HARLOW | JASON | – | Solutions.

| FAMILY NAME **as on student card** | Given Name(s) **as on student card** | Student Number | Practical Group Code (ie F3B) |
|---|---|---|---|

**PHY131H1F**

**Term Test 2 —version A**

Tuesday, November 14, 2017

Duration: 80 minutes

**Aids allowed:** A calculator with no communication ability (programmable calculators and graphing calculators are okay). A single hand-written aid-sheet prepared by the student, no larger than 8.5"x11", written on both sides. A hard-copy English translation dictionary. A ruler.

- **Completely turn off** any communication device you may have and leave it with your belongings at the front of the room.

- **DO NOT separate the sheets of your question paper.** You can, however, *carefully* tear off the blank page at the end, as it does not have to be handed in.

- Before starting, please **PRINT IN BLOCK LETTERS your name, student number, and practical group code** at the top of this page **and** on the answer sheet.

**Locate your test version letter (A or B) in the header at the top of the page and fill in the circle with the corresponding version code on your answer sheet in the "Form Code" box.** Mark in your student number by shading the circles at the top-right of the sheet, starting with a 0 if the first digit is a 9. It is not required to bubble in your surname on the lower half of the sheet.

**Scanned Area of the Answer Sheet:**

1. **Use a dark-black, soft-lead pencil or a black pen.**
2. Indicate your answer to a multiple-choice question by thoroughly filling the appropriate circle on the answer sheet and also by recording your answer on the test paper.
3. If you wish to modify an answer, erase your pencil mark thoroughly.
4. **Do not write anything else on the answer sheet.** Use the blank sheets at the end or the back of the question sheets for rough work.

The first part of the test consists of **8** multiple-choice questions, worth 2 points each, or altogether 16 points. Each multiple-choice question has one best answer, and up to four answers that are not the best. Blank, incorrect or multiple answers are awarded zero points. There may be partial credit awarded. Note that only the bubbled answer sheet is marked for the multiple choice part; anything you write in this booklet is ignored.

The second part of the test is a set of free-form questions, worth a total of 12 points. To be awarded maximum credit, you must write out fully worked solutions to all parts of the free-form questions in the space provided in this question booklet. In addition to showing your work, please put your answer(s) for each part in the boxes provided. You can use the back-side of the sheets and the blank pages at the end for your rough work which will not be graded or taken into account.

The total number of points available for the test is 28.

**Possibly helpful information for this test:**
$\pi = 3.14159$ is the ratio of the circumference to the diameter of a circle
$g = 9.80\ \text{m/s}^2$ is the acceleration due to gravity near the Earth's surface.
Air resistance may be neglected in all questions, unless otherwise stated.

Gravitational energy of a system comprising a mass $m$ located at a distance $r$ from the center of another mass $M$ is: $U = -\frac{GMm}{r}$.

When a ball of mass $m_1$, originally moving at speed $v_{1i}$, collides elastically head-on with a ball of mass $m_2$, originally at rest, the final velocities of the two balls, in the direction of the original velocity of $m_1$, are:

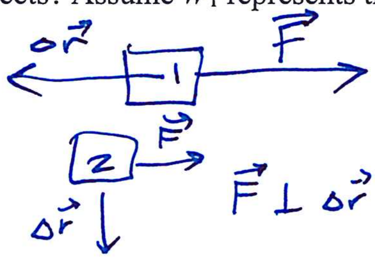
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad\qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Some definite integrals, where $a$, $b$ and $c$ are constants:

- $\int_a^b x\sqrt{c^2 - x^2}\,dx = -\frac{1}{3}(c^2 - b^2)^{\frac{3}{2}} + \frac{1}{3}(c^2 - a^2)^{\frac{3}{2}}$
- $\int_a^b x^n\,dx = \frac{1}{n+1} b^{n+1} - \frac{1}{n+1} a^{n+1}$
- $\int_a^b \cos(x)\,dx = \sin(b) - \sin(a)$
- $\int_a^b \sin(x)\,dx = -\cos(b) + \cos(a)$

**MULTIPLE CHOICE PART** (16 points total)
Choose the best answer for each question. 2 points per question, no penalty for guessing.

1. You apply the same force on three objects of the same mass. Each time, your force points north. Object 1 moves south. Object 2 moves west. Object 3 does not move. What is the correct ranking of the work you did on the objects? Assume $W_1$ represents the work you did on object 1, and similarly for objects 2 and 3.
   A. $W_1 = W_2 = W_3$
   B. $W_2 = W_3 > W_1$
   C. $W_1 > W_2 = W_3$
   D. $W_1 = W_2 > W_3$
   E. $W_1 > W_2 > W_3$




$W_1$ is negative

$\vec{F} \perp \Delta\vec{r} \Rightarrow W_2 = 0$.

$\boxed{3}\ \Delta\vec{r} = 0 \Rightarrow W_3 = 0$.

2. A certain planet has an escape speed $v_{esc}$. If another planet of the same size has twice the mass as the first planet, its escape speed will be
   A. $4\ v_{esc}$
   B. $2\ v_{esc}$
   C. $\sqrt{2}\ v_{esc}$
   D. $v_{esc}$
   E. $v_{esc} / 2$

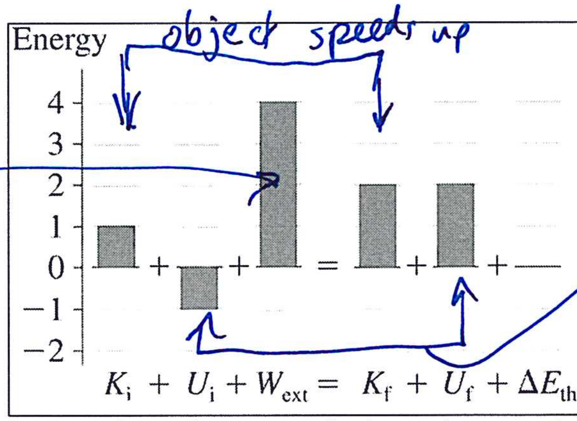
Escape $\Rightarrow K + U = 0 = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{r} = 0$

$\frac{v^2}{2} = \frac{GM}{r}$

$v_1 = \sqrt{\frac{2GM_1}{r}}$

$v_2 = \sqrt{\frac{2G(2M_1)}{r}} = \sqrt{2}\, v_1$

3. Examine the energy budget below. Which of the following scenarios is best described by the energy budget?

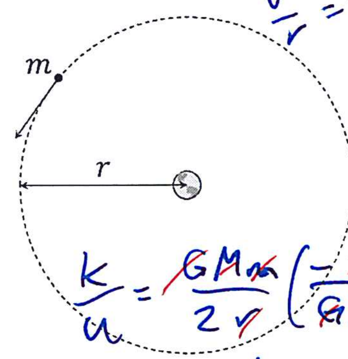



positive external work is done.

object gains potential energy.

A. Arjun does work stretching a spring. The system is the spring. ← No $U$.
B. Binyu accelerates up a hill, pulling a wagon. The system is the wagon. ✓
C. Calista releases an arrow from a stretched bow, firing the arrow horizontally. The system is the arrow. No change in $U$.
D. Dylan tries to stop a piano from rolling downhill but Dylan is not strong enough, so the piano accelerates down the hill. The system is the piano. $U$ decreases.

4. A satellite of mass $m$ is in a circular orbit of radius $r$ around a planet. ($r$ is measured from the center of the planet, not its surface.) Assume the planet is much more massive than the satellite, and that the planet is not moving. What is the ratio of the kinetic energy to the potential energy of the satellite, $K/U$? [Use the convention that potential energy goes to zero at infinity.]
A. −2
B. −1
C. −1/2
D. 1/2
E. 2




$\frac{v^2}{r} = \frac{GM}{r^2}$

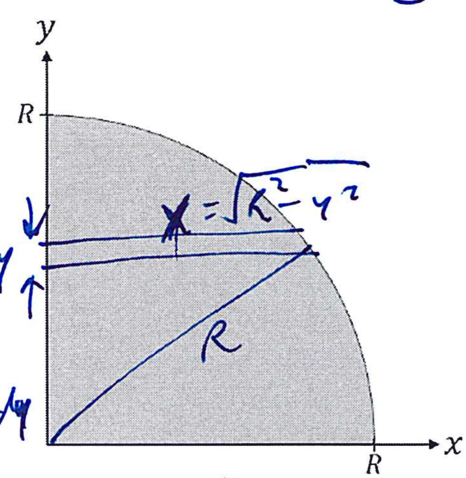
$v_{circ} = \sqrt{\frac{GM}{r}}$

$K = \frac{1}{2}mv^2 = \frac{m}{2}\left(\frac{GM}{r}\right) = \frac{GMm}{2r}$

$U = -\frac{GMm}{r}$

$\frac{K}{U} = \frac{\cancel{GMm}}{2\cancel{r}}\left(\frac{-\cancel{r}}{\cancel{GMm}}\right) = -\frac{1}{2}$

5. Assume the flat object shown has mass $M$, distributed uniformly over the surface of area $A = \frac{\pi R^2}{4}$, and the origin is at the left-bottom corner of the shape. By symmetry, one can argue that the centre of mass of this object is at coordinates $(x_{cm}, y_{cm}) = (cR, cR)$, where $c$ is some constant. Which of the following is true?
A. $c \le 0$
B. $0 < c < ½$
C. $c = ½$
D. $½ < c < 1$
E. $c \ge 1$




$dA = dy\sqrt{R^2 - y^2}$

$dm = \frac{M}{A}\,dA = \frac{4M}{\pi R^2}\,dA$

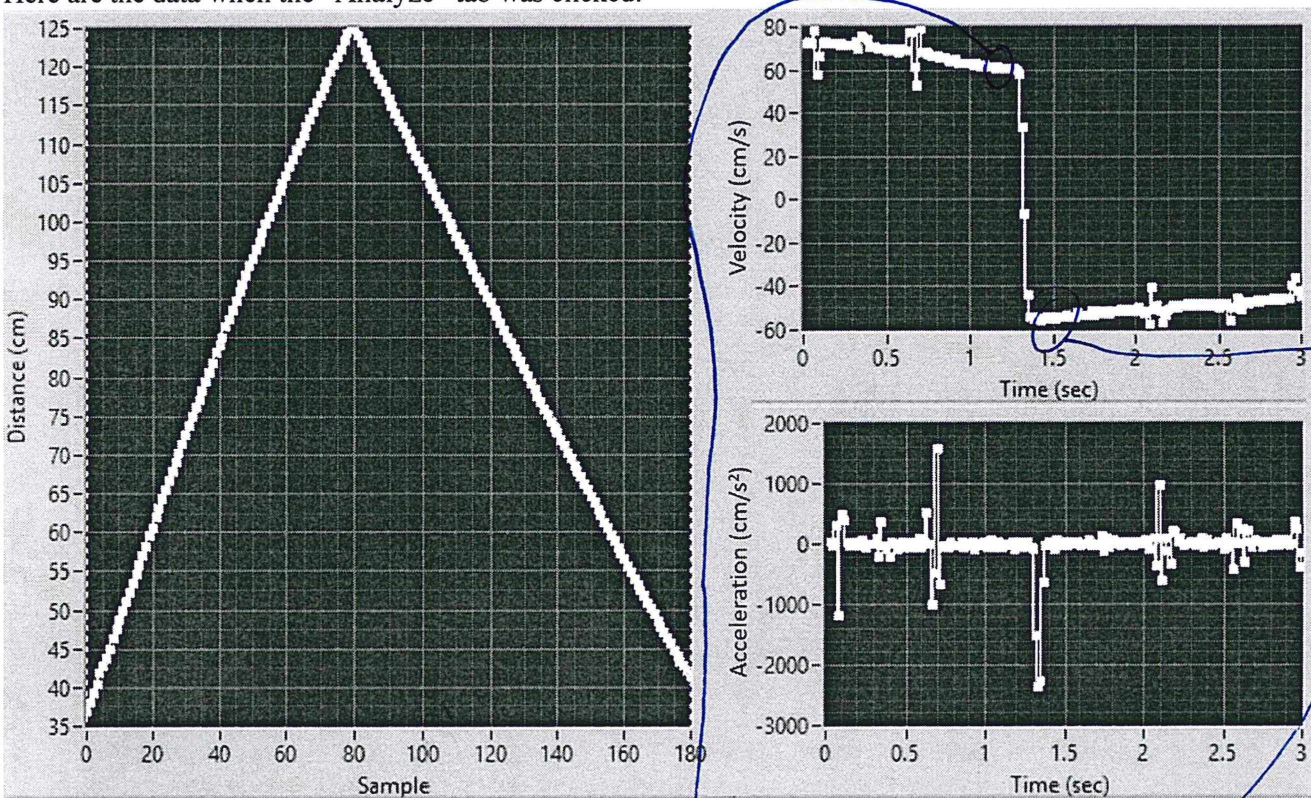
$y_{cm} = \frac{1}{M}\int_0^R y\,dm = \frac{4}{\pi R^2}\int_0^R y\sqrt{R^2 - y^2}\,dy$

$y_{cm} = \frac{4}{\pi R^2}\left[-\frac{1}{3}(0) + \frac{1}{3}(R^2)^{3/2}\right] = \frac{4}{3\pi \cancel{R^2}} \cancel{R^3} = \frac{4}{3\pi} R$

same reasoning for $x_{cm}$.

$c \approx 4/9.4 < 1/2$

**Questions 6, 7 and 8** are based on Motion Sensor Data of a cart on a track undergoing a collision with a fixed bumper. The motion sensor software, which you used in Practicals, was set to record 60 samples per second. Here are the data when the "Analyze" tab was clicked:




6. The change in the velocity of the cart between 1.2 s and 1.5 s is $\Delta v$. Based on the data shown, which of the following is closest **value** of $\Delta v$, in cm/s?
   A. 140
   B. 115
   C. 60
   D. 5
   E. 0

$v_1 = 60 \pm 5$ $\quad$ $v_2 = -55 \pm 5$

$v_2 - v_1 = -55 - 60 = -115$ cm/s

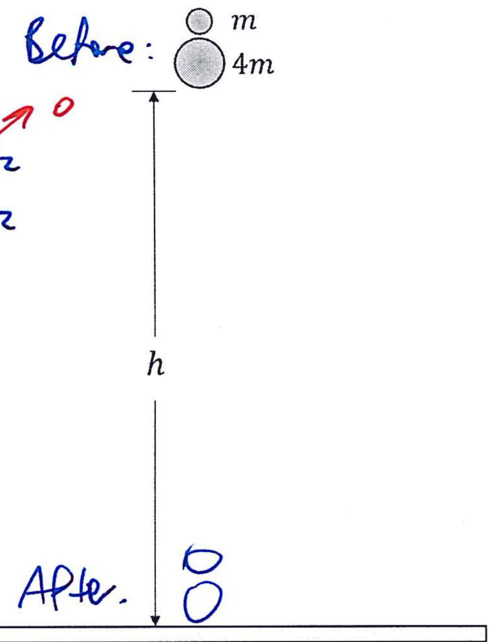
7. The change in the velocity of the cart between 1.2 s and 1.5 s is $\Delta v$. Based on the data shown, which of the following is closest **uncertainty** of $\Delta v$, in cm/s?
   A. 0.1
   B. 1.0
   C. 7
   D. 25
   E. 115

A bit more than 5 ...
1.0 is too low for these data.
25 is too high ..

8. The velocity graph appears to have a slightly negative slope before $t = 1.3$ s and a slightly positive slope after $t = 1.3$ s. What is the best explanation for this observation?
   A. The cart loses kinetic energy before the collision and gains kinetic energy after the collision. ✗
   B. The measurement uncertainty due to the motion sensor. ✗
   C. The track is not perfectly level. ← if so, slopes would be the same before & after.
   D. Friction and drag forces are doing negative work on the cart as it moves.
   E. The collision is not perfectly elastic. ✗

**FREE-FORM PART** (12 points total)
For full marks, you must clearly show all of your work and reasoning in the space provided. State any assumptions you make, and show all the steps of your calculations. Write your final answers in the boxes provided.

**A1** (2 points). A rubber ball of mass $m$ is held a very small distance above a more massive rubber ball of mass $4m$. They are both dropped from rest from an initial height, $h$, above a hard floor. The sizes of the two balls are negligible compared to $h$. What is the speed of the balls just before the larger ball hits the floor? Express your answer in terms of $h$ and $g$.




$$E_1 = E_2$$

$$\cancelto{0}{\tfrac{1}{2}mv_1^2} + mgy_1 = \tfrac{1}{2}mv_2^2 + \cancelto{0}{mgy_2}$$

$$mgy_1 = \cancel{m}gh = \tfrac{1}{2}\cancel{m}v_2^2$$

$$v_2 = \sqrt{2gh}$$

$\boxed{v = \sqrt{2gh}}$

**A2** (4 points). The larger ball collides with the floor, then, a very short amount of time later, it collides with the smaller ball head-on. To what maximum height above the floor, $h_{max}$, does the larger ball (with mass $4m$) rise after the second collision? Express your answer in terms of $h$. Assume all collisions are elastic.

Before: 2 ↓ $\sqrt{2gh}$
1 ↑ $\sqrt{2gh}$

in $m_2$ frame: add $\sqrt{2gh}$

② $v_{2i} = 0$

↑ ① $v_{1i} = 2\sqrt{2gh}$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{4m - m}{4m + m}\, 2\sqrt{2gh} = \frac{6}{5}\sqrt{2gh}$$

Ground frame subtract $\sqrt{2gh}$: $v_f = \frac{6}{5}\sqrt{2gh} - \sqrt{2gh} = \frac{1}{5}\sqrt{2gh}$ ← initial upward velocity.

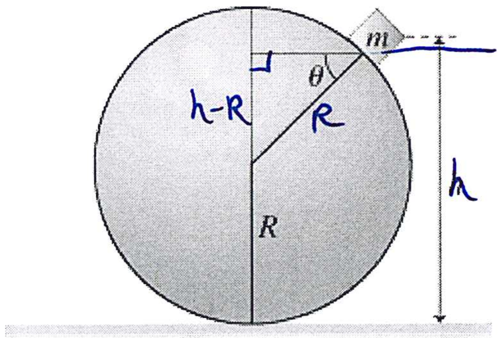
use $E_1 = E_2$

$$\tfrac{1}{2}\cancel{m}v_1^2 = \cancel{m}g\,h_{max}$$

$$h_{max} = \frac{v_1^2}{2g} = \frac{\left(\frac{1}{5}\sqrt{2gh}\right)^2}{2g} = \frac{h}{25}$$

$\boxed{h_{max} = h/25}$

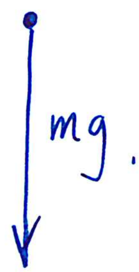
**Problem B** (6 points)
A large globe has a radius $R = 5.00$ m and a frictionless surface.
A small block of mass $m = 1.00 \times 10^{-3}$ kg starts to slide with a tiny (negligible) speed from the very top of the globe and slides along the surface of the globe. The initial height above the ground of the mass, when it is almost at rest, is $h_{\text{initial}} = 10.0$ m. At what height above the ground, $h_{\text{crit}}$, does the block leave the surface of the globe?



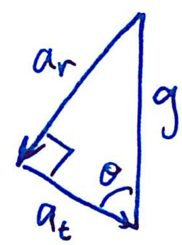

$\boxed{\sin\theta = \frac{h-R}{R}}$ , from geometry.

$n = 0$ no normal force at $h = h_{crit}$.

free body diagram:




acceleration:




$a_r$ = radial acceleration
$a_t$ = tangential acceleration

$$\sin\theta = \frac{a_r}{g}$$

$$a_r = \frac{v^2}{R} = g\sin\theta$$

$$v^2 = Rg\left(\frac{h-R}{R}\right)$$

$$\boxed{v^2 = gh - gR}$$

Conservation of Energy.

$$E_1 = E_2$$

$$\frac{1}{2}\cancel{m}v_1^2 + \cancel{m}gh_1 = \frac{1}{2}\cancel{m}v_2^2 + \cancel{m}gh_2$$

$v_1 = 0$, $h_1 = 2R$, $h_2 = h$

$$2gR = \frac{v^2}{2} + gh$$

$$2gR = \frac{1}{2}(gh - gR) + gh$$

$$4\cancel{g}R = \cancel{g}h - \cancel{g}R + 2\cancel{g}h$$

$$4R + R = h + 2h$$

$$5R = 3h$$

$$h = \frac{5}{3}R$$

$$h = \frac{5 \cdot 5.00}{3} = 8.333 \text{ m}$$

$$\boxed{h_{\text{crit}} = 8.33 \text{ m}}$$