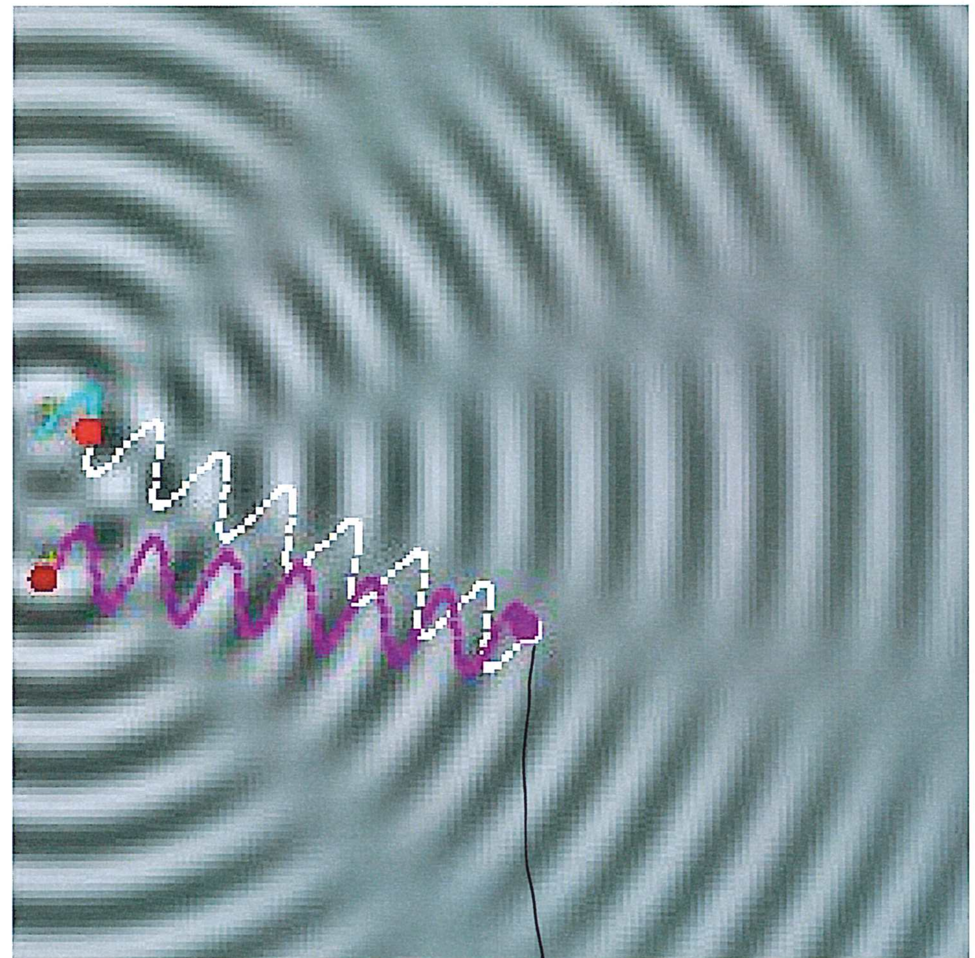
PHY131H1F - Class 23

"light touch"

Today:

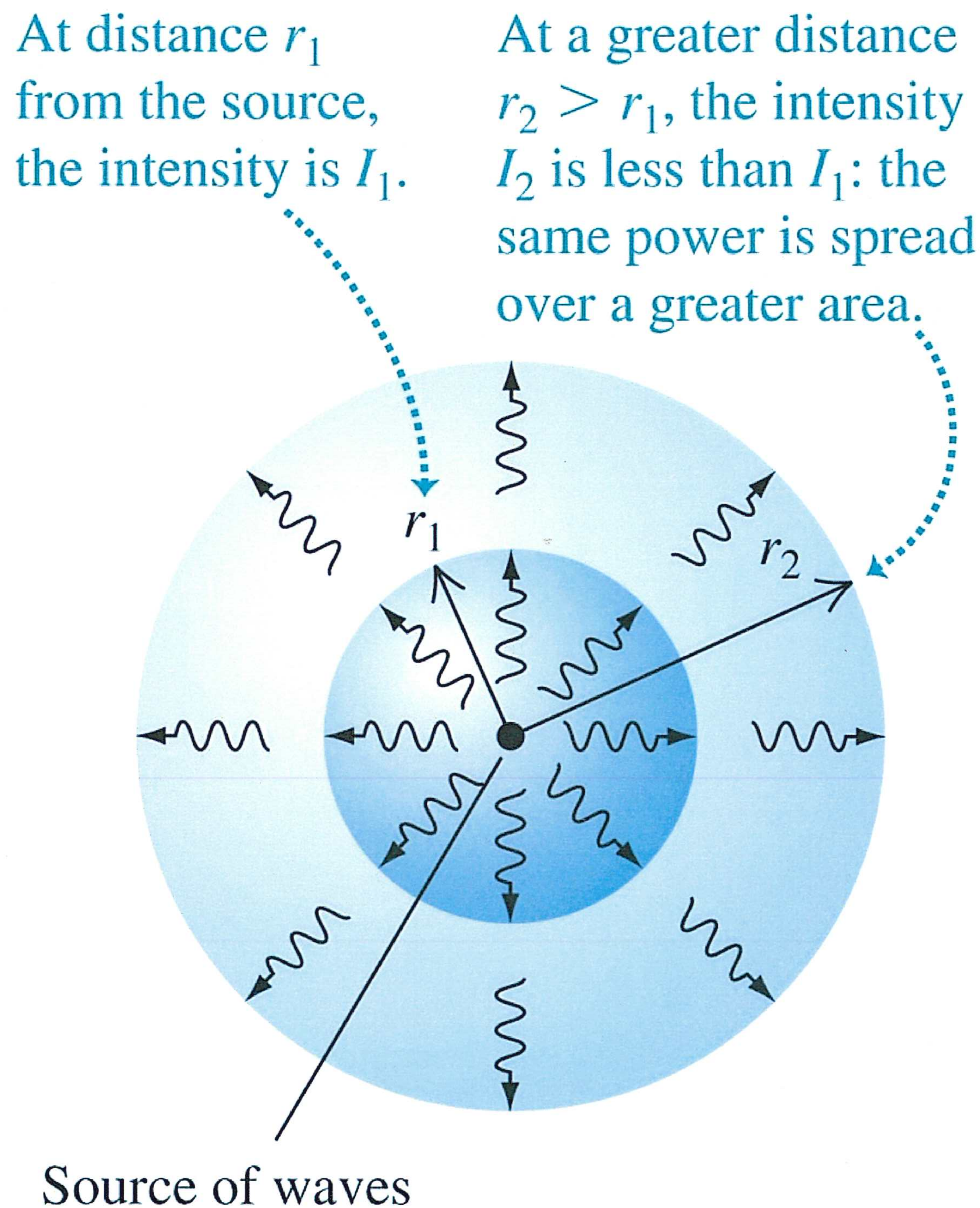
- Sound Wave Intensity and the decibel system
- Wave Interference: The Principle of Superposition
- Constructive and Destructive Interference
- Beats
- Reflection and Refraction
- Standing Waves
- Musical Instruments



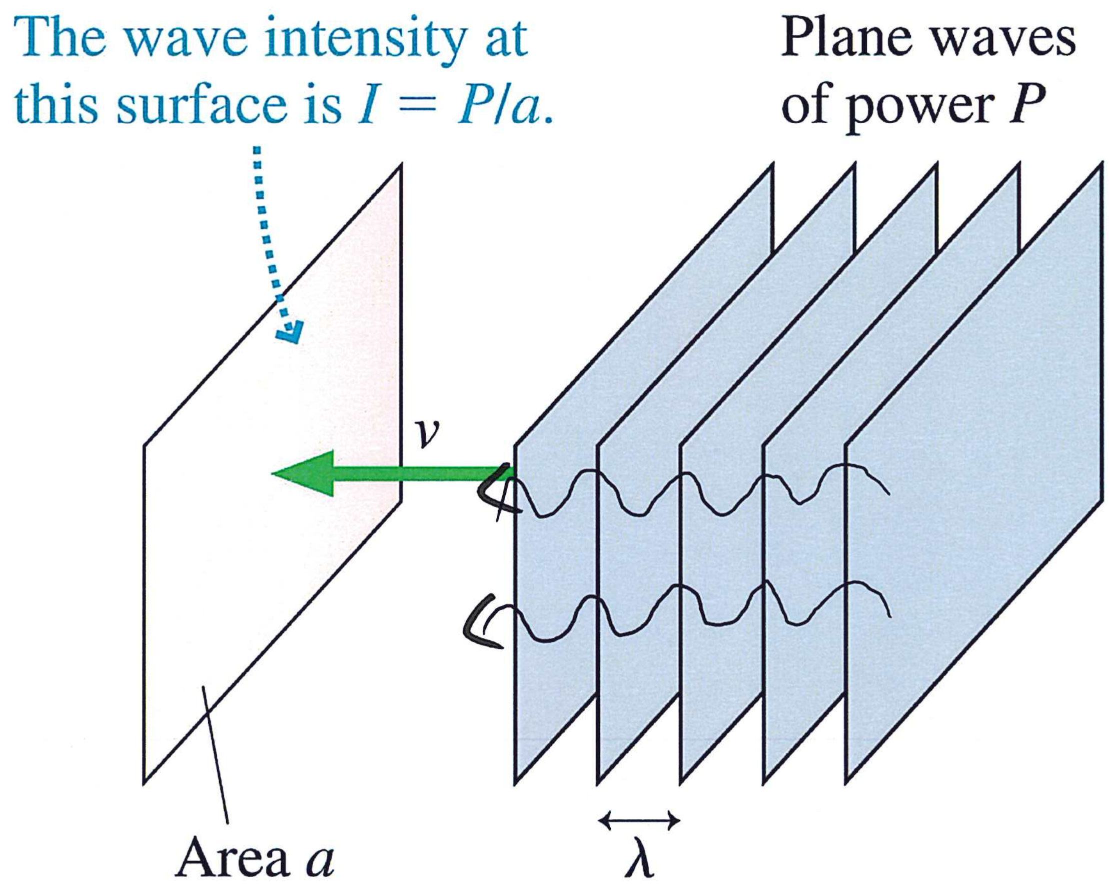
destructive interference

Wave intensity

- The *intensity* of a wave is the average power it carries per unit area.
- If the waves spread out uniformly in all directions and no energy is absorbed, the intensity I at any distance r from a wave source is inversely proportional to r^2.



Power and Intensity

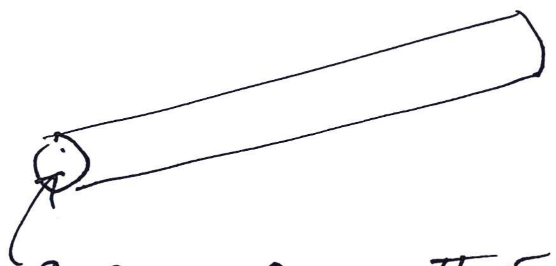


- When plane waves of power P impinge on area a, we define the **intensity** I to be:

$$I = \frac{P}{a} = \text{power-to-area ratio}$$

Example

A laser pointer emits 1.0 mW of light power into a 1.0 mm diameter laser beam. What is the intensity of the laser beam?



$$r = \frac{d}{2} = \frac{10^{-3}\,m}{2}$$

$$\text{area} = a = \pi r^2$$

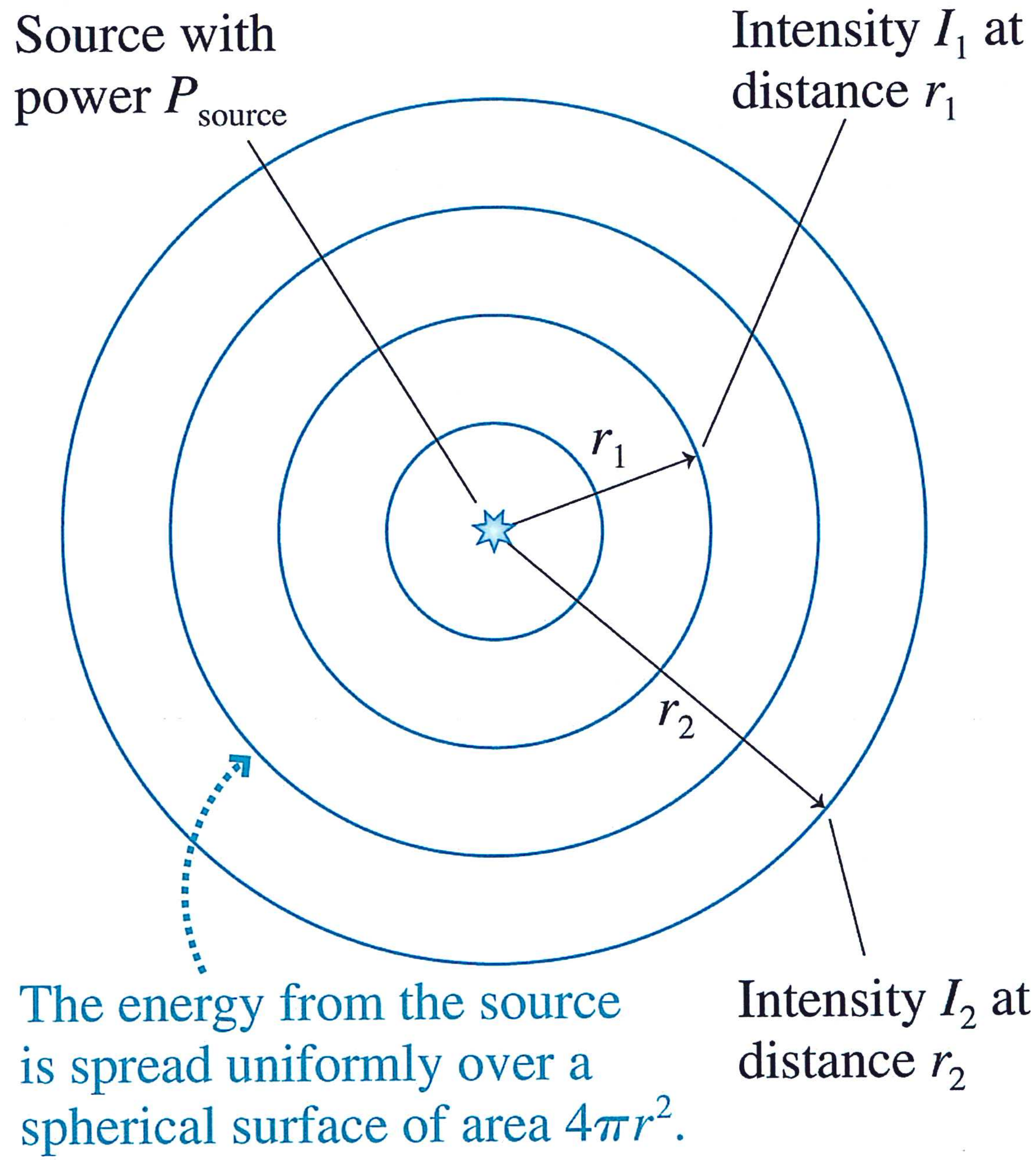
$$I = \frac{P}{a} = \frac{10^{-3}\ \text{Watts}}{\pi \left(10^{-3}/2\right)^2}$$

$$I = 1273\ W/m^2$$

Intensity of Spherical Waves

- If a source of spherical waves radiates uniformly in all directions, then the power at distance r is spread uniformly over the surface of a sphere of radius r.
- The intensity of a uniform spherical wave is:

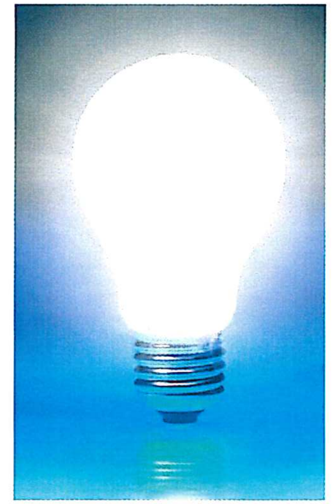
$$I = \frac{P_{\text{source}}}{4\pi r^2}$$



Example

A laser pointer has an intensity of 1273 W/m².

How far from a 100-Watt light bulb should your eye be in order to have the same intensity as shining this green laser directly into your eye?



Assume bulb has 100% efficiency → all 100 W goes into light.

$$I = \frac{P}{4\pi r^2}$$

Set
$I = 1273\ W/m^2$
$P = 100\ W$
solve for r.

$$r^2 = \frac{P}{4\pi I}$$

$$r = \sqrt{\frac{100\ W}{4\pi(1273)}} = 0.079\ m$$

→ 8 cm

Intensity and Decibels

- Human hearing spans an extremely wide range of intensities, from the *threshold of hearing* at $\approx 1 \times 10^{-12}\ \mathrm{W/m^2}$ (at midrange frequencies) to the *threshold of pain* at $\approx 10\ \mathrm{W/m^2}$.
- If we want to make a scale of loudness, it's convenient and logical to place the zero of our scale at the threshold of hearing.
- To do so, we define the **sound intensity level**, expressed in **decibels** (dB), as:

$$\beta = (10\ \mathrm{dB}) \log_{10}\left(\frac{I}{I_0}\right)$$

where $I_0 = 1 \times 10^{-12}\ \mathrm{W/m^2}$.

$10^{\log_{10} x} = x$

Solve for I:

$\frac{\beta}{10} = \log_{10}(I/I_0)$

$10^{\beta/10} = 10^{\log_{10}(I/I_0)} = \frac{I}{I_0}$

$I = I_0\, 10^{\beta/10}$

Sound Intensity Levels – Representative Values

Source	Sound Intensity Level, β (dB)	Intensity, I (W/m^2)
Military jet aircraft 30 m away	140	10^2
Threshold of pain	120	1
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Threshold of hearing at 1000 Hz	0	10^{-12}

Learning Catalytics Question

- A sound level of 10 decibels has 10 times more intensity than a sound level of zero decibels.
- A sound level of 20 decibels has ____ times more intensity than a sound level of zero decibels.

A. 10
B. 20
C. 50
D. 100
E. 200

Every additive step of +10 db in sound intensity level corresponds to a multiplicative step of ×10 in intensity.

Learning Catalytics Question

- When you turn up the volume on your ipod, the sound originally entering your ears at 50 decibels is boosted to 80 decibels. By what factor is the intensity of the sound has increased?

A. 1 (no increase)
B. 30
C. 100
D. 300
E. 1000

$80 - 50 = 30 \text{ dB}$

$\frac{30}{10} = 3$ "steps"

$10^3 = 1000$ times the intensity.

The Principle of Superposition

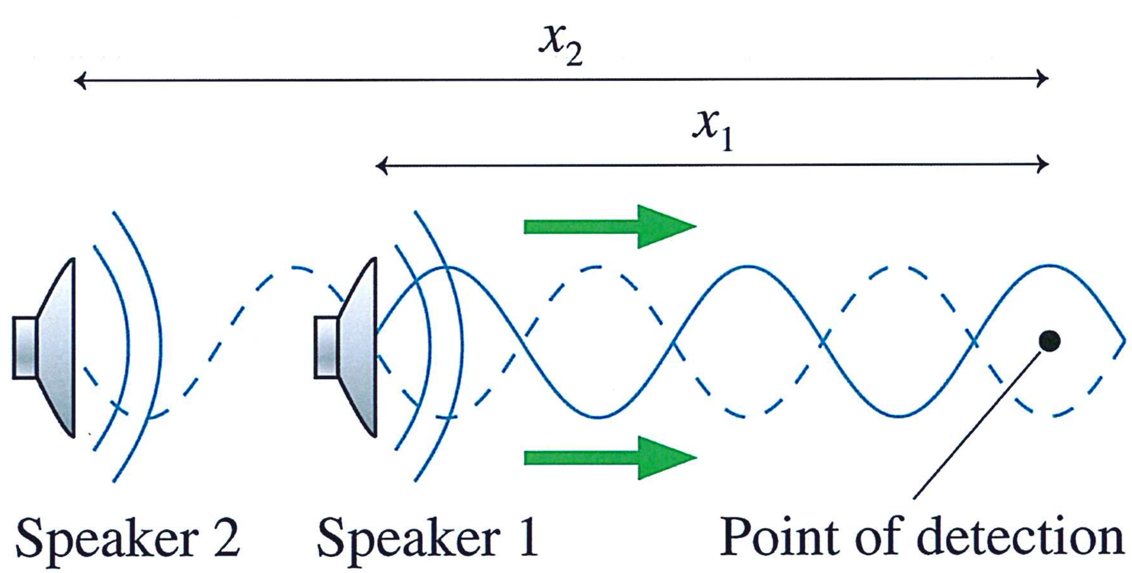
If two or more waves combine at a given point, the resulting disturbance is the *sum* of the disturbances of the individual waves.

$$y = y_1 + y_2$$

Wave Interference

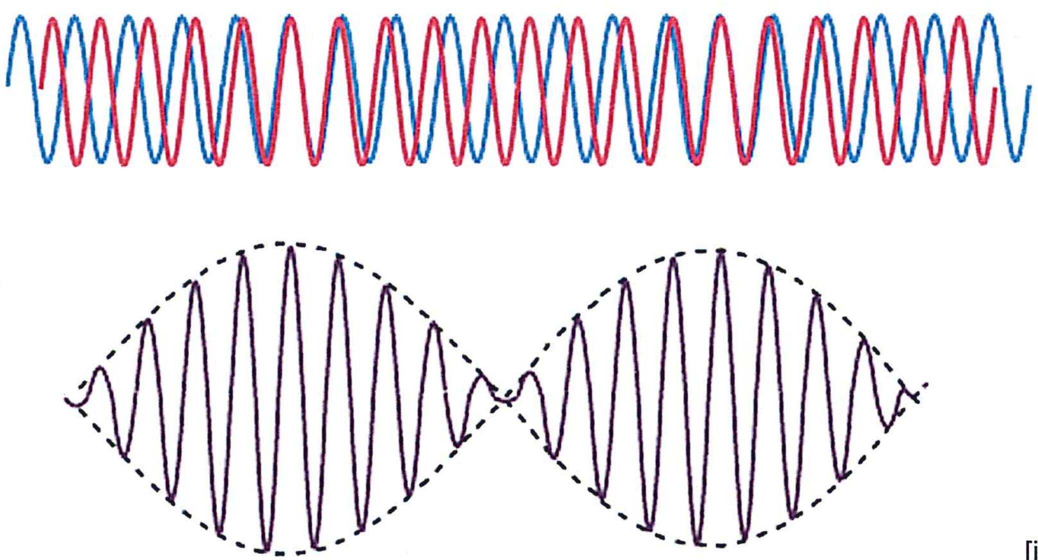
- The pattern resulting from the superposition of two waves is called interference. Interference can be
- ***constructive***, meaning the disturbances **add** to make a resultant wave of **larger** amplitude, or
- ***destructive***, meaning the disturbances **cancel**, making a resultant wave of **smaller** amplitude.

Two overlapped sound waves



Beats

- Periodic variations in the loudness of sound due to interference
- Occur when two waves of similar, but not equal frequencies are superposed.
- Provide a comparison of frequencies
- Frequency of beats is equal to the **difference** between the frequencies of the two waves.

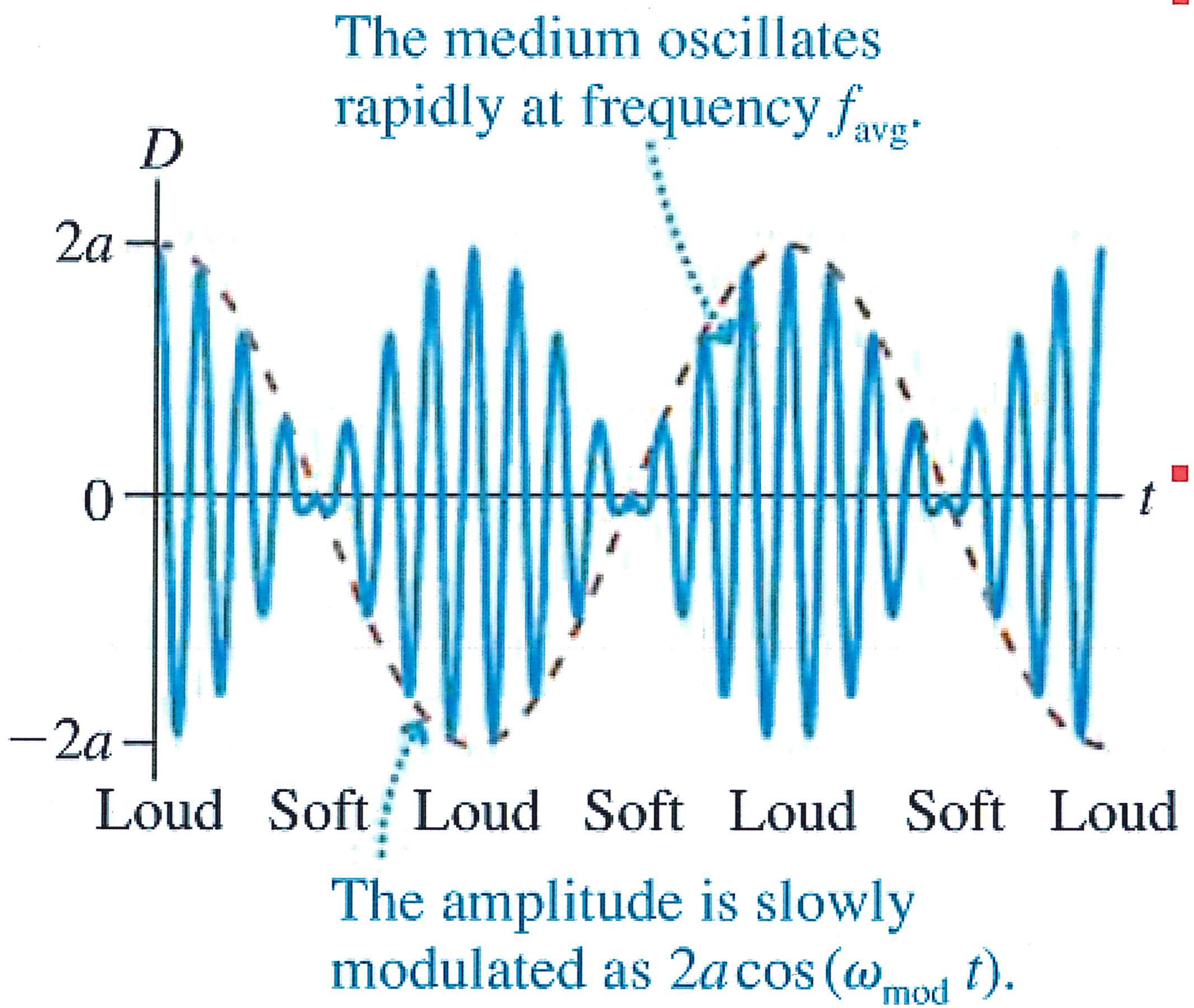


[image from http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html]

Beats

- Applications
  - Piano tuning by listening to the disappearance of beats from a known frequency and a piano key
  - Tuning instruments in an orchestra by listening for beats between instruments and piano tone

Beats



- The amplitude is slowly modulated with a frequency $f_{mod} = (f_1 - f_2)/2$ (red-dashed line)
- Beats are heard at $f_{beat} = 2f_{mod} = f_1 - f_2$

$f_{beat} = |f_1 - f_2|$

Preclass question from this morning

$f_{beat} = |f_1 - f_2|$

The tension in each of two strings is adjusted so that both vibrate at exactly 666 Hz. The tension in one of the strings (2) is then increased slightly. As a result, six beats per second are heard when both strings vibrate. What is the new frequency of the string that was tightened?

$f_{beat} = 6$ Hz $= 6$ beats per second

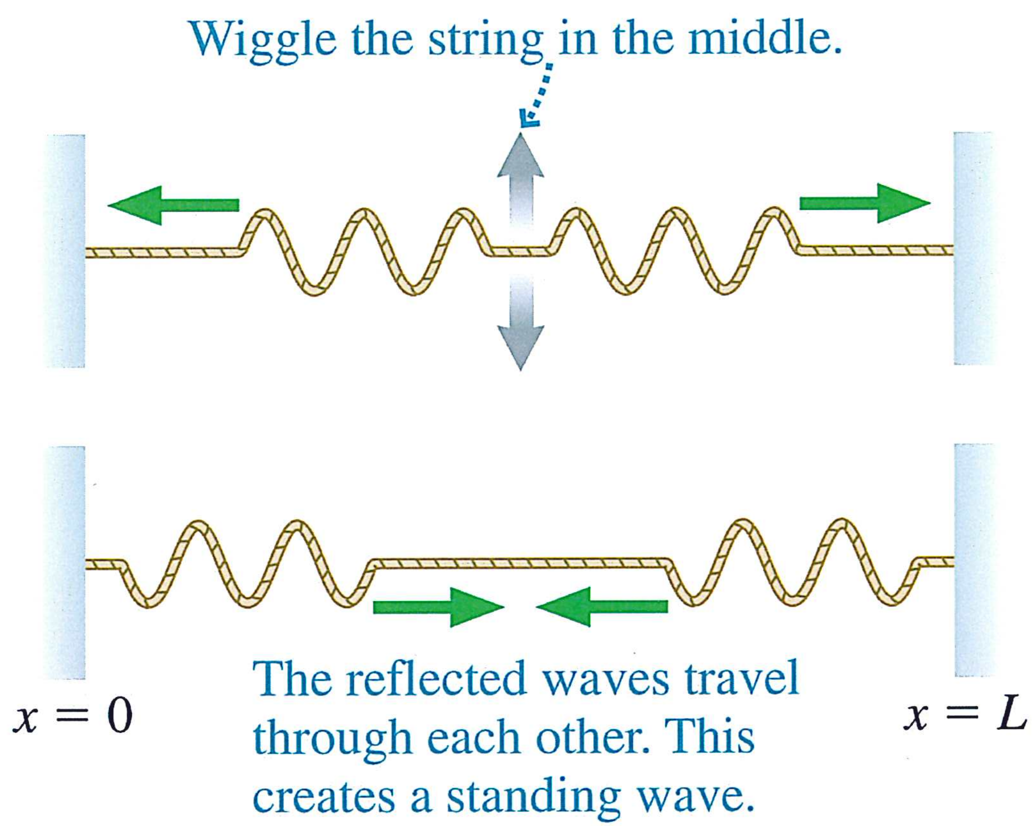
$6 = |f_1 - f_2|$ $f_1 = 666$ Hz

$f_2 = 660$ Hz or 672 Hz

For standing wave, f is prop. to v_{wave}.

$v_{wave} \uparrow$, $f \uparrow$ $v_{wave} = \sqrt{F/\mu}$

Standing Waves on a String



Reflections at the ends of the string cause waves of *equal amplitude and wavelength* to travel in opposite directions along the string, which results in a standing wave.

The Mathematics of Standing Waves

According to the principle of superposition, the net displacement of a medium when waves with displacements D_R and D_L are present is

$$y(x,t) = y_R + y_L = a\cos(kx - \omega t) + a\cos(kx + \omega t)$$

We can simplify this by using a trigonometric identity, and arrive at:

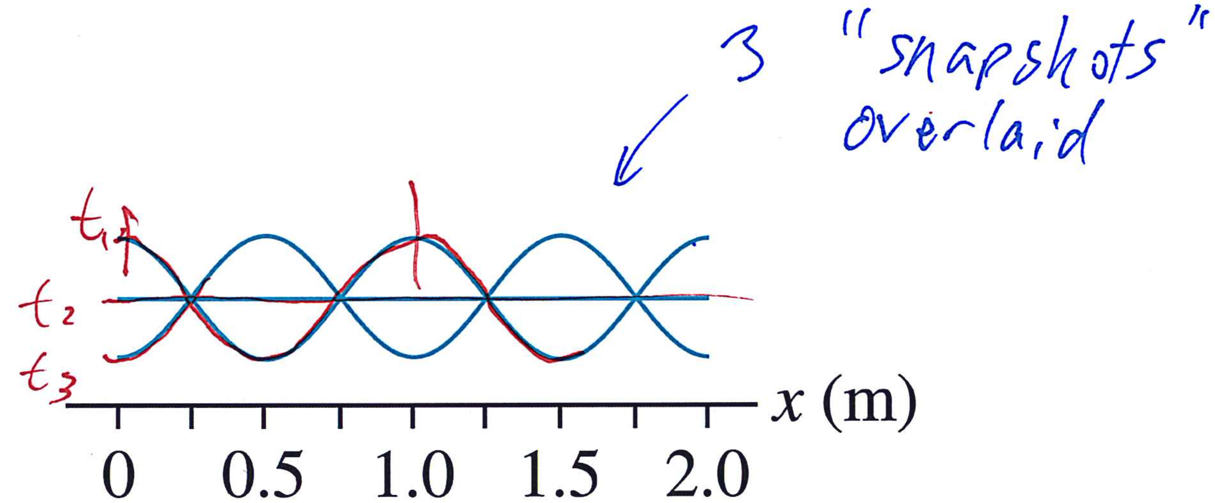
$$y(x,t) = A(x)\sin(\omega t)$$

where $$A(x) = 2a\sin(kx)$$

For a standing wave, the pattern is not propagating!

Learning Catalytics Question

What is the wavelength of this standing wave?



A. 0.25 m.

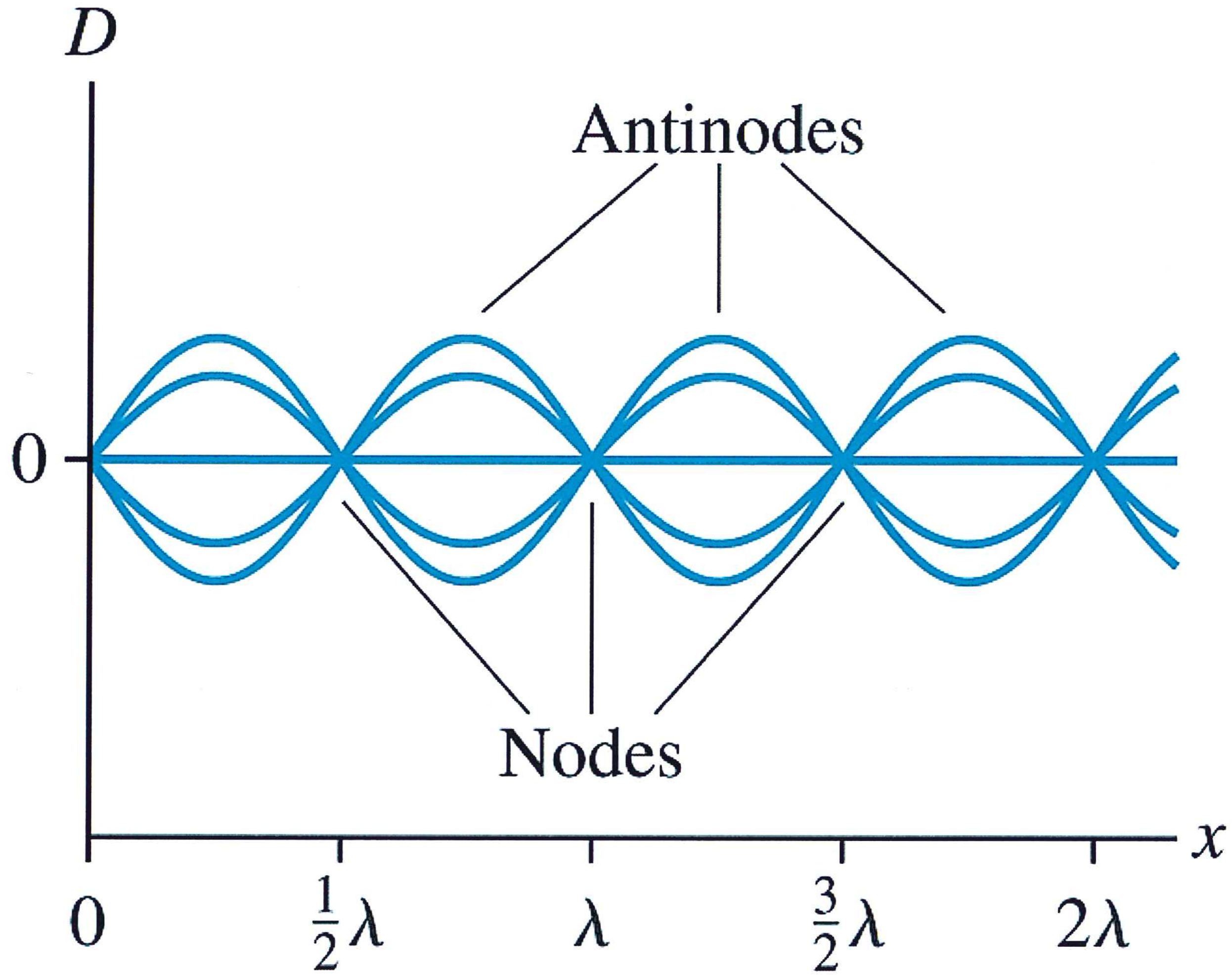
B. 0.5 m.

C. 1.0 m.

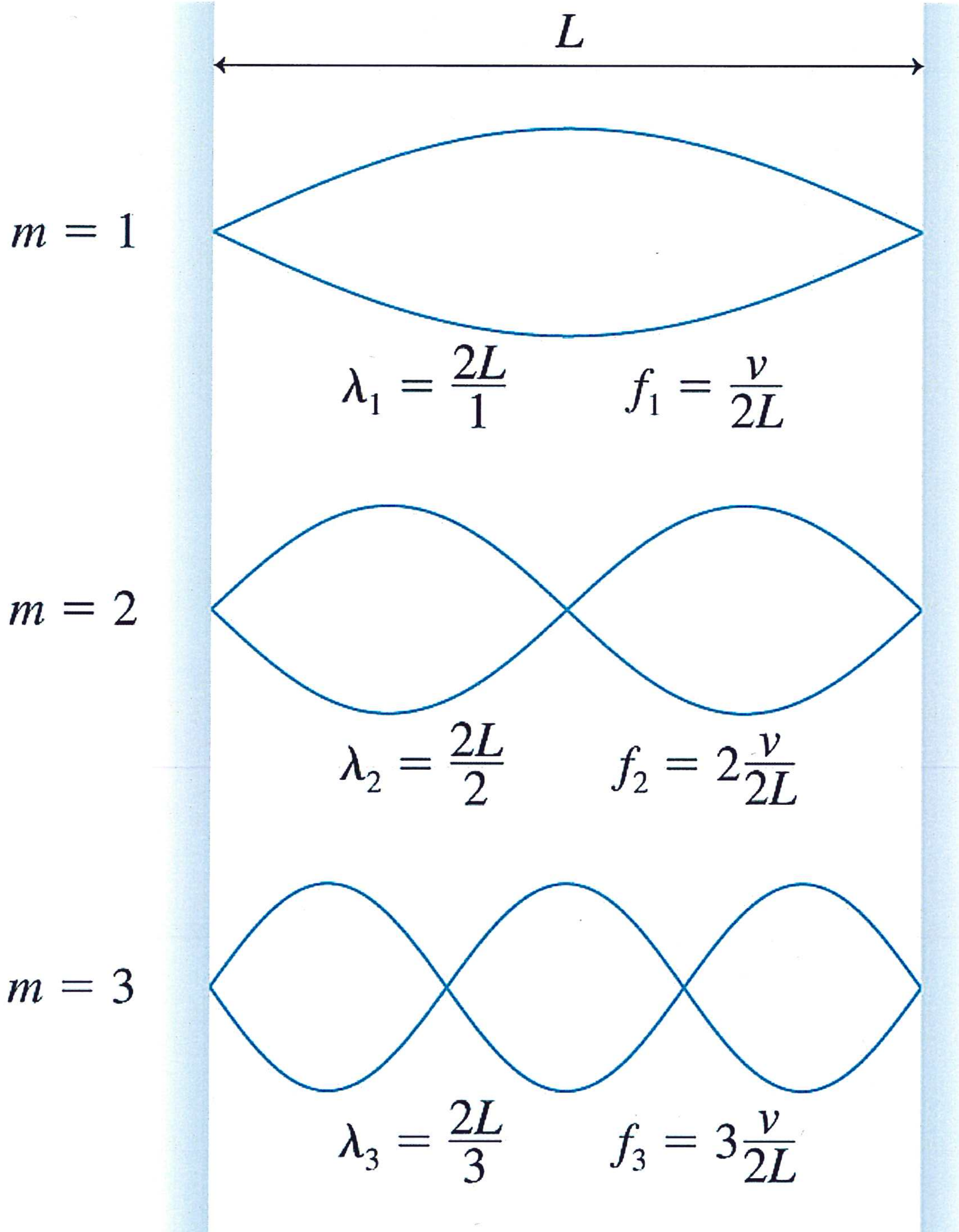
D. 2.0 m.

E. Standing waves don't have a wavelength.

Node Spacing on a String



The nodes and antinodes are spaced $\lambda/2$ apart.



On a string of length L with fixed end points,

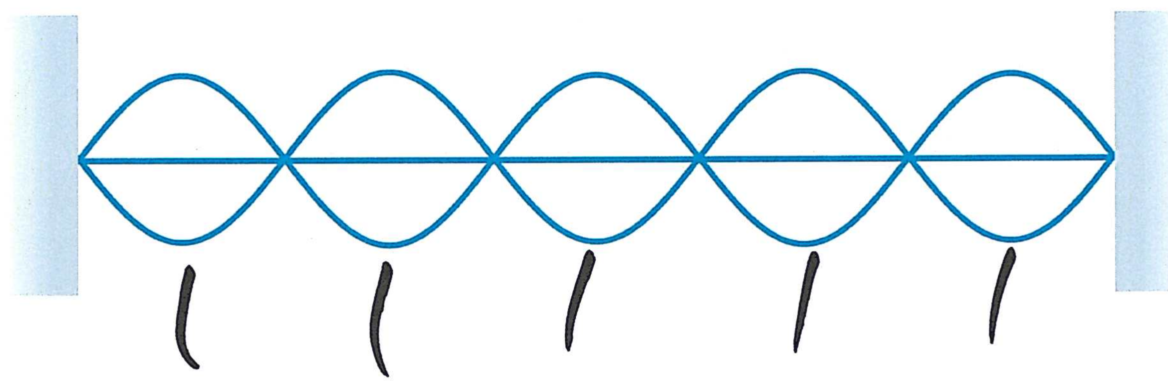
$y(0,t)=0$ and $y(L,t)=0$

Only oscillations with specific wavelengths are allowed.

- m is called the mode number
- $m=1$ is the "fundamental".
- $m=2$ is the "second harmonic"

Learning Catalytics Question

What is the mode number of this standing wave?



$m = 5$

count antinodes

Class 24 (last class) is Tomorrow!

- No Pre-Class Video or Pre-Class quiz
- Homework 11 on Chapter 14 is for practice for the final exam – it's not worth marks but I suggest you try it anyway for studying
- If you haven't done it, please check your utoronto email, respond to the course_evaluations email and evaluate us!
- We are skipping section 14.8 on Doppler Shift for this course.
- Tomorrow my plan is finish up to section 14.7, then I will do some course review and give some advice about the final exam.
- Professor Wilson and I will be giving back-to-back "Exam Jam" lectures on Friday from 1:00-3:00pm in SS2117. I have posted a hand-out for Exam Jam on my slides, and I will post any written notes from Exam Jam on the portal after Friday.