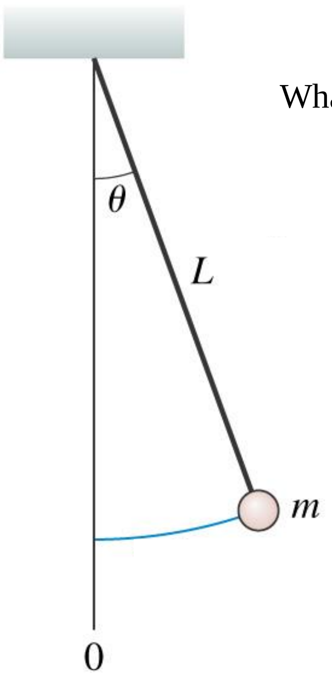
PHY131H1F - Class 21

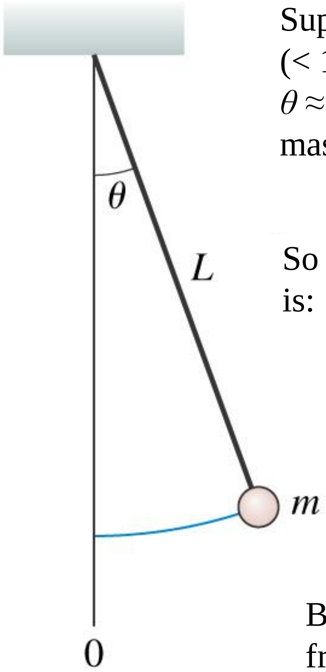
Today:

- **Today, finishing Chapter 13:**
- Simple Pendulum
- Circular motion and S.H.M.
- Energy in S.H.M.
- Damped Harmonic Motion
- Driven Oscillations and Resonance.

From http://www.cavatoyota.com/blog/what-are-shock-absorbers/
:
To test your vehicle's shock absorbers, simply push down on the each corner of the vehicle and observe its bounce. The vehicle should bounce up and return to its center resting position. If it continues to bounce, the shock absorber **should be replaced**.



What is the net torque on this pendulum?



Suppose we restrict a pendulum's oscillations to small angles (< 10°). Then we may use the **small angle approximation** $\sin\theta \approx \theta$, where θ is measured in radians. The net torque on the mass is

$$\Sigma\tau = I\frac{d^2\theta}{dt^2} = -mgL\theta$$

So the simple harmonic motion equation for θ as a function of time is:

$$\frac{d^2\theta}{dt^2} = -\frac{mgL}{I}\theta$$

The solution to this is $\theta = A\cos(\omega t + \phi_0)$, where A and ϕ_0 are arbitrary, and the **frequency** of oscillations (in rad/s) is:

$$\omega = \sqrt{\frac{mgL}{I}}$$

But the rotational inertia of a point mass m a distance L from the rotation axis is $I = mL^2$, so

$$\omega = \sqrt{\frac{g}{L}}$$

Learning Catalytics Question 1

Two pendula have the same length, but different mass. The force of gravity, $F=mg$, is larger for the larger mass. Which will have the longer period?

A. the larger mass

B. the smaller mass

C. neither

Example.

Luke and Leia have a combined mass of 120 kg and both grasp a rope of length 30 m that is attached to a beam above them. The beam is half-way across a 10 m horizontal gap, and they want to swing across. If they start from rest and swing down and up, just reaching the other side, how long does this take?

Mass on Spring versus Pendulum

	Mass on a Spring	Pendulum
Condition for S.H.M.	Small oscillations (Hooke's Law is obeyed)	Small angles
Angular frequency	$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{L}}$
Period	$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$

Learning Catalytics Question 2

A person swings on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead, the person stands on the swing, the natural frequency of the swing is

A. greater

B. the same

C. smaller

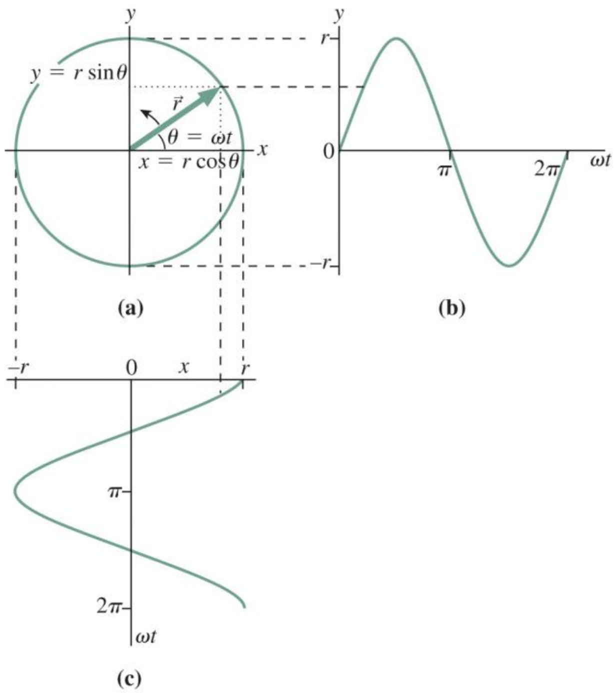
Learning Catalytics Question 3

A grandfather clock at high altitudes runs

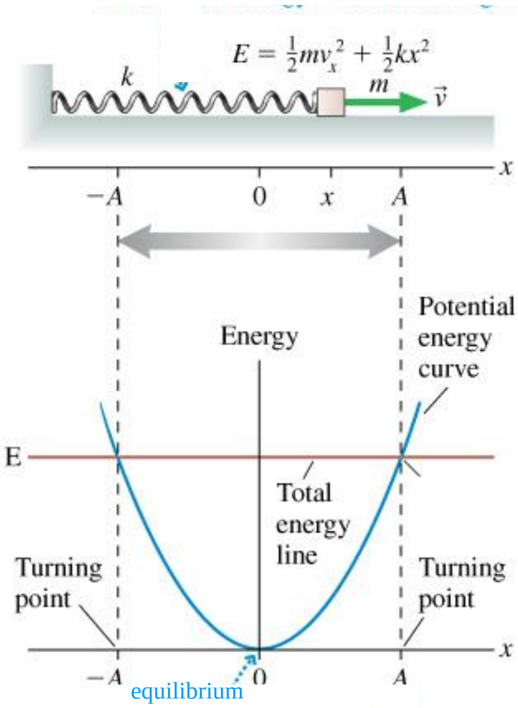
A. fast.
B. slow.
C. normally as it does at sea level.

Image from https://www.1-800-4clocks.com/Bulova-Vickery-Wall-Chimes-Clock_C4329_CUV

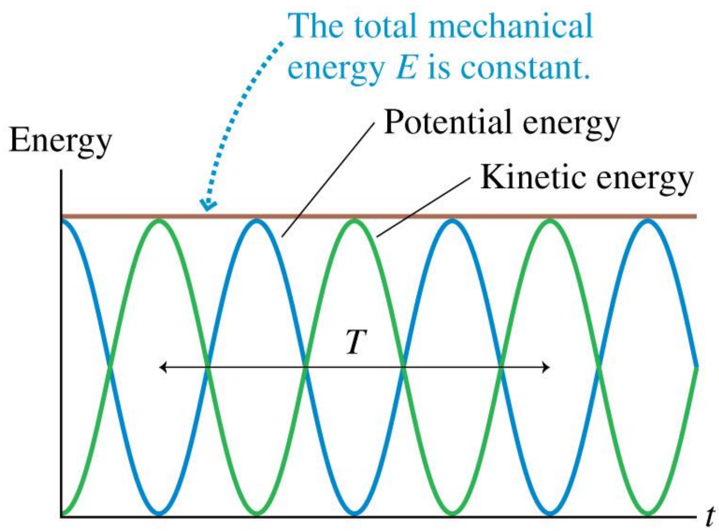
Uniform Circular Motion	Simple Harmonic Motion



Energy of a mass on a spring

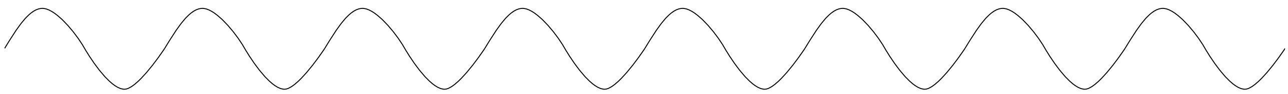


$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(v_{max})^2 \quad \text{(conservation of energy)}$$

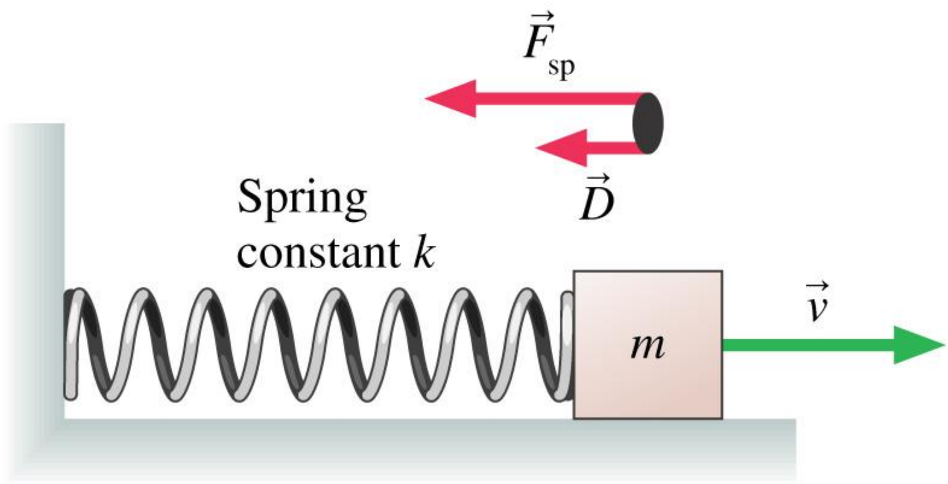


Simple Harmonic Motion (SHM)

- If the net force on an object is a linear restoring force (ie a mass on a spring, or a pendulum with small oscillations), then the position as a function of time is related to cosine: $x = A\cos(\omega t + \phi_0)$



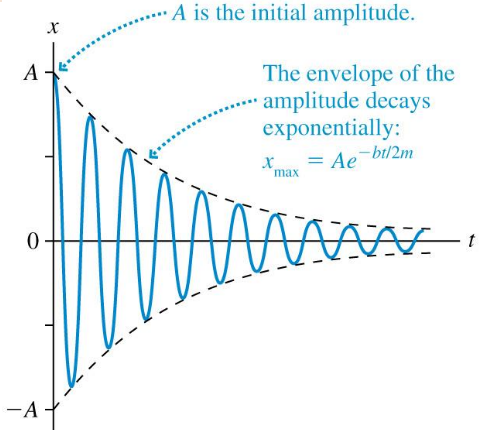
- Cosine is a function that goes forever, but in real life, due to friction or drag, all oscillations eventually slow down.



Damped Oscillations

When a mass on a spring experiences the force of the spring as given by Hooke's Law, as well as a drag force of magnitude $|D|=bv$, the solution is

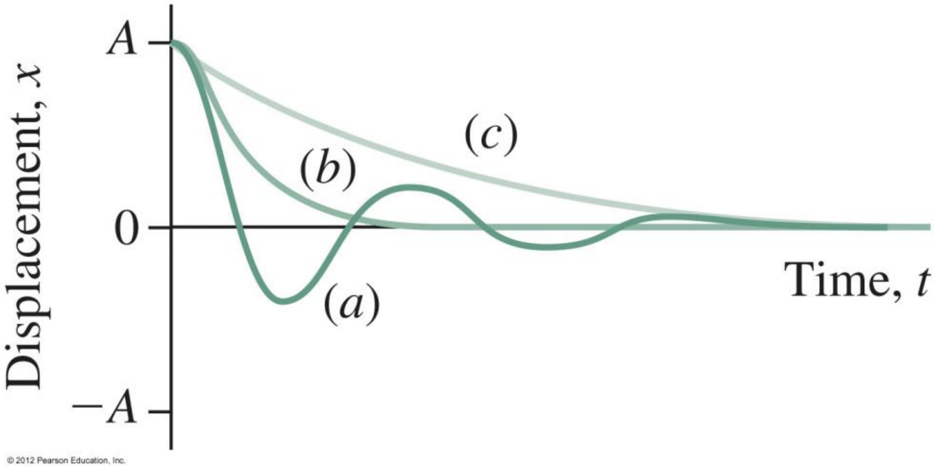
$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0) \qquad \text{(damped oscillator)}$$



Example 13.6.

A car's suspension acts like a mass-spring system with m = 1200 kg and $k = 5.8 \times 10^4$ N/m. Its shock absorbers provide a damping constant of b = 230 kg/s. After the car hits a bump in the road, how many oscillations will it make before the amplitude drops to half its initial value?

Damped Harmonic Motion



(a) underdamped
(b) critically damped, and
(c) overdamped oscillations.

Driven Oscillations and Resonance

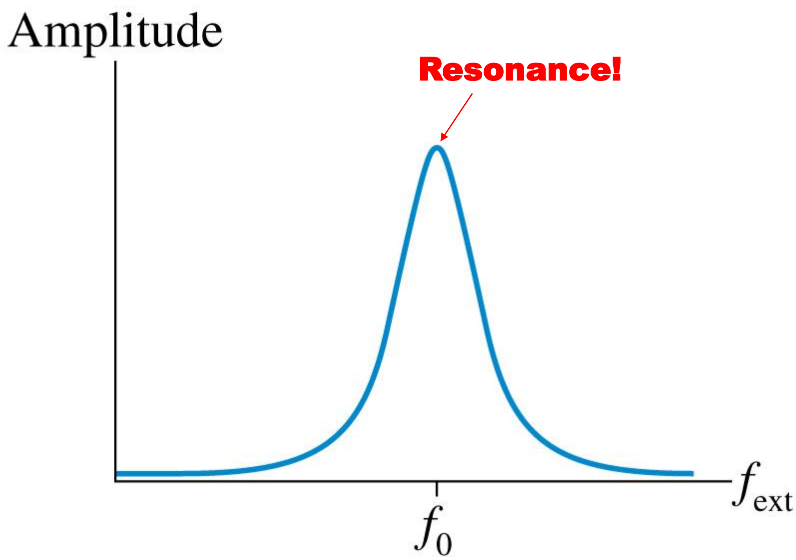
• Consider an oscillating system that, when left to itself, oscillates at a frequency f_0. We call this the **natural frequency** of the oscillator.

• Suppose that this system is subjected to a *periodic* external force of frequency f_{ext}. This frequency is called the **driving frequency**. Driven systems oscillate at f_{ext}.

• The amplitude of oscillations is generally not very high if f_{ext} differs much from f_0.

• As f_{ext} gets closer and closer to f_0, the amplitude of the oscillation rises dramatically.

14.8 Externally Driven Oscillations



Before Class 22 on Monday

- If you haven't done it, please check your utoronto email, respond to the course_evaluations email and evaluate this course!
- Please start reading Chapter 14 on Waves and/or watch the Preclass 22 video. Note that we will only cover the first seven sections of chapter 14 for this course (No Doppler shift)

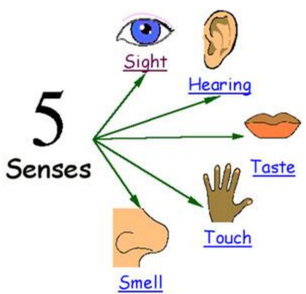


Image from http://freger.weebly.com/the-five-senses.html

- Something to think about over the weekend: Two of the five senses depend on **waves** in order to work: which two?