PHY131 - Test. 2. Marking Scheme. Fall 2017

Possibly helpful information for this test:
$\pi = 3.14159$ is the ratio of the circumference to the diameter of a circle
$g = 9.80\ \text{m/s}^2$ is the acceleration due to gravity near the Earth's surface.
Air resistance may be neglected in all questions, unless otherwise stated.

Gravitational energy of a system comprising a mass m located at a distance r from the center of another mass M is: $U = -\frac{GMm}{r}$.

When a ball of mass m_1, originally moving at speed v_{1i}, collides elastically head-on with a ball of mass m_2, originally at rest, the final velocities of the two balls, in the direction of the original velocity of m_1, are:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

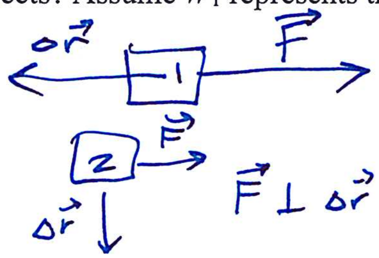
Some definite integrals, where a, b and c are constants:

- $\int_a^b x\sqrt{c^2 - x^2}\,dx = -\frac{1}{3}(c^2 - b^2)^{\frac{3}{2}} + \frac{1}{3}(c^2 - a^2)^{\frac{3}{2}}$
- $\int_a^b x^n dx = \frac{1}{n+1} b^{n+1} - \frac{1}{n+1} a^{n+1}$
- $\int_a^b \cos(x)\,dx = \sin(b) - \sin(a)$
- $\int_a^b \sin(x)\,dx = -\cos(b) + \cos(a)$

+2 for correct.
SOME WRONG ANSWERS:
+1 wrong
0 more wrong
(Professor's discretion)

MULTIPLE CHOICE PART (16 points total)
Choose the best answer for each question. 2 points per question, no penalty for guessing.

1. You apply the same force on three objects of the same mass. Each time, your force points north. Object 1 moves south. Object 2 moves west. Object 3 does not move. What is the correct ranking of the work you did on the objects? Assume W_1 represents the work you did on object 1, and similarly for objects 2 and 3.
   - ~~A.~~ $W_1 = W_2 = W_3$
   - B. $W_2 = W_3 > W_1$
   - +1 C. $W_1 > W_2 = W_3$
   - ~~D.~~ $W_1 = W_2 > W_3$
   - +1 E. $W_1 > W_2 > W_3$



W_1 is negative

$\vec{F} \perp \Delta\vec{r} \Rightarrow W_2 = 0$.

3 $\Delta\vec{r} = 0$
$\Rightarrow W_3 = 0$

2. A certain planet has an escape speed v_{esc}. If another planet of the same size has twice the mass as the first planet, its escape speed will be
   - ~~A.~~ $4\ v_{esc}$
   - +1 B. $2\ v_{esc}$
   - C. $\sqrt{2}\ v_{esc}$
   - ~~D.~~ v_{esc}
   - ~~E.~~ $v_{esc} / 2$

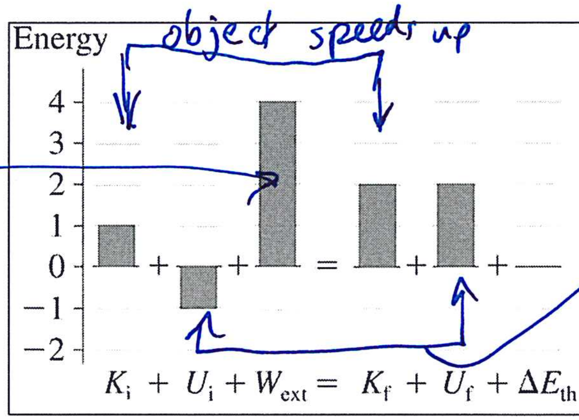
Escape $\Rightarrow K + U = 0 = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{r} = 0$

$\frac{v^2}{2} = \frac{GM}{r}$

$v_1 = \sqrt{\frac{2GM_1}{r}}$

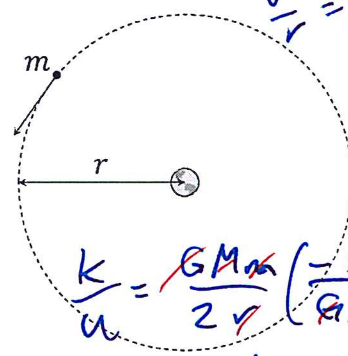
$v_2 = \sqrt{\frac{2G(2M_1)}{r}} = \sqrt{2}\, v_1$

3. Examine the energy budget below. Which of the following scenarios is best described by the energy budget?



+1 → A. Arjun does work stretching a spring. The system is the spring. ← No U.
B. Binyu accelerates up a hill, pulling a wagon. The system is the wagon. ✓
~~C.~~ Calista releases an arrow from a stretched bow, firing the arrow horizontally. The system is the arrow. No change in U.
~~D.~~ Dylan tries to stop a piano from rolling downhill but Dylan is not strong enough, so the piano accelerates down the hill. The system is the piano. U decreases.

4. A satellite of mass m is in a circular orbit of radius r around a planet. (r is measured from the center of the planet, not its surface.) Assume the planet is much more massive than the satellite, and that the planet is not moving. What is the ratio of the kinetic energy to the potential energy of the satellite, K/U ? [Use the convention that potential energy goes to zero at infinity.]



+1 → A. −2
+1 → B. −1
C. −1/2
~~D.~~ 1/2
~~E.~~ 2

$\frac{v^2}{r} = \frac{GM}{r^2}$

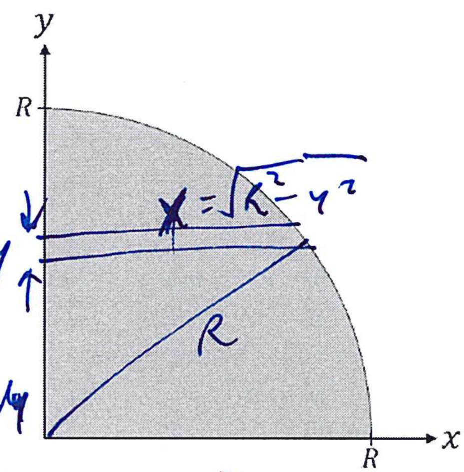
$v_{circ} = \sqrt{\frac{GM}{r}}$

$K = \frac{1}{2}mv^2 = \frac{m}{2}\left(\frac{GM}{r}\right) = \frac{GMm}{2r}$

$U = -\frac{GMm}{r}$

$\frac{K}{U} = \frac{\cancel{GMm}}{2\cancel{r}}\left(\frac{-\cancel{r}}{\cancel{GMm}}\right) = -\frac{1}{2}$

5. Assume the flat object shown has mass M, distributed uniformly over the surface of area $A = \frac{\pi R^2}{4}$, and the origin is at the left-bottom corner of the shape. By symmetry, one can argue that the centre of mass of this object is at coordinates $(x_{cm}, y_{cm}) = (cR, cR)$, where c is some constant. Which of the following is true?



~~A.~~ $c \leq 0$
B. $0 < c < ½$
+1 → C. $c = ½$
+1 → D. $½ < c < 1$
~~E.~~ $c \geq 1$

$dA = dy\sqrt{R^2 - y^2}$

$dm = \frac{M}{A}dA = \frac{4M}{\pi R^2}dA$

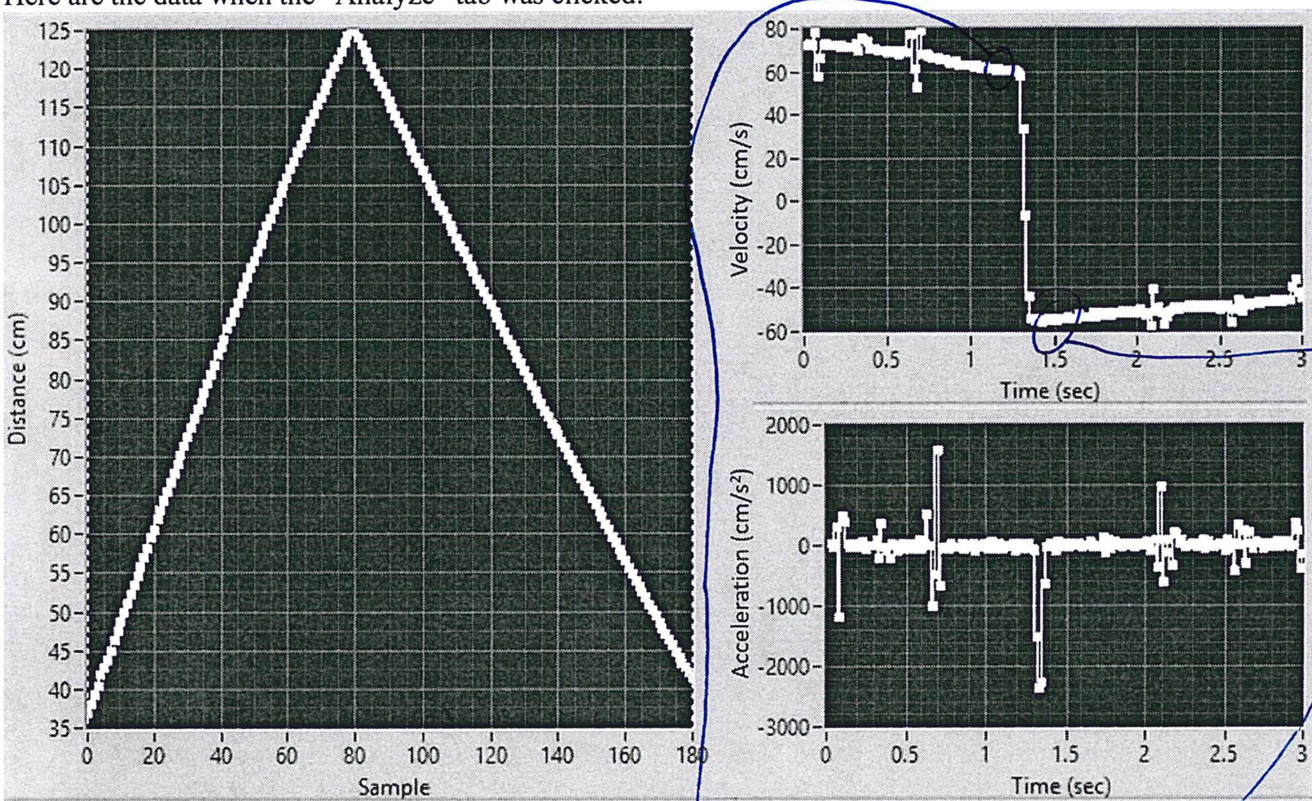
$y_{cm} = \frac{1}{M}\int_0^R y\,dm = \frac{4}{\pi R^2}\int_0^R y\sqrt{R^2 - y^2}\,dy$

$y_{cm} = \frac{4}{\pi R^2}\left[-\frac{1}{3}(0) + \frac{1}{3}(R^2)^{3/2}\right] = \frac{4}{3\pi \cancel{R^2}}R^{\cancel{3}} = \frac{4}{3\pi}R$

same reasoning for x_{cm}.

$c \approx 4/9.4 < 1/2$

Questions 6, 7 and 8 are based on Motion Sensor Data of a cart on a track undergoing a collision with a fixed bumper. The motion sensor software, which you used in Practicals, was set to record 60 samples per second. Here are the data when the "Analyze" tab was clicked:



6. The change in the velocity of the cart between 1.2 s and 1.5 s is Δv. Based on the data shown, which of the following is closest **value** of Δv, in cm/s?
   A. 140
   B. 115
   C. 60
   D. 5
   E. 0

$v_1 = 60 \pm 5$ $\quad$ $v_2 = -55 \pm 5$

$v_2 - v_1 = -55 - 60 = -115 \text{ cm/s}$

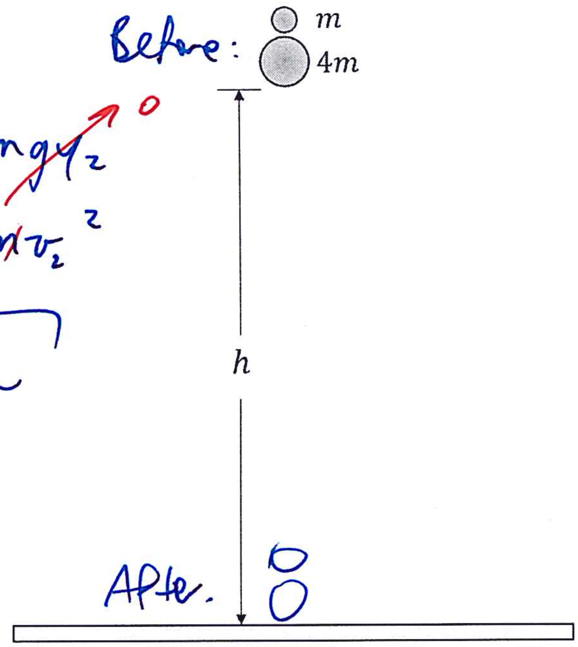
7. The change in the velocity of the cart between 1.2 s and 1.5 s is Δv. Based on the data shown, which of the following is closest **uncertainty** of Δv, in cm/s?
   A. 0.1
   +1 → B. 1.0
   C. 7
   D. 25
   E. 115

A bit more than 5 ...
1.0 is too low for these data.
25 is too high ..

8. The velocity graph appears to have a slightly negative slope before $t = 1.3$ s and a slightly positive slope after $t = 1.3$ s. What is the best explanation for this observation?
   A. The cart loses kinetic energy before the collision and gains kinetic energy after the collision.
   B. The measurement uncertainty due to the motion sensor.
   +1 → C. The track is not perfectly level. ← if so, slopes would be the same before & after.
   D. Friction and drag forces are doing negative work on the cart as it moves.
   E. The collision is not perfectly elastic.

FREE-FORM PART (12 points total)
For full marks, you must clearly show all of your work and reasoning in the space provided. State any assumptions you make, and show all the steps of your calculations. Write your final answers in the boxes provided.

A1 (2 points). A rubber ball of mass m is held a very small distance above a more massive rubber ball of mass $4m$. They are both dropped from rest from an initial height, h, above a hard floor. The sizes of the two balls are negligible compared to h. What is the speed of the balls just before the larger ball hits the floor? Express your answer in terms of h and g.



$E_1 = E_2$

$\cancel{\frac{1}{2}mv_1^2}^{0} + mgy_1 = \frac{1}{2}mv_2^2 + \cancel{mgy_2}^{0}$

$mgy_1 = mgh = \frac{1}{2}mv_2^2$

$v_2 = \sqrt{2gh}$

+1 for trying to conserve energy.

2 $\boxed{v = \sqrt{2gh}}$ +1 for answer.

A2 (4 points). The larger ball collides with the floor, then, a very short amount of time later, it collides with the smaller ball head-on. To what maximum height above the floor, h_{max}, does the larger ball (with mass $4m$) rise after the second collision? Express your answer in terms of h. Assume all collisions are elastic.

Before: 2 $\downarrow \sqrt{2gh}$, 1 $\uparrow \sqrt{2gh}$

+1 for realizing basketball reverses direction.

in m_2 frame: add $\sqrt{2gh}$

② $v_{2i} = 0$

① $v_{1i} = 2\sqrt{2gh}$

$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{4m - m}{4m + m} 2\sqrt{2gh} = \frac{6}{5}\sqrt{2gh}$

Ground frame subtract $\sqrt{2gh}$: $v_f = \frac{6}{5}\sqrt{2gh} - \sqrt{2gh} = \boxed{\frac{1}{5}\sqrt{2gh}}$ ← initial upward velocity.

+2 for finding basketball speed is down by $(1/5)$.

+1 for p conservation

use $E_1 = E_2$

$\frac{1}{2}mv_1^2 = mgh_{max}$

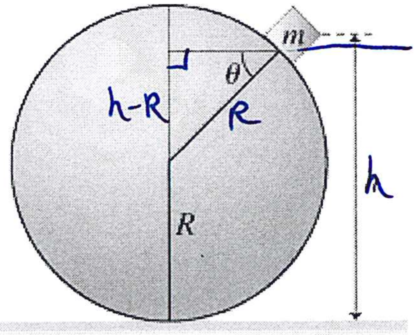
$h_{max} = \frac{v_1^2}{2g} = \frac{\left(\frac{1}{5}\sqrt{2gh}\right)^2}{2g} = \frac{h}{25}$

$\boxed{h_{max} = h/25}$ 4

+1 for final answer.

6

Problem B (6 points)
A large globe has a radius $R = 5.00$ m and a frictionless surface.
A small block of mass $m = 1.00 \times 10^{-3}$ kg starts to slide with a tiny (negligible) speed from the very top of the globe and slides along the surface of the globe. The initial height above the ground of the mass, when it is almost at rest, is $h_{\text{initial}} = 10.0$ m. At what height above the ground, h_{crit}, does the block leave the surface of the globe?

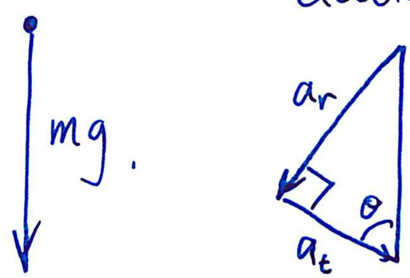


$\boxed{\sin\theta = \frac{h-R}{R}}$, from geometry. +1

$n = 0$ no normal force at $h = h_{crit}$

free body diagram: +1

acceleration:



a_r = radial acceleration
a_t = tangential acceleration

$\sin\theta = \frac{a_r}{g}$

$a_r = \frac{v^2}{R} = g\sin\theta$

$v^2 = Rg\left(\frac{h-R}{R}\right)$ +1

$\boxed{v^2 = gh - gR}$

Conservation of Energy.

$E_1 = E_2$

$\frac{1}{2}\cancel{m}v_1^2 + \cancel{m}gh_1 = \frac{1}{2}\cancel{m}v_2^2 + \cancel{m}gh_2$

$v_1 = 0$, $h_1 = 2R$, $h_2 = h$

+1 $\boxed{2gR = \frac{v^2}{2} + gh}$

$2gR = \frac{1}{2}(gh - gR) + gh$

$4\cancel{g}R = \cancel{g}h - \cancel{g}R + 2\cancel{g}h$

$4R + R = h + 2h$

$5R = 3h$

$h = \frac{5}{3}R$

$h = \frac{5 \cdot 5.00}{3} = 8.333$ m

+1 for final answer

-1 if units are missing

$h_{\text{crit}} =$ 8.33 m

-1 point deducted for wrong # of significant figures.