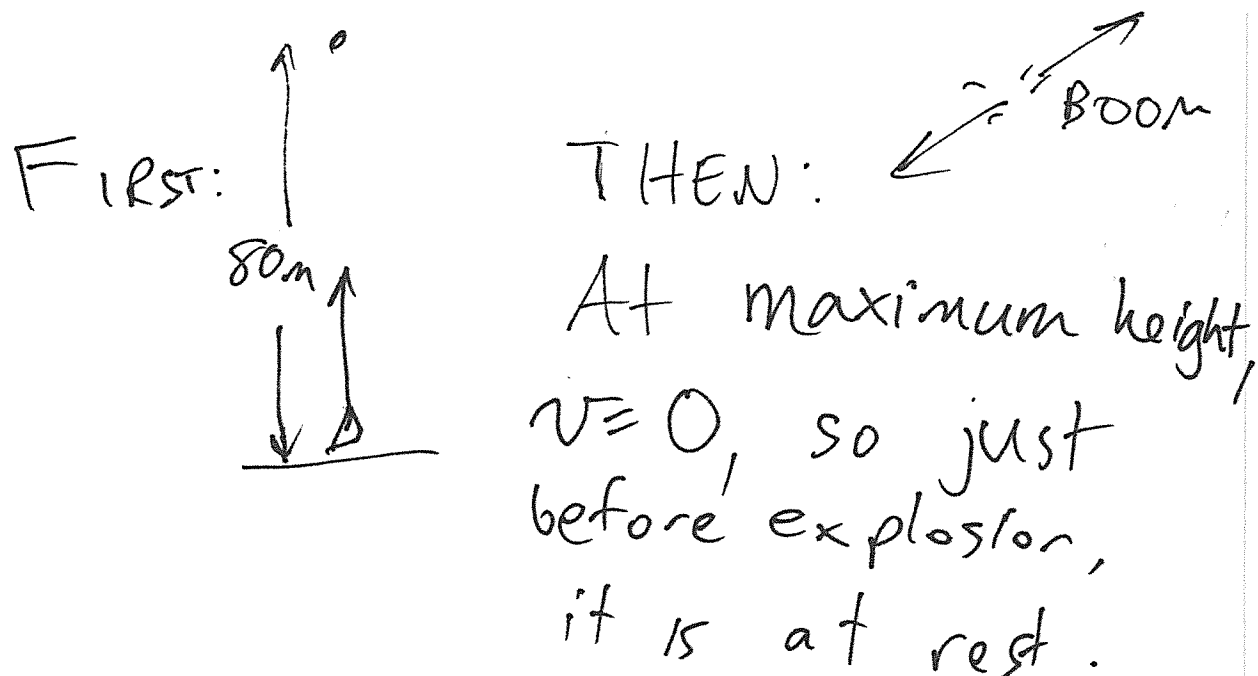


**Problem C**

A fireworks rocket is fired vertically upward. At its maximum height of 80.0 m, it explodes and breaks into two pieces. The first piece has mass  $m_1 = 1.40$  kg, and the second piece has mass  $m_2 = 0.28$  kg. In the explosion, 860 J of chemical energy is converted to kinetic energy of the two fragments.

1. (4 points) What is the speed of each fragment just after the explosion?



Explosion: Ch. 9 cons. of momentum.

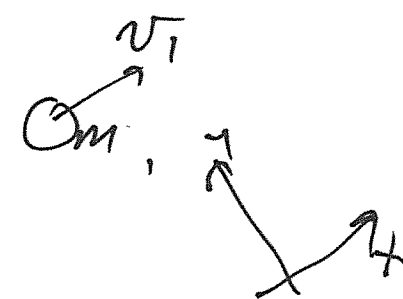
BEFORE:



$$v_i = 0$$

$$\vec{p}_i = 0$$

AFTER:



$$\vec{p}_f = \vec{p}_i$$

Let's Define  $+x$  to be the direction  $m_1$  goes, right after explosion. ①

**Problem C**

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1. (4 points) What is the speed of each fragment just after the explosion?

$$(P_f)_x = m_1 v_1 - m_2 v_2 = 0$$

$$P_{fy} = 0$$

$$m_1 v_1 = m_2 v_2 \quad (1)$$

Need another eq.

Before:  $K_i = 0$

$$\text{After: } K_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = 860 \text{ J} \quad (2)$$

2 eqs, 2 unknowns.

$$(1) \Rightarrow v_1 = \frac{m_2}{m_1} v_2$$

plug into (2):

$$\frac{1}{2} m_1 \left( \frac{m_2}{m_1} v_2 \right)^2 + \frac{1}{2} m_2 v_2^2 = 860$$

$$\text{Solve for } v_2: \quad \frac{v_2^2}{2} \left( \frac{m_2^2}{m_1} + m_2 \right) = 860$$

$$v_2 = \sqrt{2(860) \left( \frac{0.28^2}{1.4} + 0.28 \right)}$$

$$v_2 = 71.5 \text{ m/s}$$

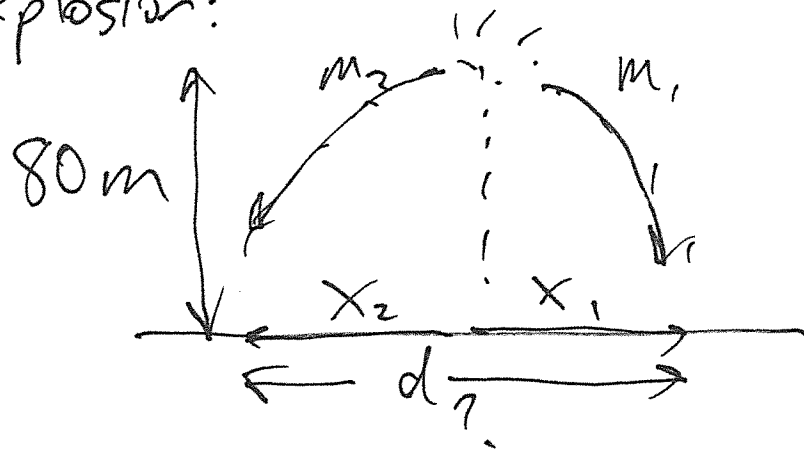
$$(1): \quad v_1 = \frac{0.28}{1.4} (71.5) = 14.3 \text{ m/s}$$

**Problem C**

A fireworks rocket is fired vertically upward. At its maximum height of 80.0 m, it explodes and breaks into two pieces. The first piece has mass  $m_1 = 1.40$  kg, and the second piece has mass  $m_2 = 0.28$  kg. In the explosion, 860 J of chemical energy is converted to kinetic energy of the two fragments.

2. (2 points) It is observed that the two fragments hit the ground at the same time. What is the distance between the points on the ground where they land? [Assume that the ground is level and that  $g = 9.80$  m/s<sup>2</sup> is a constant throughout the motion.]

After explosion:



fall to ground in same time

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$

↑  $v_{0y} = 0$  for both.

→ They must go sideways.

$$v_1 = 14.3 \text{ m/s, to the right}$$

$$v_2 = 71.5 \text{ m/s, to the left.}$$

$$d = x_1 + x_2$$

We need time for fall...

$$h = 80 \text{ m} = \frac{1}{2}gt^2$$

**Problem C**

A fireworks rocket is fired vertically upward. At its maximum height of 80.0 m, it explodes and breaks into two pieces. The first piece has mass  $m_1 = 1.40$  kg, and the second piece has mass  $m_2 = 0.28$  kg. In the explosion, 860 J of chemical energy is converted to kinetic energy of the two fragments.

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$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(80)}{9.8}} = 4.04061 \text{ s}$$

$$\text{Next: } x_1 = v_1 t \quad (a_x = 0)$$

$$= 14.3 (4.0)$$

$$= 57.8 \text{ m}$$

$$x_2 = v_2 t = 71.5 (4.04) = 289 \text{ m}$$

$$d = x_1 + x_2 = \boxed{347 \text{ m}}$$

Question 8

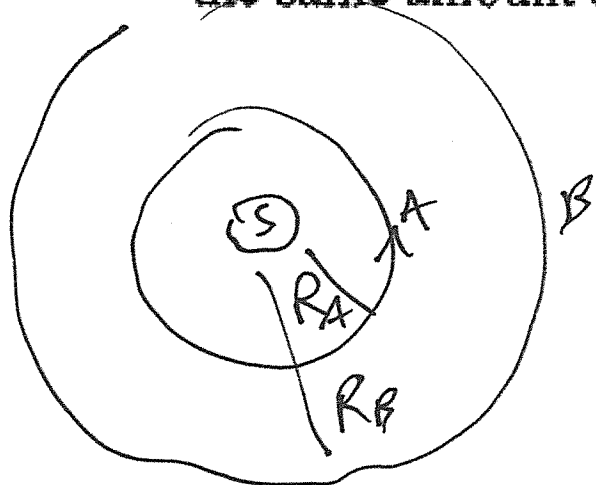
Ch. 8

Two planets having equal masses are in circular orbit around a star. Planet A has a smaller orbital radius than planet B. Which statement is true? (Note that mechanical energy is defined as kinetic energy plus potential energy.) *Same mass*

$$R_A < R_B$$

$$K, U, E = K + U$$

- (A) Planet A has less potential energy than planet B, and both planets have the same amount of kinetic energy and mechanical energy.
- (B) Planet A has more kinetic energy, less potential energy, and less mechanical energy than planet B.
- (C) Planet A has more kinetic energy, more potential energy, and more mechanical energy than planet B.
- (D) Planet A has more kinetic energy and less potential energy than planet B, and both planets have the same amount of mechanical energy.
- (E) Planet A has more kinetic energy and more potential energy than planet B, and both planets have the same amount of mechanical energy.



$$K_A = \frac{1}{2} m v_A^2 = \frac{1}{2} m \frac{GM}{R_A}$$

$$K_B = \frac{1}{2} m v_B^2 = \frac{1}{2} m \frac{GM}{R_B}$$

$$R_A < R_B, \quad K_A > K_B$$

Ch. 8 Circ. orbit:  $v = \sqrt{\frac{GM}{r}}$

$$U_A = -\frac{GMm}{R_A}, \quad U_B = -\frac{GMm}{R_B}$$

$$U_A < U_B$$

$$K_A + U_A = \frac{1}{2} \frac{GMm}{R_A} - \frac{GMm}{R_A} = -\frac{GMm}{2R_A}$$

$$E_A < E_B$$

(5)

Question 9

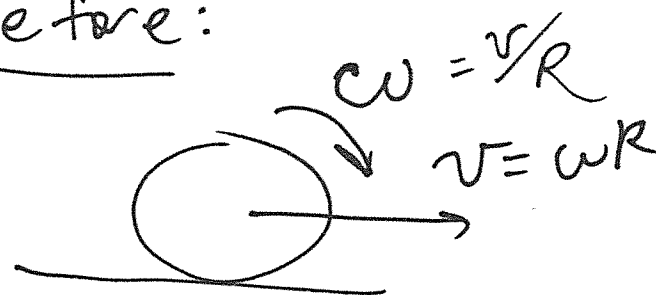
A hoop and a solid disk roll along the ground without slipping. They have the same velocity and the same radius. They both come to the same hill and roll up the hill without slipping. Which makes it further up the hill?

$$I = CMR^2, \quad C = 1 \text{ for hoop, } C = \frac{1}{2} \text{ for disk.}$$

- (A) The hoop.      (B) The disk.      (C) They travel the same distance.  
(D) More information is needed to answer this question.

Conservation of Energy.

Before:

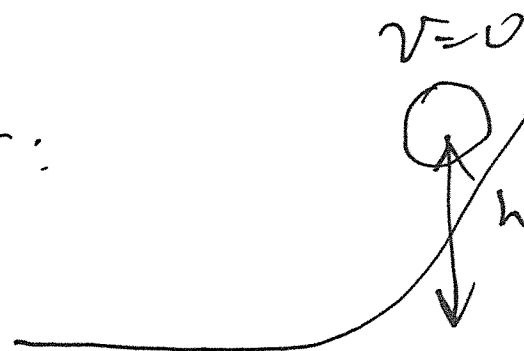


$$E_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh_i \rightarrow 0$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}(CMR^2)\left(\frac{v}{R}\right)^2$$

$$E_i = \frac{1}{2}mv^2(1+C)$$

After:



$$E_f = mgh$$

$$E_f = E_i$$

$$mgh = \frac{1}{2}mv^2(1+C)$$

$$h = \frac{v^2}{g}(1+C)$$

greater  $C$ ,  
greater  $h$ .

(6)

**Question 11**

As you are leaving a building, the door opens outward. If the hinges on the door are on your right, what is the direction of the angular velocity vector of the door as you open it?

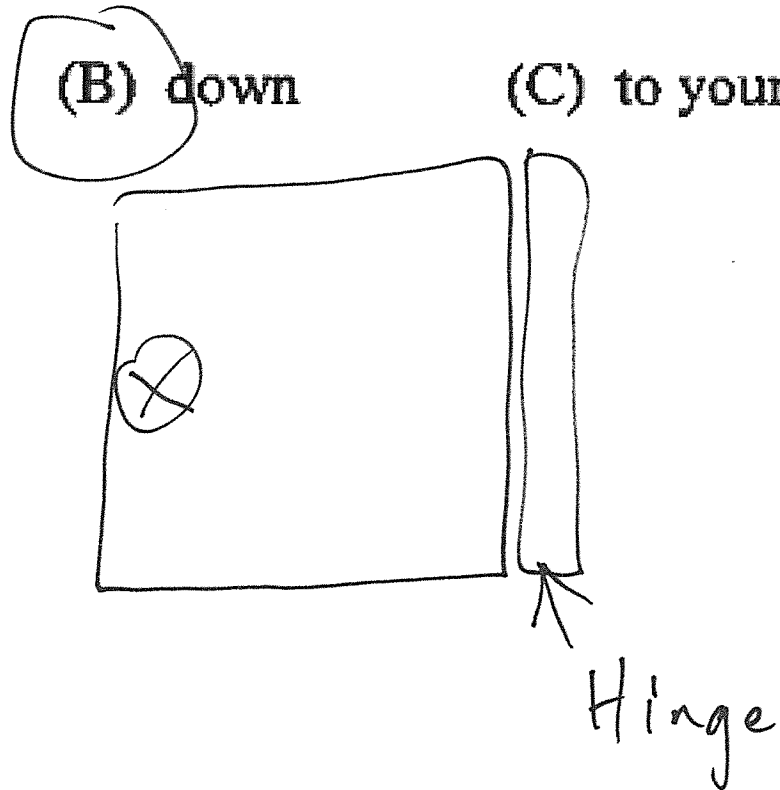
(A) up

(B) down

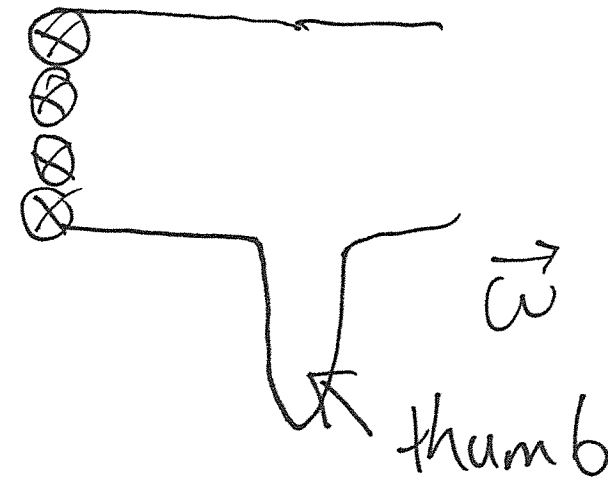
(C) to your left

(D) to your right

(E) forwards



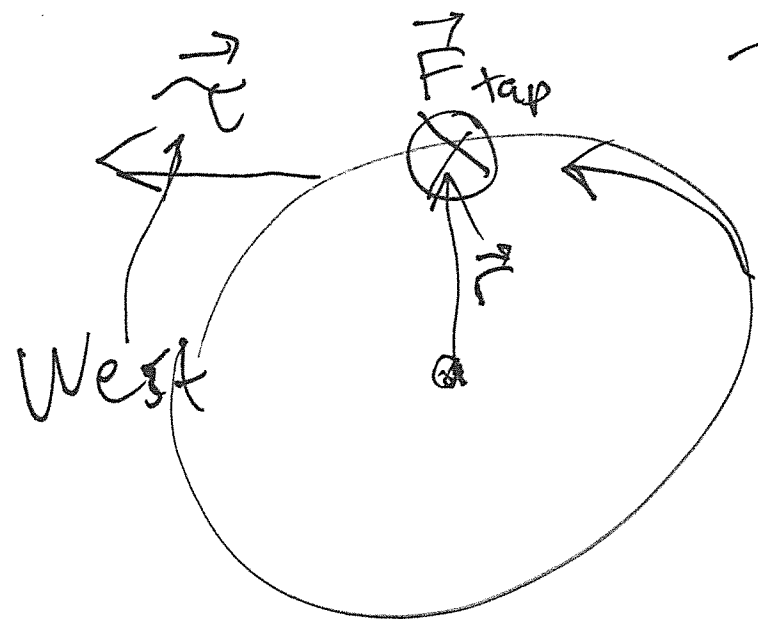
RHR for rotation:



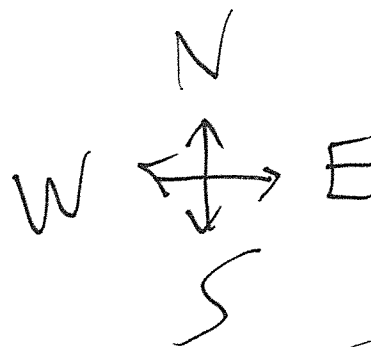
**Question 7**

A wheel is spinning in a horizontal circle, rotating counter-clockwise when viewed from above. It rotates around its centre of mass, so there is no precession due to gravity. You give the wheel a light tap downward, applied on its north-most side. Assume that the magnitude of the tap is small enough so that the magnitude of the angular velocity does not change. What is the direction of the wheel's angular velocity after your tap?

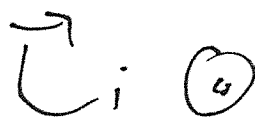
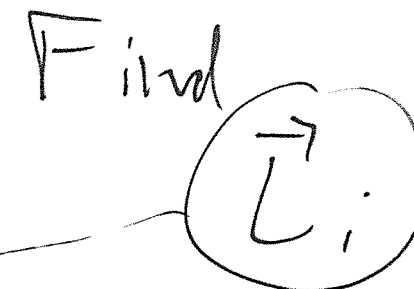
- (A) Down and east. (B) Down and west. (C) Up and east. **(D) Up and west.**  
(E) None of the other answers are correct.



Top view



Precession



up + a little west

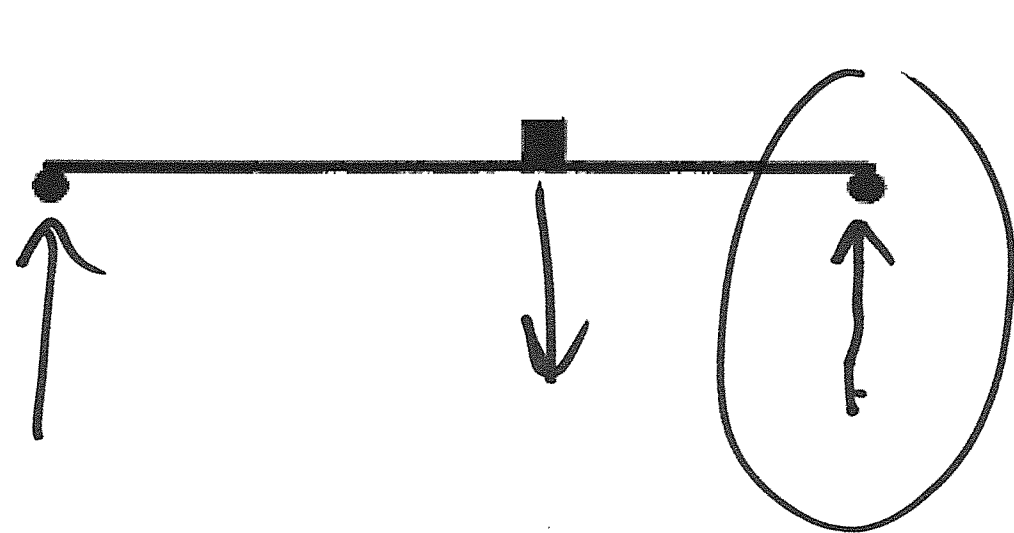
$\vec{\tau} = \vec{r} \times \vec{F}$   
change to  $\vec{L}$ .



**Question 3**

I place a heavy block at the 60-cm mark of a ruler which is 1-m long. I hold the ruler up by balancing it on my left finger at the 0-cm mark and my right finger at the 100-cm mark. Which finger exerts a larger force on the ruler?

- (A) Left finger.
- (B) Right finger.
- (C) They exert the same force.
- (D) More information is needed to answer this question.



by symmetry

**PROBLEM 2 [10 points]**

Ch. 12

A ladder (length  $L$  and mass  $M$ ) is leaning to the right. One end rests on the ground, the other end is attached to a rope (length  $L$  and negligible mass). The other end of the rope is attached to the ceiling directly above the point where the ladder touches the ground (as shown below). If the height of the ceiling is also  $L$ , what is the minimum coefficient of static friction between the ladder and the floor for this to be stable?

Equilateral triangle:  $60^\circ$  angles  
all around.

Set  $f_s = f_{s,max} = \mu_s n$   
solve for  $\mu_s$

NO PROBLEM.

① Define  $x, y$ , rotation axis

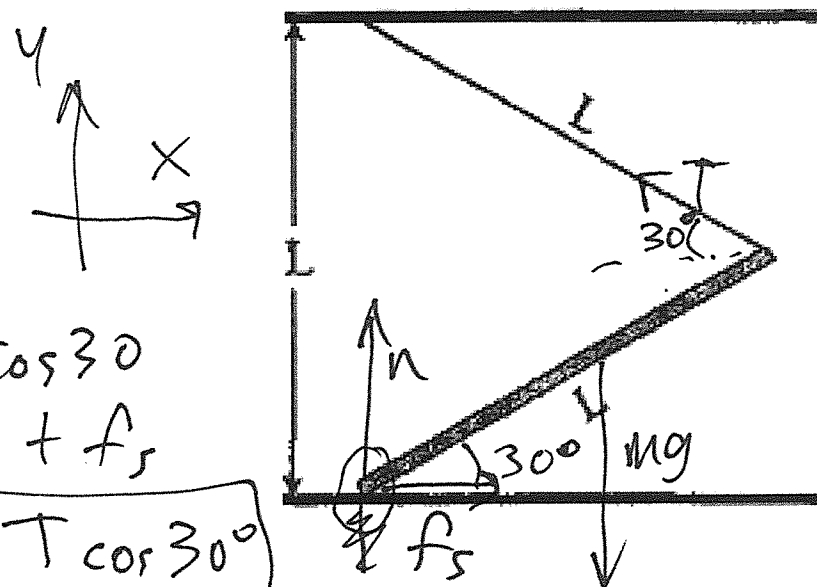
Unknowns:  $T, n, \mu_s$   
 $m$

$$(\sum F_{net})_x = -T \cos 30^\circ + f_s$$

$$\Rightarrow f_s = \mu_s n = T \cos 30^\circ$$

$$(1) (\sum F_{net})_y = 0 = T \sin 30^\circ + n - mg$$

$$(2) n = mg - T \sin 30^\circ$$

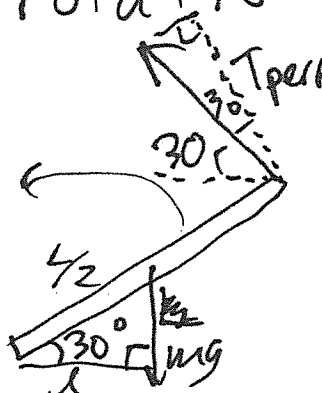


**PROBLEM 2 [10 points]**

A ladder (length  $L$  and mass  $M$ ) is leaning to the right. One end rests on the ground, the other end is attached to a rope (length  $L$  and negligible mass). The other end of the rope is attached to the ceiling directly above the point where the ladder touches the ground (as shown below). If the height of the ceiling is also  $L$ , what is the minimum coefficient of static friction between the ladder and the floor for this to be stable?



Set bottom of ladder = rotation axis.  $\tau_{fs} = 0$



$$\tau_n = 0$$

$$\tau_T = T_{\perp} L$$

$$\tau_T = +T \cos 30^\circ L$$

$$\tau_{mg} = -L mg = -\frac{L}{2} \cos 30^\circ mg$$

$$\tau_{net} = T \cos 30^\circ L - \frac{L}{2} \cos 30^\circ mg = 0$$

$$T - \frac{mg}{2} = 0$$

$$T = \frac{mg}{2} \quad (3)$$

3 eqs, 3 unknowns.

Eliminate  $T, n$ , (hopefully  $m$ )

solve for  $\mu_s$

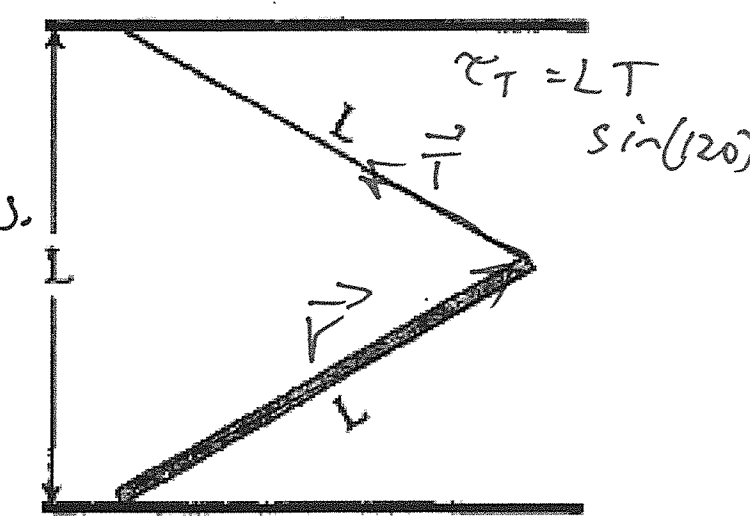
Plug (3) into (2):  $n = mg - \frac{mg}{2} \sin 30^\circ$

$$n = mg \left( 1 - \frac{\sin 30^\circ}{2} \right)$$

plug into 1:

$$\mu_s = \frac{T \cos 30^\circ}{n} = \frac{\frac{mg}{2} \cos 30^\circ}{mg \left( 1 - \frac{\sin 30^\circ}{2} \right)}$$

$$\mu_s = \frac{\cos 30^\circ}{2 \left( 1 - \frac{\sin 30^\circ}{2} \right)} = \boxed{0.577}$$



Ch. 13 problem : S.H.M.

**PROBLEM 1 [10 points]**

A  $2.000 \pm 0.001$  kg block is attached to a horizontal spring of spring constant  $150 \pm 5$  N/m. The block is free to oscillate horizontally on a frictionless table, but can only move in the north-south directions. The mass is initially at rest when I give it a quick tap. If my tap imparts  $35 \pm 3$  kg m/s of impulse to the block in the southward direction, what is the position of the mass  $3.00 \pm 0.01$  seconds after the tap relative to its starting position?

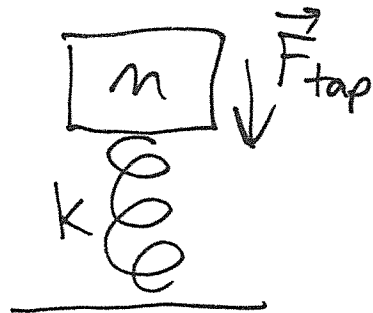
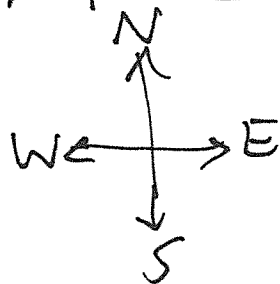
I choose to ignore all of the uncertainties. So ..

$$m = 2 \text{ kg} \quad k = 150 \text{ N/m}$$

$$\text{Impulse} = J = 35 \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

$$t = 3 \text{ s}$$

Top view:



IDEA: Step 1: Give an impulse, which gives an initial velocity.  
→ The block starts at equilibrium, so, this will be  $v_{\text{max}}$  for S.H.M.

Step 2: use S.H.M. equation:

$$x = A \cos(\omega t + \phi)$$

Note: if  $x = 0$  at  $t = 0$ , and  $v_x$  is positive at  $t = 0$ , then

$$\phi = -\pi/2$$

**PROBLEM 1 [10 points]**

A  $2.000 \pm 0.001$  kg block is attached to a horizontal spring of spring constant  $150 \pm 5$  N/m. The block is free to oscillate horizontally on a frictionless table, but can only move in the north-south directions. The mass is initially at rest when I give it a quick tap. If my tap imparts  $35 \pm 3$  kg m/s of impulse to the block in the southward direction, what is the position of the mass  $3.00 \pm 0.01$  seconds after the tap relative to its starting position?

Step 1  
Impulse  $J = \Delta p = p_f - p_i$   $\begin{matrix} \nearrow 0 \\ \text{at} \\ \text{rest.} \end{matrix}$

$$\Rightarrow m v = J, v_{\max} = \frac{J}{m}$$

Step 2  $E_{\text{total}} = \frac{1}{2} k A^2 = \frac{1}{2} m v_{\max}^2$

Solve for A:

$$A = \frac{m}{k} v_{\max}^2 = \frac{m}{k} \left( \frac{J}{m} \right)^2$$

$$A = \frac{J^2}{km} = \frac{35^2}{150(2)} = 4.083 \text{ m}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{150}{2}} = 8.66 \frac{\text{rad}}{\text{s}}$$

$$x = A \cos(\omega t + \phi)$$

$$x = 4.083 \cdot \cos\left((8.66 \times 3) - \frac{3.14}{2}\right)$$

NOTE: These are radians  
set your calculator to  
RAD mode!

$$x = 3.06 \text{ m}$$