PHY131H1F - Class 21

Today:

- **Today, finishing Chapter 13:**
- Simple Pendulum
- Circular motion and S.H.M.
- Energy in S.H.M.

in depth

- Damped Harmonic Motion
- Driven Oscillations and Resonance.

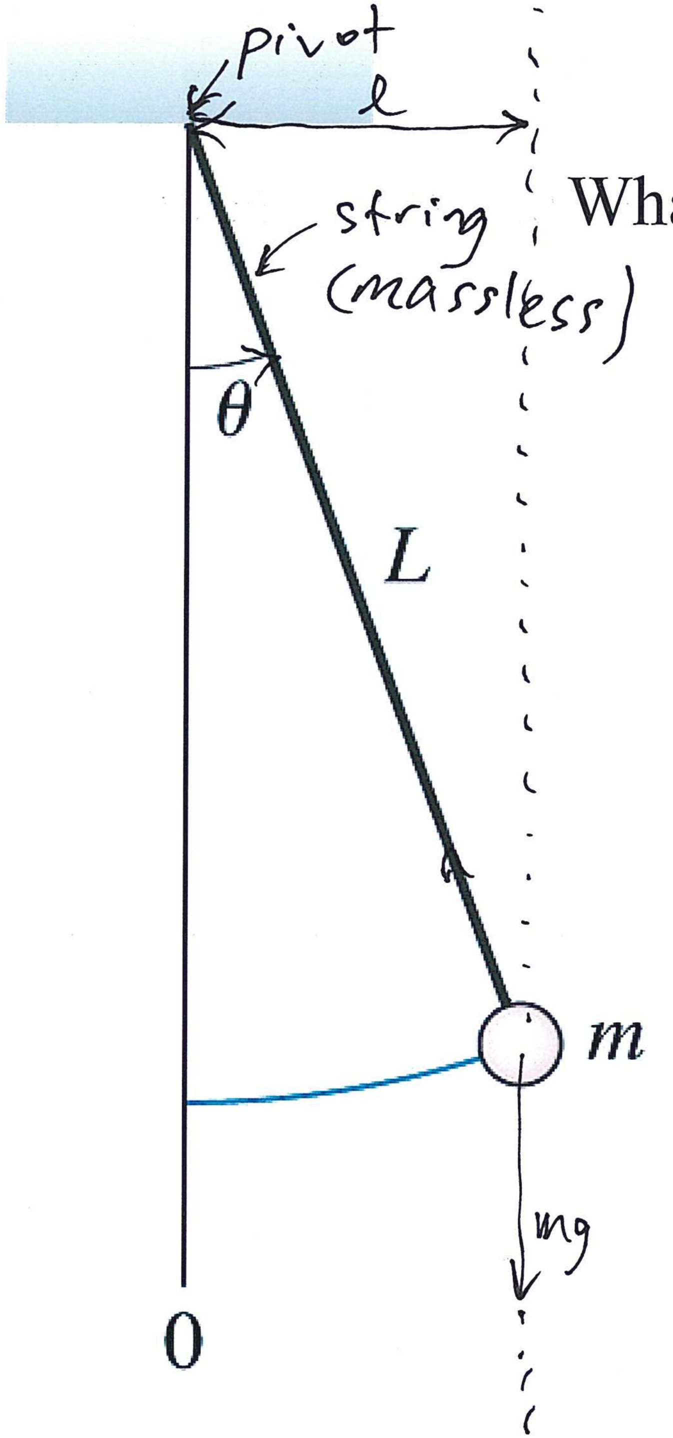
"light touch"

From http://www.cavatoyota.com/blog/what-are-shock-absorbers/ :
To test your vehicle's shock absorbers, simply push down on the each corner of the vehicle and observe its bounce. The vehicle should bounce up and return to its center resting position. If it continues to bounce, the shock absorber **should be replaced**.

Bonus Point for Over 65% Course Evaluation Response Rate

- The end of the evaluation period for this semester is Thu. Dec. 7: 8 more days!
- If, by the end of the course evaluation period, at least 65% of the students enrolled in this course have completed the course evaluations, then **every student in the course will have 1% bonus added to their final course mark.**
- Results so far (as of 9:06am today):

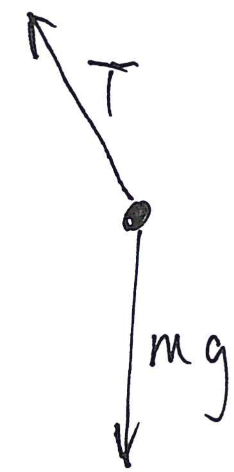
	Invited	Percentage	Responded	
L0101	558	57%	318	
L5101	213	53%	113	
	771		431	55.9%
			Need	9.1%
				70 students!



What is the net torque on this pendulum?

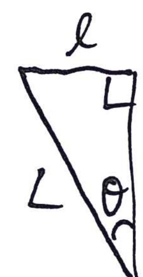
object is a mass on a string.

f.b.d. of m:



τ of tension = 0 since force line passes through pivot (zero lever arm)

Net torque = $\tau_{gravity}$.



$\sin\theta = l/L$

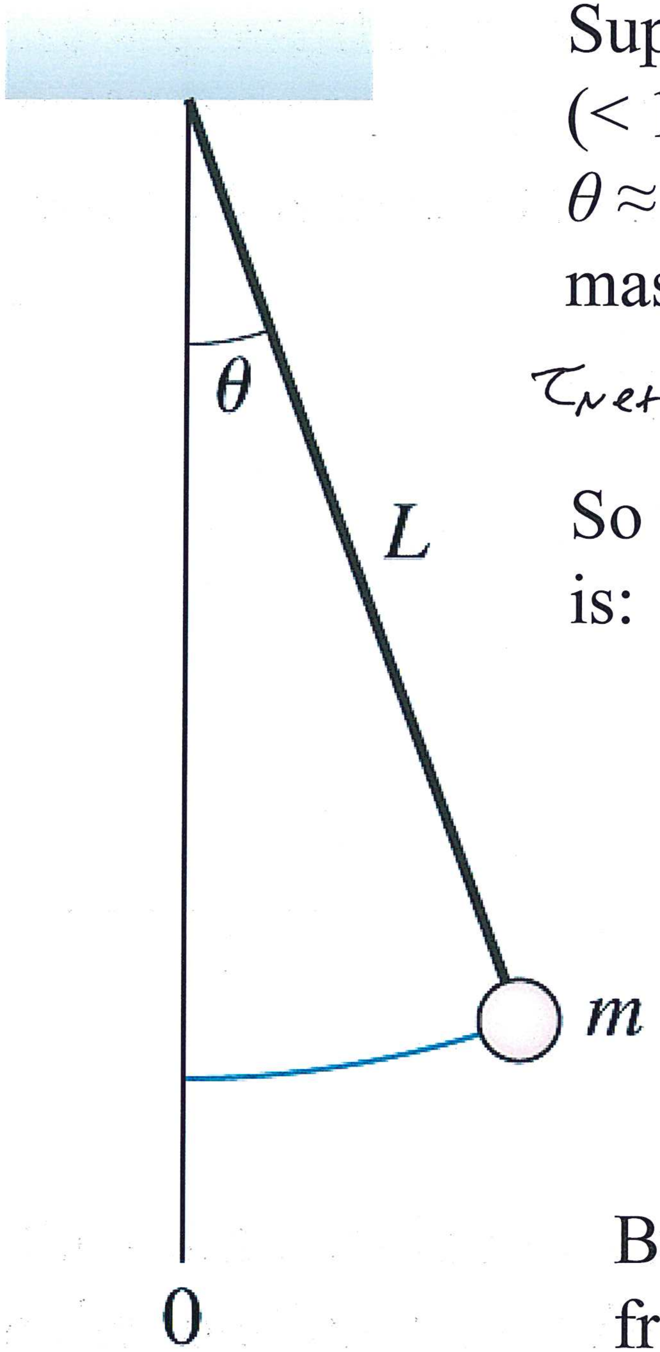
$|\tau_g| = l\,mg = mgL\sin\theta$

dir?

→ clockwise.

→ negative

$$\boxed{\tau_{net} = -mgL\sin\theta}$$



Suppose we restrict a pendulum's oscillations to small angles ($< 10°$). Then we may use the **small angle approximation** $\sin\theta \approx \theta$, where θ is measured in radians. The net torque on the mass is

$\tau_{net} = I\alpha$

$$\Sigma\tau = I\frac{d^2\theta}{dt^2} \approx -mgL\theta$$

So the simple harmonic motion equation for θ as a function of time is:

$$\frac{d^2\theta}{dt^2} = -\underbrace{\frac{mgL}{I}}_{\text{Constant}}\theta$$

The solution to this is $\theta = A\cos(\omega t + \phi_0)$, where A and ϕ_0 are arbitrary, and the **frequency** of oscillations (in rad/s) is:

$$\text{Constant} \rightarrow \omega = \sqrt{\frac{mgL}{I}} = \sqrt{\frac{\cancel{m}g\cancel{L}}{\cancel{m}L^{\cancel{2}}}} = \sqrt{g/L}$$

But the rotational inertia of a point mass m a distance L from the rotation axis is $I = mL^2$, so

$$\boxed{\omega = \sqrt{\frac{g}{L}}}$$

Preclass 21 Quiz due this morning

1. multiple choice

A frictionless pendulum clock on the surface of the earth has a period of 1.00 s. On a distant planet, the length of the pendulum must be shortened slightly to have a period of 1.00 s. What is true about the acceleration due to gravity on the distant planet?

A. The gravitational acceleration on the planet is slightly greater than g.

B. The gravitational acceleration on the planet is slightly less than g.

C. The gravitational acceleration on the planet is equal to g.

$\omega = \frac{2\pi}{T}$ $\quad$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

L, g both decrease,

L/g = same

Learning Catalytics Question 1

Two pendula have the same length, but different mass. The force of gravity, $F=mg$, is larger for the larger mass. Which will have the longer period?

A. the larger mass

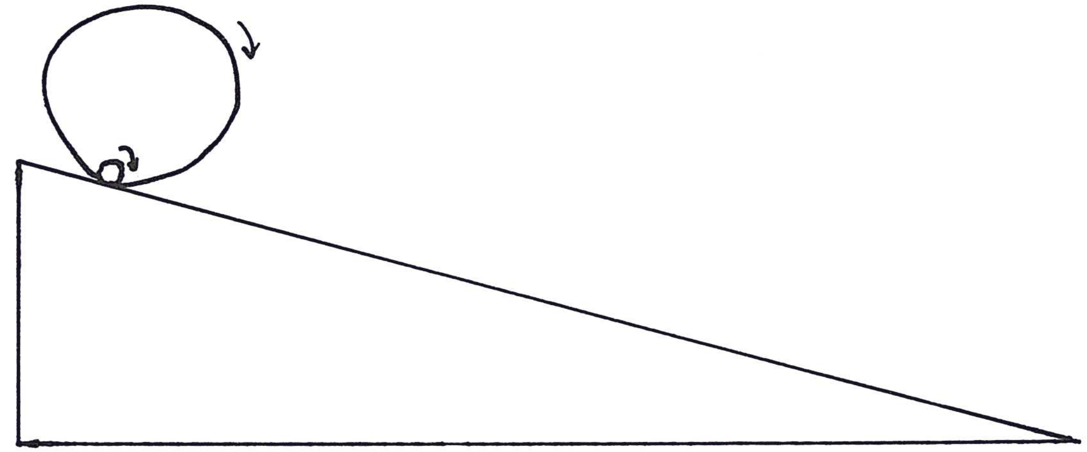
B. the smaller mass

C. neither

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$g = 9.8$

Review Quiz



A marble and a bowling ball are released from rest at the top of an incline. They roll without slipping. Which reaches the bottom first?

Table 10.2 (on 1st page of final exam)
Solid Sphere: $I = \frac{2}{5} mR^2$

A. The marble.

B. The bowling ball.

C. Both will reach the bottom at about the same time.

$I = c\, mR^2$

Preclass 21 Quiz due this morning

2. multiple choice

A simple pendulum swings back and forth with a period of 2.20 seconds. What is the mass of the swinging bob at the end of the pendulum?

A. 2.85 kg

B. 1.66 kg

C. 0.454 kg

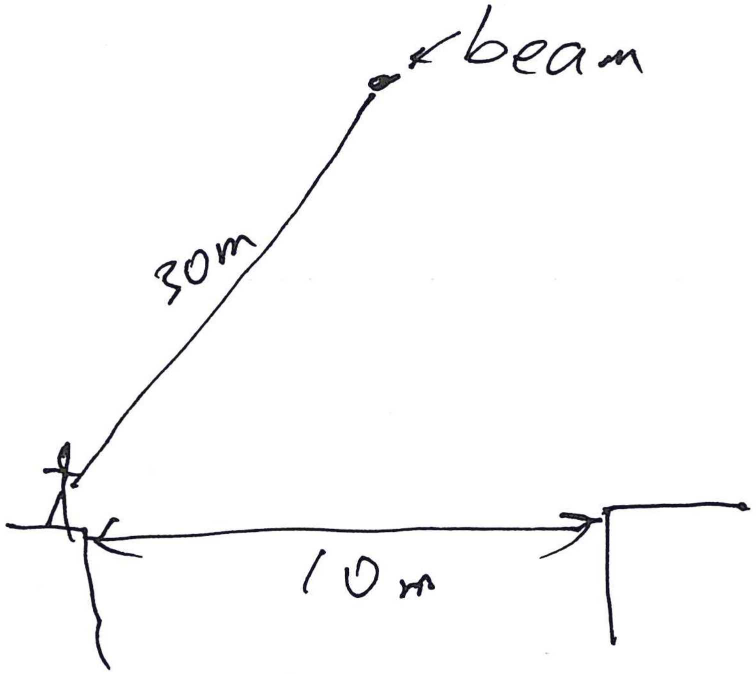
D. 0.150 kg

E. It cannot be determined from the information given.

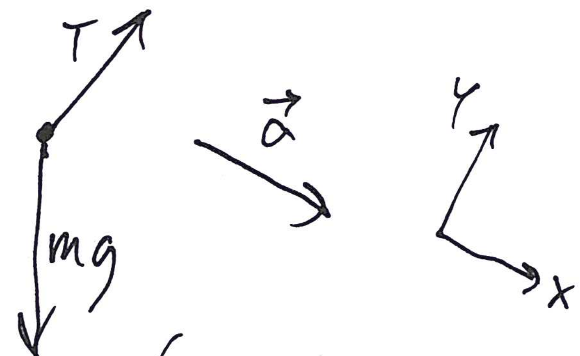
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Example.

Luke and Leia have a combined mass of 120 kg and both grasp a rope of length 30 m that is attached to a beam above them. The beam is half-way across a 10 m horizontal gap, and they want to swing across. If they start from rest and swing down and up, just reaching the other side, how long does this take?



f.bd. at start: (after jumping → no normal



$(F_{Net})_y = 0$

$(F_{Net})_x = mg \sin\theta$

Not a constant force.

No easy solution → not constant acceleration

CRUDE APPROXIMATION.

→ simple pendulum, $\sin\theta \approx \theta$

S.H.M. oscillation

frequency

$$\omega = \frac{2\pi}{T} = \sqrt{g/L}$$

$$T = 2\pi\sqrt{L/g}$$

T = time to go twice across gap.

$\frac{T}{2}$ is time to cross once.

$$t = \frac{T}{2} = \pi\sqrt{L/g} = 3.14\sqrt{\frac{30}{9.8}}$$

$$\boxed{t = 5.5\ s}$$

Mass on Spring versus Pendulum

	Mass on a Spring	Pendulum
Condition for S.H.M.	Small oscillations (Hooke's Law is obeyed)	Small angles
Angular frequency	$\omega = \sqrt{\dfrac{k}{m}}$	$\omega = \sqrt{\dfrac{g}{L}}$
Period	$T = 2\pi\sqrt{\dfrac{m}{k}}$	$T = 2\pi\sqrt{\dfrac{L}{g}}$

Learning Catalytics Question ~~2~~ 3

A person swings on a swing. When the person sits ~~still~~, the swing oscillates back and forth at its natural frequency. If, instead, the person stands on the swing, the natural frequency of the swing is

$$\omega = \sqrt{g/L}$$

A. greater

B. the same

C. smaller

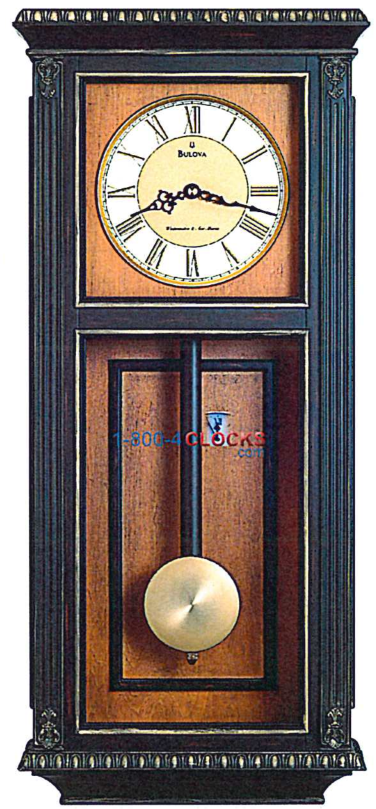
standing reduces effective distance between centre of mass & pivot.

$L \downarrow \quad \omega \uparrow$

Learning Catalytics Question [redacted] 4

A grandfather clock at high altitudes runs

A. fast.
B. slow.
C. normally as it does at sea level.

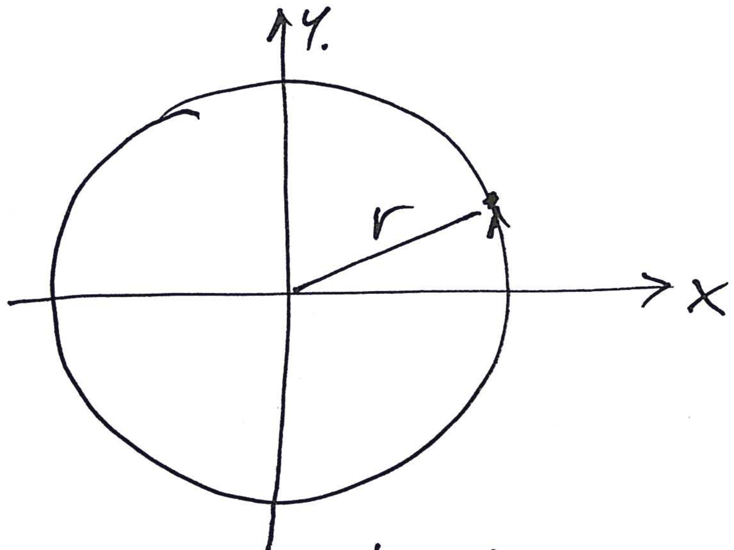


$g \downarrow$

$$T = 2\pi\sqrt{\frac{L}{g}}, \quad T\uparrow$$

Image from https://www.1-800-4clocks.com/Bulova-Vickery-Wall-Chimes-Clock_C4329_CUV

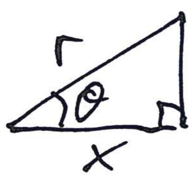
Uniform Circular Motion



ω = angular velocity = constant.

$v_t = \omega r$

what is x?



$\cos\theta = \frac{x}{r}$

$x = r\cos\theta$

$x = r\cos(\omega t)$

↑ constant.

$\theta_f = \theta_i + \omega t$

$\theta_i = 0$

$\theta = \omega t$

Simple Harmonic Motion

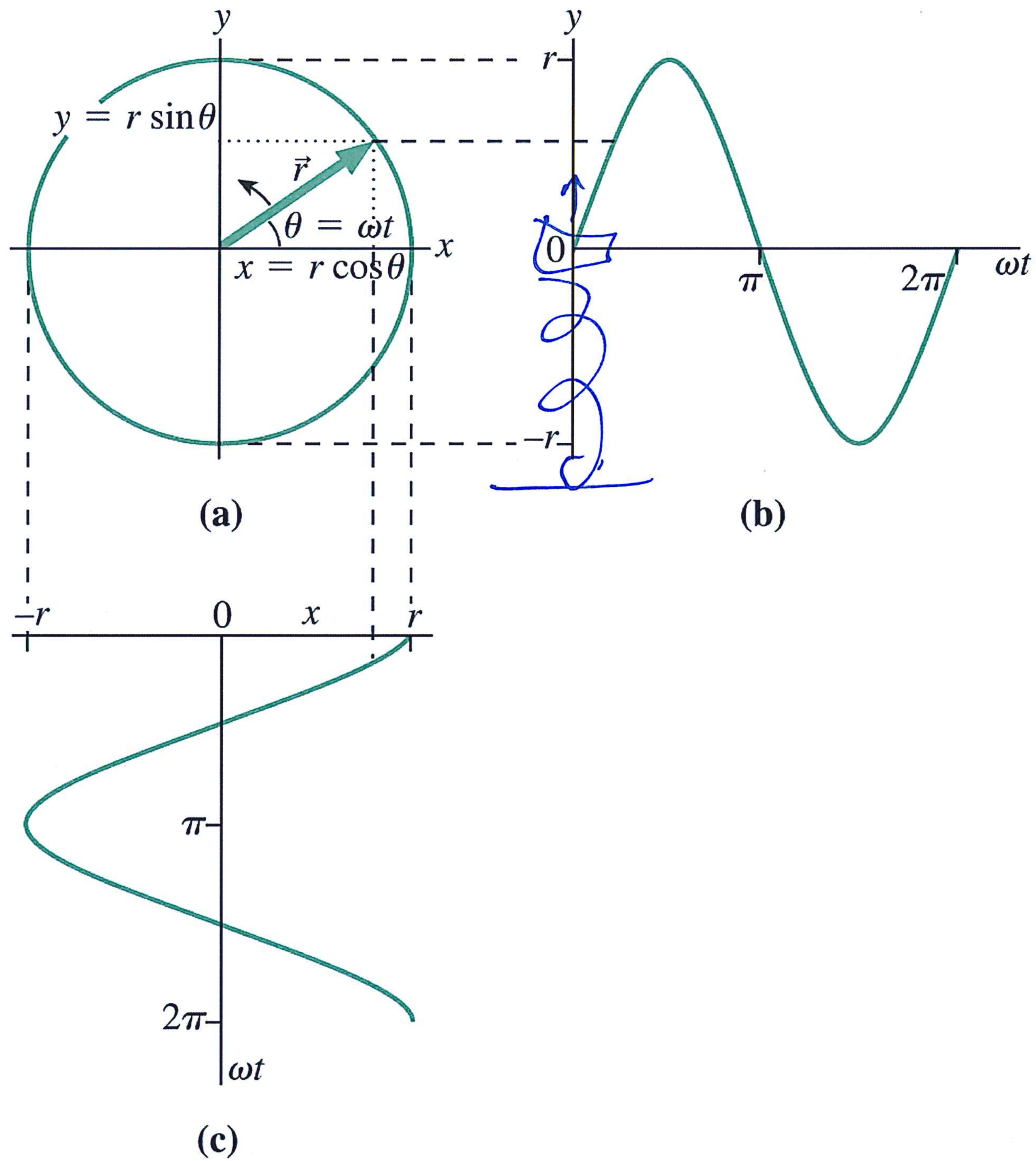
start: $a_x = -\omega^2 x$

Solution:

$x = A\cos(\omega t)$

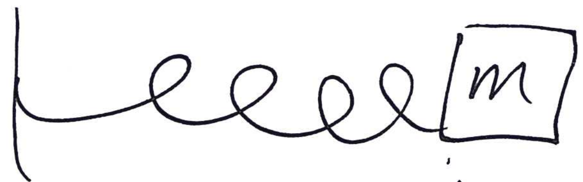
↑ constant.

Mathematically, S.H.M. is identical to one component of uniform circular motion.



y
$y = r\sin\theta$
$\vec{r}$
$\theta = \omega t$
x
$x = r\cos\theta$
(a)
y
r
0
−r
π
2π
ωt
(b)
−r
0
x
r
π
2π
ωt
(c)

Energy of a mass on a spring



$x=0$ = spring equilibrium.

position: $\boxed{x = A\cos(\omega t)}$

velocity: $\boxed{v_x = -A\omega\sin(\omega t)}$

Energy: $E = \frac{1}{2}m v_x^2 + \frac{1}{2}k x^2$

positive positive

$$E = \frac{1}{2}m\left[-A\omega\sin(\omega t)\right]^2 + \frac{1}{2}k\left[A\cos(\omega t)\right]^2$$

use: $\omega^2 = \frac{k}{m}$ &

$$E = \frac{1}{2}A^2\left[m\omega^2\sin^2(\omega t) + k\cos^2(\omega t)\right]$$

$$= \frac{1}{2}A^2 k \underbrace{\left[\sin^2(\omega t) + \cos^2(\omega t)\right]}_{=1}$$

$$\boxed{E_{tot} = \frac{1}{2}k A^2}$$

excercise for student.

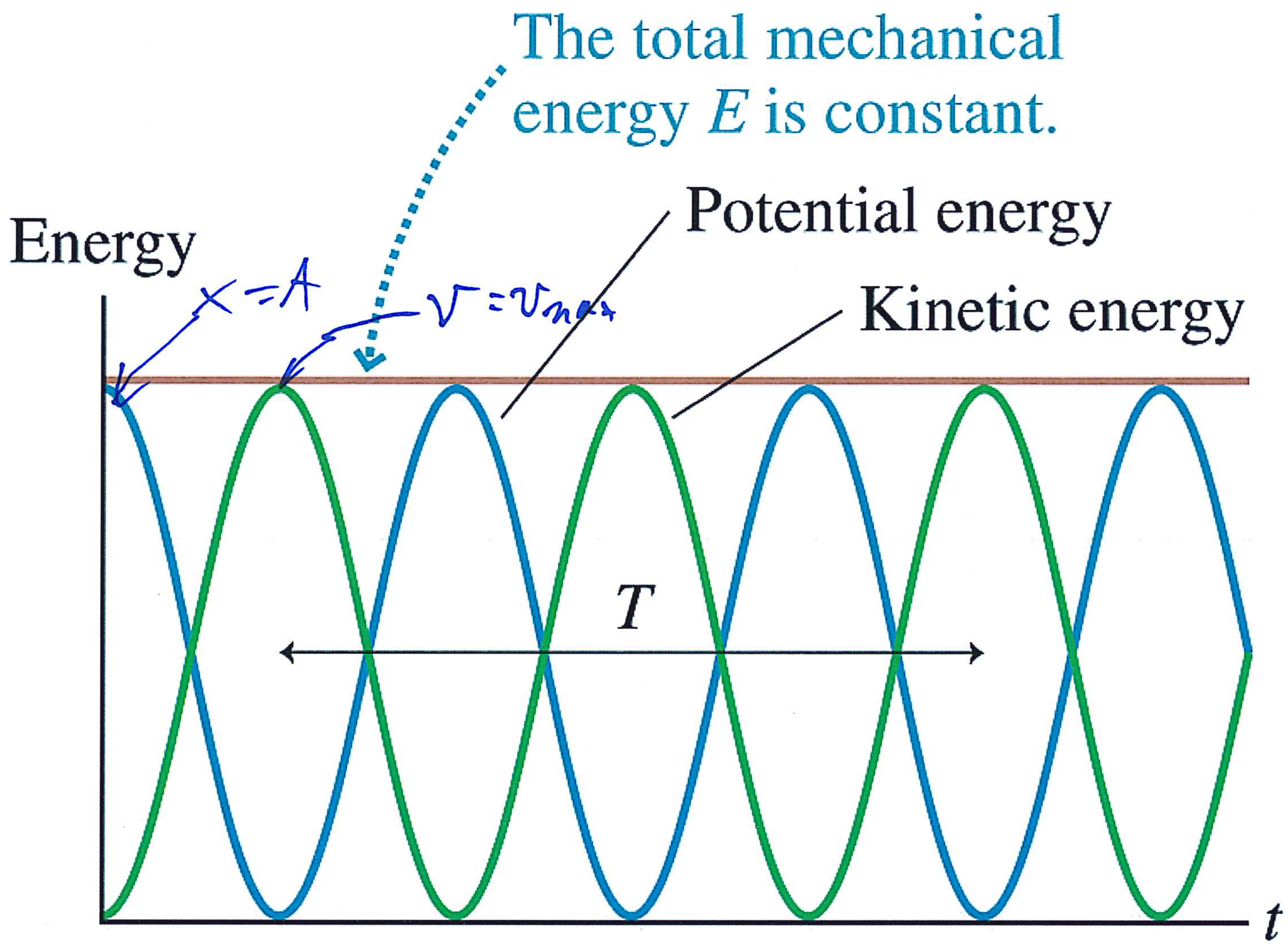
this *also* is

WRONG! (sorry).

$$\cancel{E_{tot} = \frac{1}{2}k(v_{max})^2}$$

$$\boxed{E_{tot} = \frac{1}{2}m(v_{max})^2}$$

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(v_{\max})^2 \quad \text{(conservation of energy)}$$



Preclass 21 Quiz due this morning

3. multiple choice

If we double only the amplitude of a vibrating ideal mass-and-spring system, the mechanical energy of the system

A. increases by a factor of $\sqrt{2}$.

B. increases by a factor of 2.

C. increases by a factor of 3.

D. increases by a factor of 4.

E. does not change.

$E = \frac{1}{2} k A^2$

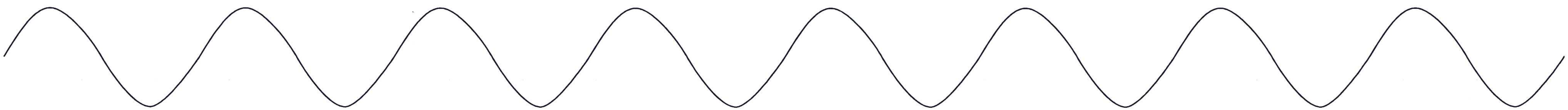
$A_1 = 2A$

$E_1 = 4E$

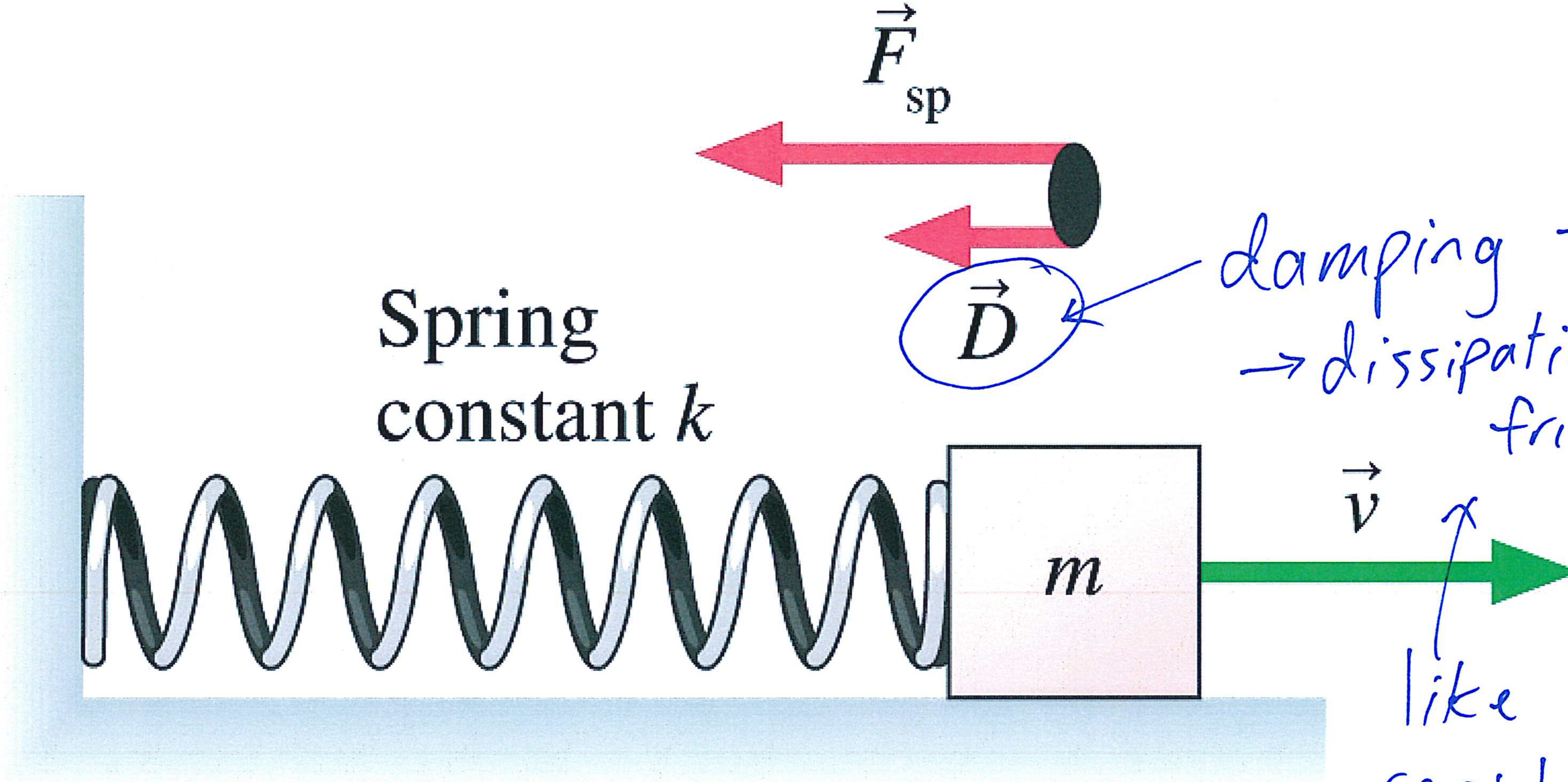
Simple Harmonic Motion (SHM)

- If the net force on an object is a linear restoring force (ie a mass on a spring, or a pendulum with small oscillations), then the position as a function of time is related to cosine:

$$x = A\cos(\omega t + \phi_0)$$



- Cosine is a function that goes forever, but in real life, due to friction or drag, all oscillations eventually slow down.



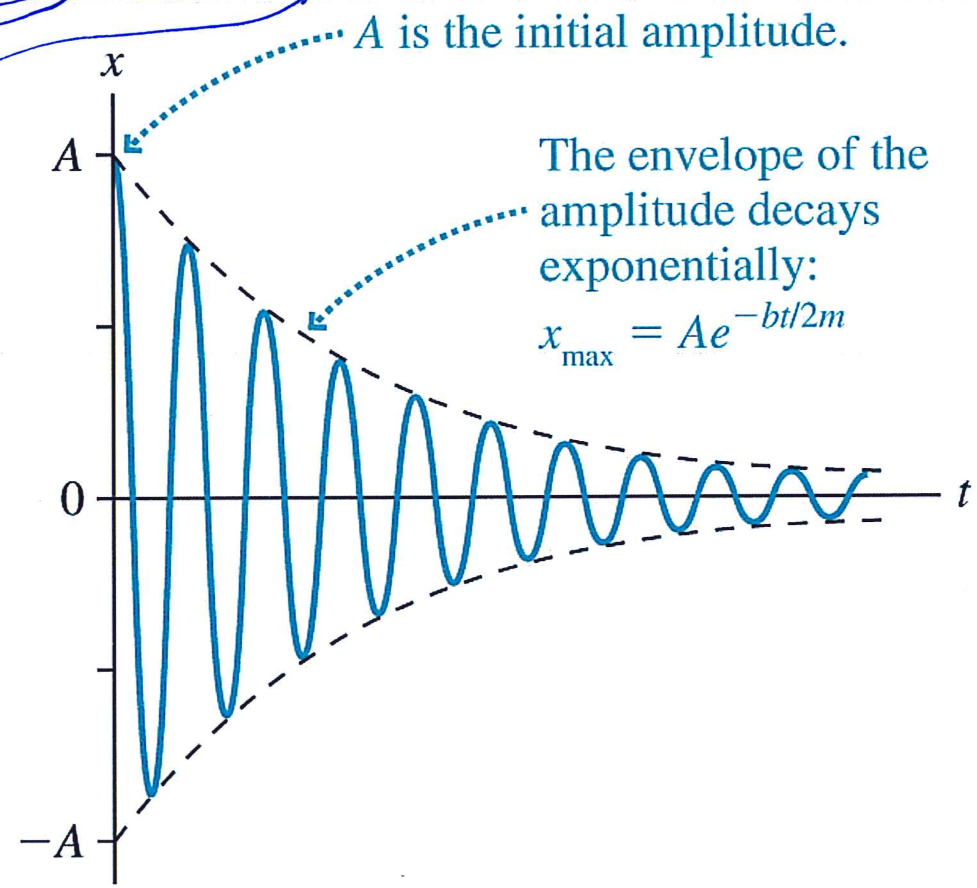
damping force
→ dissipative
frictional
force.

like air
resistance.

Damped Oscillations

When a mass on a spring experiences the force of the spring as given by Hooke's Law, as well as a drag force of magnitude $|D|=bv$, the solution is

$$x(t) = \underbrace{Ae^{-bt/2m}}_{\text{Decaying amplitude}}\underbrace{\cos(\omega t + \phi_0)}_{\text{S.H.M.}} \qquad \text{(damped oscillator)}$$



Preclass 21 Comments

- *"Is b just a constant that is dependent on the system?"*
- **Harlow answer:** Yes. It depends on the thickness of the fluid, shape of the object, etc.

- *"What are the units of b for the damping force? If F=b*dx/dt, then is b in kg/s?"*
- **Harlow answer:** Yes!

- *"Just to clarify: the object's mass does not have an effect on the period of a simple pendulum?"*
- **Harlow answer:** Yes – usually when gravity is the driving force, the mass is not important for the motion.

- *"How do we remember all these equations??"*
- **Harlow answer:** Write them on your aid sheet!

- *"Are rigorous proofs ever used in physics?"*
- **Harlow answer:** Yes – it's called math! Theoretical physicists use and publish proofs all the time!

- *"Please please please add the practice feature in mastering physics assignments"*
- **Harlow answer:** I very much want to do this, and I thought I did! I need help from a student during office hours to look at two problem sets side by side and figure out why you can't practice for some.

- *"SHM is how I thought I was going to do my homework, damped oscillations model how I actually do it. [sometimes one can lie for the sake of seeing one's comment reach the lecture slides]"*

Driven Oscillations and Resonance

• Consider an oscillating system that, when left to itself, oscillates at a frequency f_0. We call this the **natural frequency** of the oscillator.

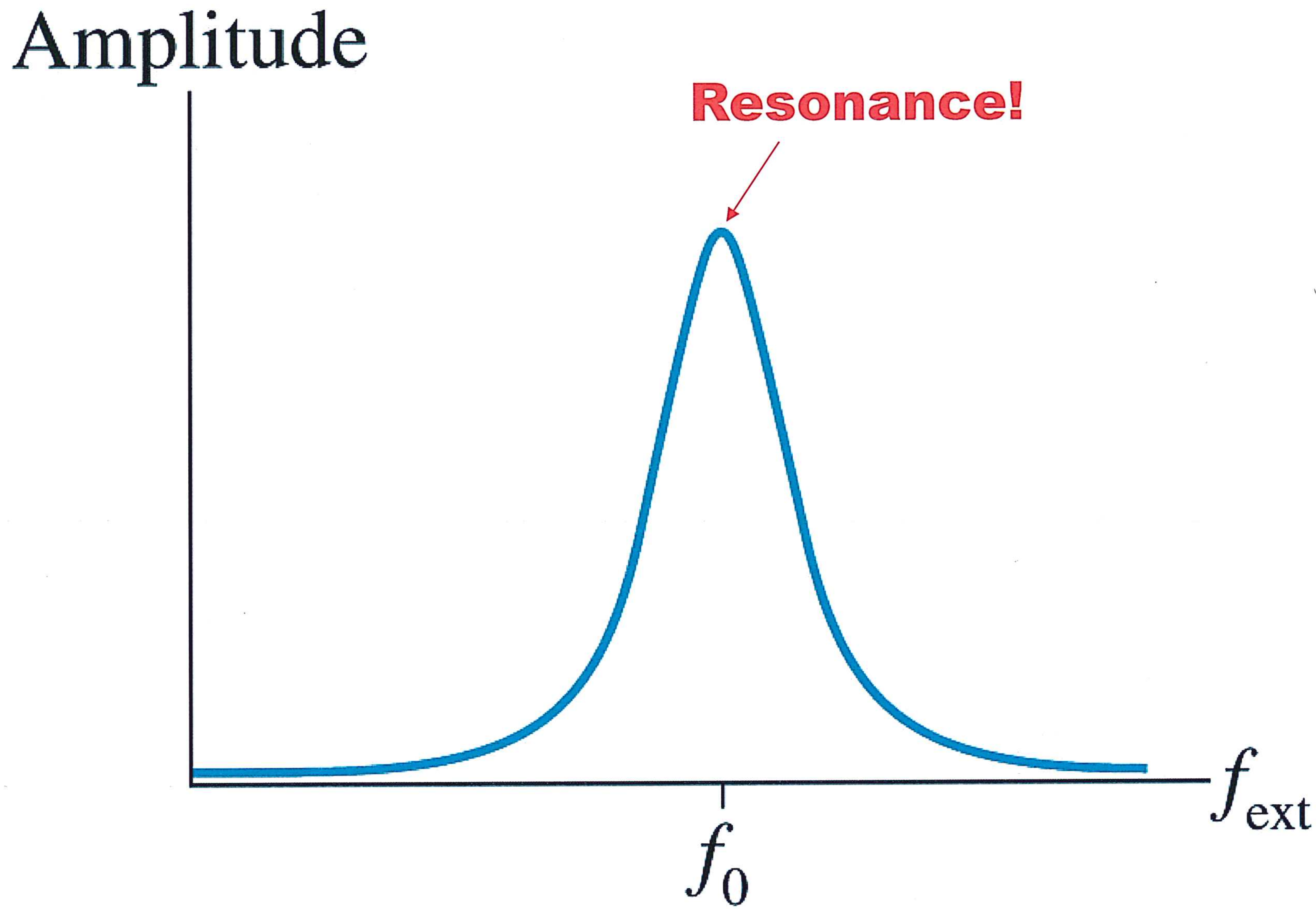
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

• Suppose that this system is subjected to a *periodic* external force of frequency f_{ext}. This frequency is called the **driving frequency**. Driven systems oscillate at f_{ext}.

• The amplitude of oscillations is generally not very high if f_{ext} differs much from f_0.

• As f_{ext} gets closer and closer to f_0, the amplitude of the oscillation rises dramatically.

14.8 Externally Driven Oscillations



Before Class 22 on Monday

- If you haven't done it, please check your utoronto email, respond to the course_evaluations email and evaluate this course!
- Please start reading Chapter 14 on Waves and/or watch the Preclass 22 video. Note that we will only cover the first seven sections of chapter 14 for this course (No Doppler shift)
- Something to think about over the weekend: Two of the five senses depend on **waves** in order to work: which two?

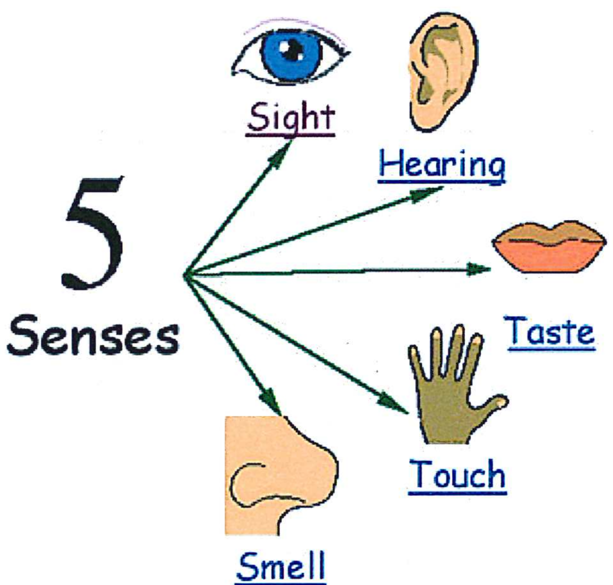


Image from http://freger.weebly.com/the-five-senses.html