PHY131H1F - Class 21
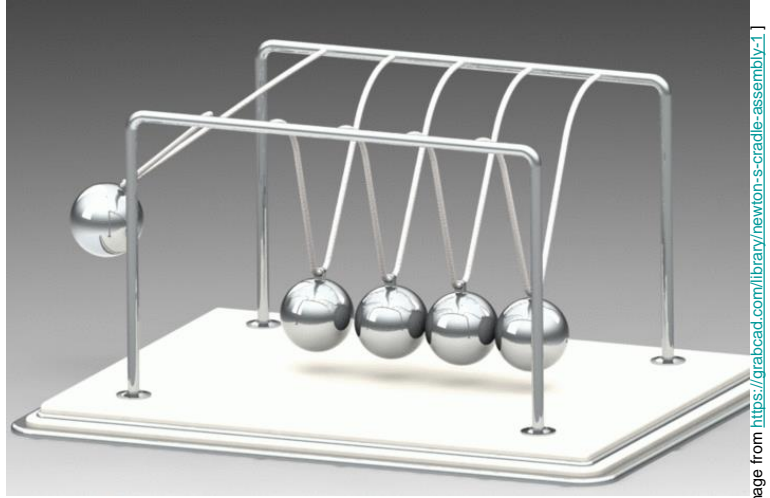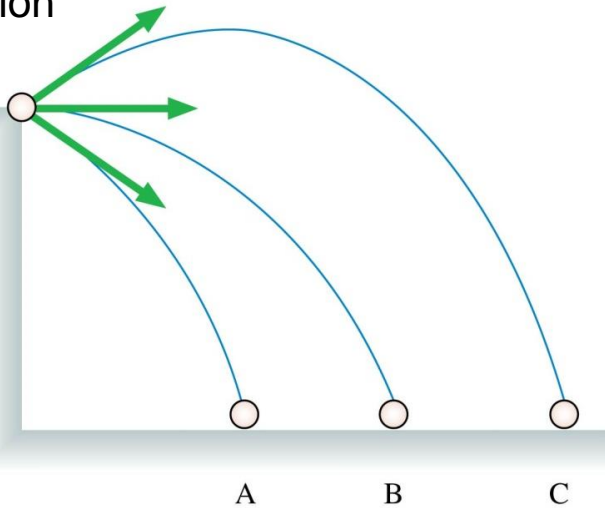
Today:

7.6 Skills for analyzing processes using the work-energy principle

7.7 Collisions

Learning Catalytics Question

Three balls are thrown from a cliff with the same speed but at different angles. Which ball has the greatest speed just before it hits the ground?

A   B   C

A.   Ball A.
B.   Ball B.
C.   Ball C.
D.   All balls have the same speed.

Learning Catalytics Question

A hockey puck sliding on smooth ice at 4 m/s comes to a 1-m-high hill. Will it make it to the top of the hill?

4 m/s

1 m

$K_i = \frac{1}{2} m v_i^2$

$= m \frac{4^2}{2}$

$K_i = 8m$

$U_f = mg y_f$

$= m(9.8)1$

$U_f = 9.8\,m$

A. Yes.

B. No.

C. Can't answer without knowing the mass of the puck.

D. Can't say without knowing the angle of the hill.

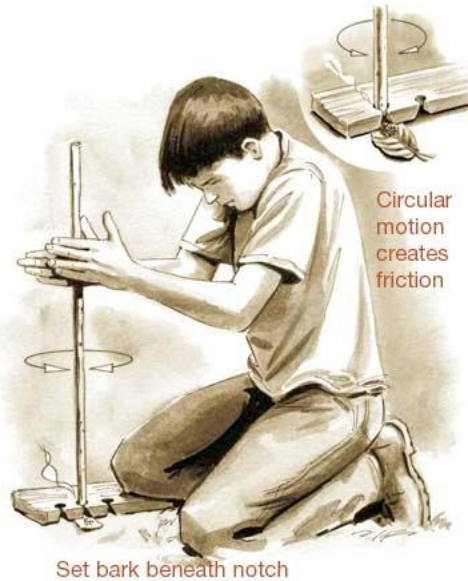$K_i + U_i = K_f + U_f$

$K_f = K_i - U_f = -1.8m$

Learning Catalytics Thoughts from last time…

- *"Will practical material be on the test"*
- **Harlow answer:** Yes! I am working with Professors Wilson and Sealfon on making the midterm. Prof. Sealfon is suggesting some Practicals questions.

- *"If I lifted a ball up, then brought it back down to its initial position, I would have done no work on the ball right? "*
- **Harlow answer:** Correct. You do positive work on the way up, negative work on the way down, and the total work is zero.

- *"if I would like to enter the physics department, did I make a mistake taking 131 instead of 151?"*
- **Harlow answer:** Nope. PHY131+PHY132 are acceptable first year courses for all of the Physics Programs including the Specialist, and they also form an acceptable prerequisite for all the second year PHY courses. (But you must have MAT135+MAT136, of course!)

Internal Energy

- Dissipative forces transform macroscopic energy (kinetic), into internal thermal energy.

- Internal energy is the microscopic energy due to random vibrational and rotational motion of atoms and molecules.

- For kinetic friction:

$$\Delta U_{\text{int}} = f_k \, d$$

Circular
motion
creates
friction

Set bark beneath notch

Conservation of Energy

- If no external work is done on the system, and no heat is exchanged between the system and its environment, then:

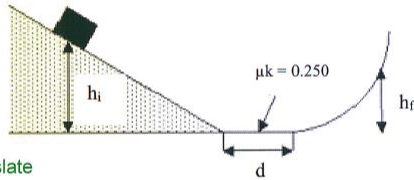$$\Delta U_{\text{int}} = |f_k \, d|$$

$$K_i + U_i = K_f + U_f + \Delta U_{\text{int}}$$

- K is the kinetic energy of the system.

- U is the total potential energy of the system, (recall ΔU is $-1\times$ the work done by conservative forces)

- ΔU_{int} is the increase internal thermal energy of the system due to kinetic friction.

[Doc Cam Examples]
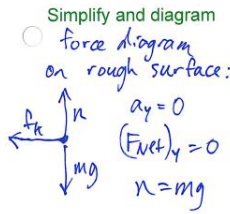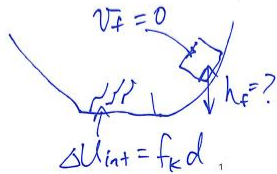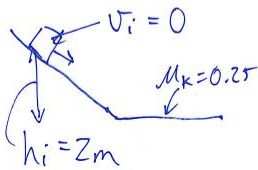
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- **From a past PHY131 Final exam:**

- A small box of mass $m = 10.0$ kg is released from rest at an initial height of $h_i = 2.00$ meters on a frictionless incline as shown. At the bottom of the ramp, it encounters a rough surface with length $d = 1.00$ m and $\mu_k = 2.50 \times 10^{-1}$, and then a **frictionless** circular rise.

- At what height h_f does the box stop on the circular rise?

$\mu_k = 0.250$

h_i

h_f

d

Sketch and translate

Initial   Final

$v_i = 0$ $v_f = 0$

$\mu_k = 0.25$ $h_f = ?$

$h_i = 2m$ $\Delta U_{int} = f_k d$

Simplify and diagram

force diagram on rough surface:

n

f_k

mg

$a_y = 0$

$(F_{net})_y = 0$

$n = mg$

$$K_i + U_{gi} = K_f + U_{gf} + \Delta U_{int}$$
$$\overset{0}{} \qquad \overset{0}{}$$

$$\Delta U_{int} = |f_k d| = \mu_k n d$$

$$\Delta U_{int} = \mu_k mg d$$

Represent mathematically

$$E_i = E_f$$

$$mgh_i = mgh_f + \mu_k mgd$$

$$h_i = h_f + \mu_k d$$

Solve and Evaluate   Solve for h_f

$$h_f = h_i - \mu_k d$$
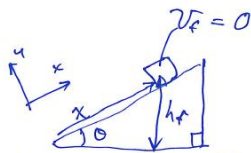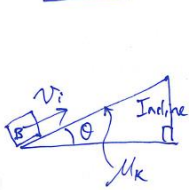
$$= 2 - (0.25)(1) = \boxed{1.75 \, m}$$

A little less high than original height.

- **Ch.7 Example:** A box of mass, m, starts sliding with initial speed v up an incline of angle θ above the horizontal. The coefficient of kinetic friction between the incline and the box is μ_k. How far along the incline does the box go before it stops?
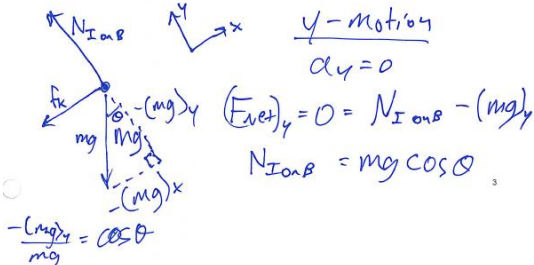
Sketch and translate

Initial   Final

v_i $v_f = 0$

θ Incline θ h_f

μ_k

Friction creates thermal ΔU_{int} as it slides

$\Delta U_{int} = f_k d = f_k x$

Simplify and diagram

Force Diagram

$N_{I\,on\,B}$

f_k

mg

$-(mg)_y$

$-(mg)_x$

$\dfrac{-(mg)_y}{mg} = \cos\theta$

y-Motion

$a_y = 0$

$(F_{net})_y = 0 = N_{I\,on\,B} - (mg)_y$

$N_{I\,on\,B} = mg\cos\theta$

Let's Use Energy:

$$K_i + U_{gi} = K_f + U_{gf} + \Delta U_{int}$$
$$y_i = 0$$

Represent mathematically

$$\tfrac{1}{2}mv_i^2 + 0 + 0 = mgh_f + f_k x$$

$$\tfrac{1}{2}mv_i^2 = mgh_f + \mu_k mg\cos\theta \, x$$

Let's eliminate h_f, and solve for x:

x h_f $\sin\theta = \dfrac{h_f}{x}$
θ

$$\boxed{h_f = x\sin\theta}$$

Solve and Evaluate

$$\frac{v_i^2}{2} = g(x\sin\theta) + \mu_k g\cos\theta \, x$$

$$\frac{v_i^2}{2} = x\left[g\sin\theta + g\mu_k\cos\theta\right]$$

$$\boxed{x = \frac{v_i^2}{2g(\sin\theta + \mu_k\cos\theta)}}$$

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Quick LC Question 1 of 3:

- Two objects collide. All external forces on the objects are negligible.
- If the collision is "elastic", that means it conserves

A. Momentum $p=mv$

B. Kinetic energy $E = ½\, mv^2$

C. Both

D. Neither

Quick LC Question 2 of 3:

- Two objects collide. All external forces on the objects are negligible.
- If the collision is "inelastic", that means it conserves

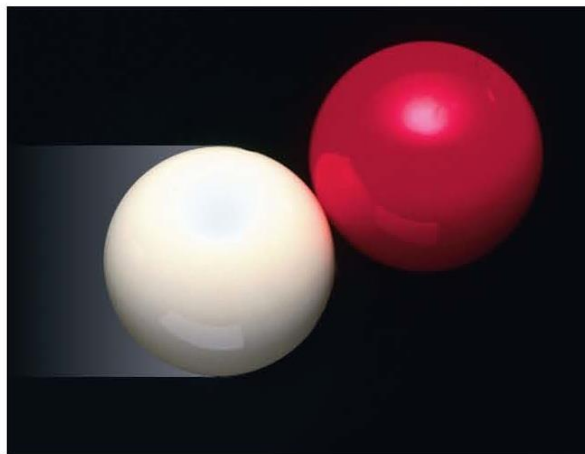A. Momentum $p=mv$

B. Kinetic energy $E = ½\, mv^2$

C. Both

D. Neither

Quick LC Question 3 of 3:

- Two objects collide. All external forces on the objects are negligible.
- If the collision is "totally inelastic", that means
A. momentum is not conserved.
B. the final kinetic energy is zero.
C. the objects stick together.
D. one of the objects ends with zero velocity.

Elastic Collisions

A perfectly elastic collision conserves both momentum and mechanical energy.

Elastic Collision in 1 Dimension when ball 2 is initially at rest.

Consider a ~~head-on,~~ perfectly elastic collision of a ball of mass m_1 having initial velocity v_{1i}, with a ball of mass m_2 that is initially at rest.

Before: ① $\xrightarrow{v_{1i}}$ ② K_i

During: ①②   During the collision energy is stored as elastic potential energy.

After: ① $\xrightarrow{v_{1f}}$ ② $\xrightarrow{v_{2f}}$ $K_i = K_f$

The balls' velocities after the collision are v_{1f} and v_{2f}.

Elastic Collision in 1 Dimension when ball 2 is initially at rest.

Unknowns:
v_{1f}, v_{2f}

Momentum conservation: $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i}$

Kinetic energy conservation: $\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2} m_1 v_{1i}^2$

There are two equations, and two unknowns: v_{1f} and v_{2f}.
Solving for the unknowns gives:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$ (Head-on elastic collision with ball 2 initially at rest.)

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Elastic Collision in 1 Dimension when ball 2 is initially at rest.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

(Head-on elastic collision with ball 2 initially at rest.)

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

- These equations come in especially handy, because you can always switch into an inertial reference frame in which ball 2 is initially at rest!

- Both of these equations will appear in the **"Possibly helpful information for this test"** section on the first page of the second midterm and the final exam.

Demonstration and Example

- A 0.50 kg basketball and a 0.05 kg tennis ball are stacked on top of each other, and then dropped from a height of 0.82 m above the floor.

- How high does the tennis ball bounce?

- Assume all perfectly elastic collisions.

Segment 1: freefall of both balls as they fall, $v_i = 0$.

Segment 2: Elastic collision of basketball with **floor**. Tennis ball continues downward, unaffected.

Segment 3: Elastic collision of upward moving basketball (1) with downward moving tennis ball (2).

Segment 4: freefall of upward moving tennis ball.
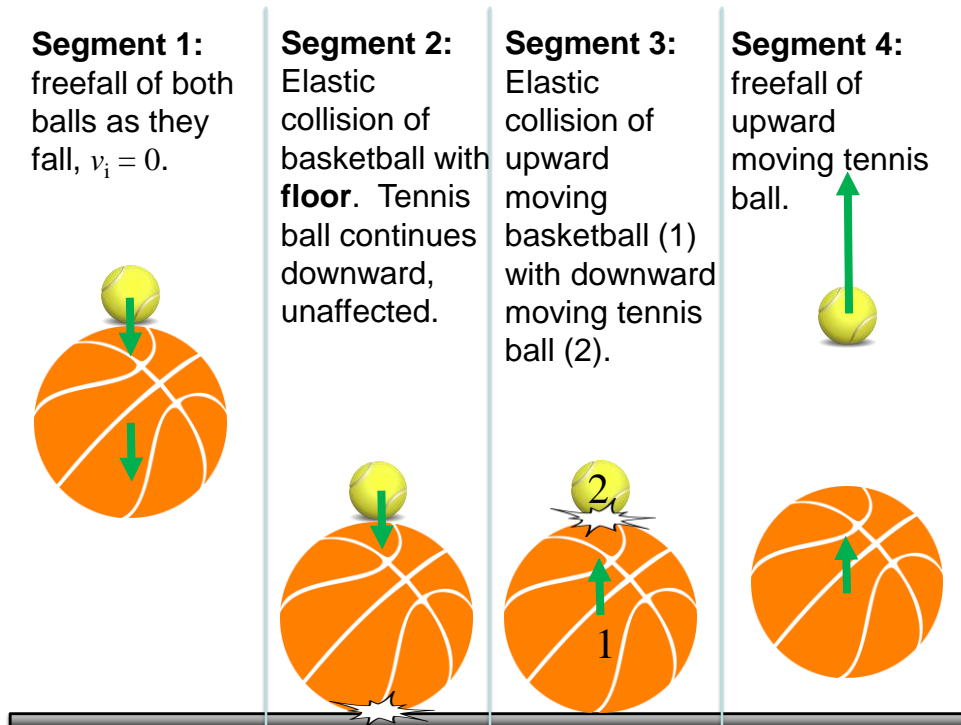
Demonstration and Example

- Divide motion into segments.
- **Segment 1:** free-fall of both balls from a height of h = 0.82 m. Use conservation of energy: $K_i + U_{gi} = K_f + U_{gf}$

$$0 + mgh = \tfrac{1}{2} m\, v_f^2 + 0$$

$v_f = \pm[2gh]^{\frac{1}{2}} = -4.0 \text{ m/s}$, for both balls.

- **Segment 2:** basketball bounces elastically with the floor, so its new velocity is +4.0 m/s.

Demonstration and Example

- **Segment 3:** A 0.50 kg basketball moving upward at 4.0 m/s strikes a 0.05 kg tennis ball, initially moving downward at 4.0 m/s.
- Their collision is perfectly elastic.
- What is the speed of the tennis ball immediately after the collision?

[Doc Cam Notes]

▪A 0.50 kg basketball moving upward at 4.0 m/s strikes a 0.05 kg tennis ball, initially moving downward at 4.0 m/s. Their collision is perfectly elastic. What is the speed of the tennis ball immediately after the collision?

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

(Elastic collision with ball 2 initially at rest.)

Sketch and translate

Initial.

② ↓ $v_{2i} = -4$ m/s
① ↑ $v_{1i} = +4$ m/s

Final

② ↑ $v_{2f} = ?$
① ↑ v_{1f}
↑
Don't care about.

Simplify and diagram

Let's go to ball 2 reference frame, in which it is at rest. To do this, we will add +4 m/s to y-velocities. (We must remember to subtract 4 m/s at the end to get back to lab frame).

Represent mathematically

Ball 2 frame:
Initial.

② $v_{2i}' = 0$
① $v_{1i}' = 4 + 4 = 8$ m/s.

Solve and Evaluate

$$v_{2f}' = \frac{2m_1}{m_1 + m_2} v_{1i}' = \frac{2(0.5) 8}{0.5 + 0.05}$$

$$v_{2f}' = +14.5 \text{ m/s}$$

Lab frame:

$$v_{2f} = v_{2f}' - 4 \text{ m/s}$$

$$\boxed{v_{2f} = +10.5 \text{ m/s}}$$

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Demonstration and Example

- **Segment 4:** freefall of tennis ball on the way up. $v_{i2} = +10.5$ m/s.

- Use conservation of energy:

$K_i + U_{gi} = K_f + U_{gf}$

$\frac{1}{2}\, m\, v_i^2 + 0 = 0 + mgh$

$h = v_i^2/(2g) = 5.6$ m.

- So the balls were dropped from 0.82 m, but the tennis ball rebounds up to 5.6 m! (Assuming no energy losses.)