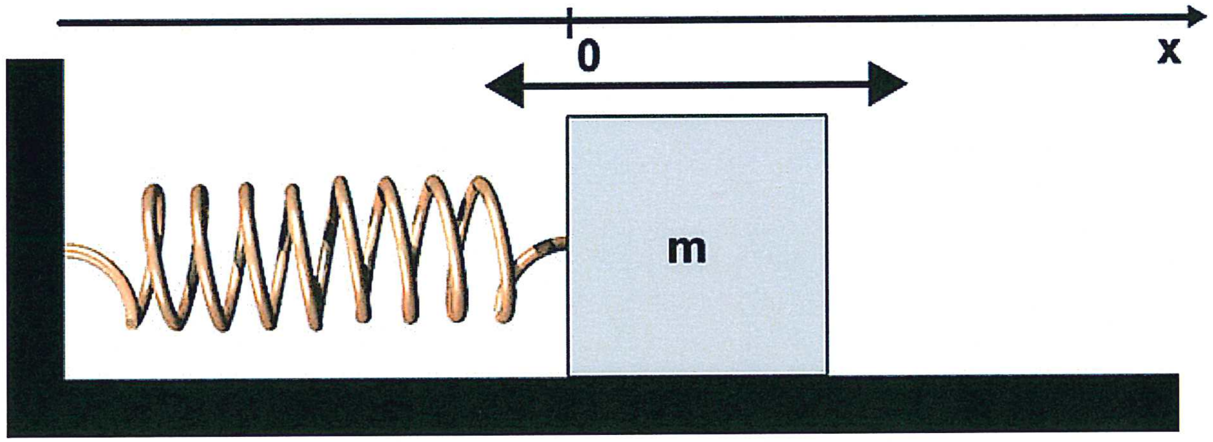
- A and ϕ_0 are set by the initial conditions: x_0 (initial position) and v_0 (initial velocity).
- A turns out to be related to the total energy of the spring oscillator system: $E = \frac{1}{2} k A^2$.

Learning Catalytics Question 6

Which of the following quantities in the description of **simple harmonic motion** is *not* determined by the initial position and velocity of the mass?

A. the amplitude, A
B. the phase constant, ϕ_0
C. the angular frequency, ω $= \sqrt{\frac{k}{m}}$

Learning Catalytics Question 7

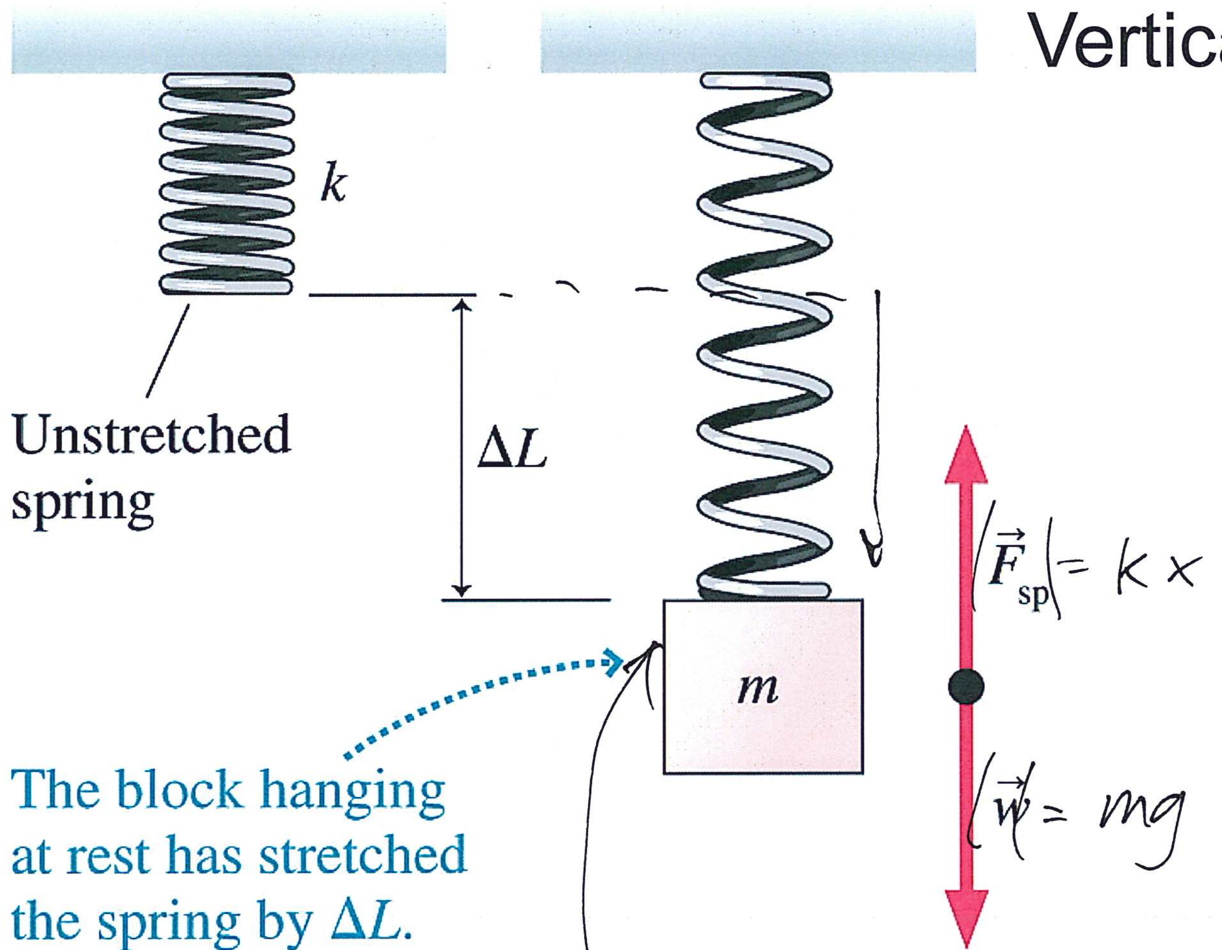


A mass is oscillating on a spring in S.H.M. When it passes through its equilibrium point, an external "kick" suddenly decreases its speed, but then it continues to oscillate. As a result of this slowing, the frequency of the oscillation

A. goes up
B. goes down
C. stays the same

$$\omega = \sqrt{\frac{k}{m}}$$

Vertical Mass on a Spring



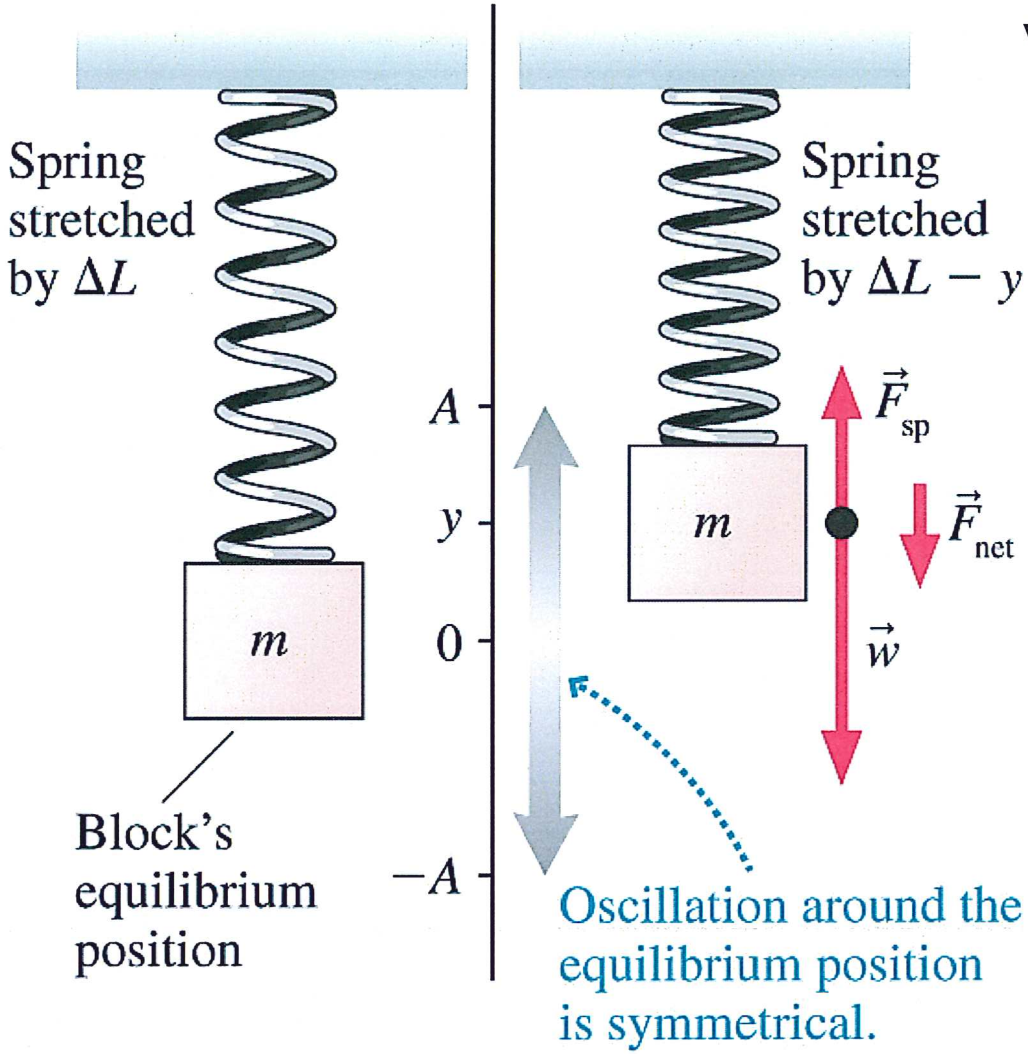
New equilibrium position of mass when $kx = mg$

$x = \Delta L$

$k\Delta L = mg$

$$\Delta L = \frac{mg}{k}$$

Vertical Mass on a Spring



Set $y=0$ to be block's new equilibrium.

$$y = A\cos(\omega t + \phi_0)$$

$$\omega = \sqrt{k/m}$$

Before Class 21 on Wednesday

- Please finish reading Chapter 13 on Oscillations, and/or watch the Preclass Video 21.

- Problem Set 9 on Chapters 10 and 11 is due tonight at 11:59pm.
- Something to think about over the weekend: If you double the mass of a mass on a spring, how does this change the frequency? If you double the mass of a swinging pendulum, how does this change the frequency? What is the difference here?