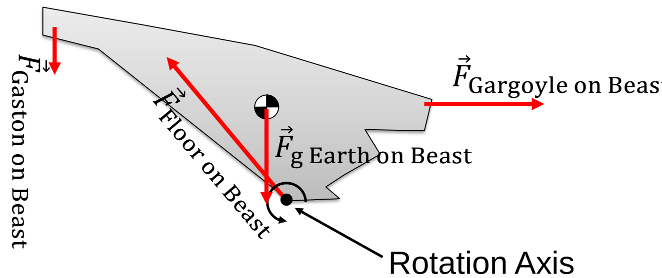
PHY131H1F - Hour 25

Today, we finish up Chapter 8:

8.5 Static Equilibrium Problems

8.6 Stability

Beast = “system”



Static Equilibrium Problems

- In equilibrium, an object has no net force and no net torque.
- Draw an extended free-body diagram that shows where each force acts on the object.
- Set up x and y axes, and choose a rotation axis. All of these choices should be done to simplify your calculations.
- Each force has an x and y component and a torque. Sum all of these up.
- Three equations which you can use are:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

Learning Catalytics Question.

An object could be in static equilibrium when

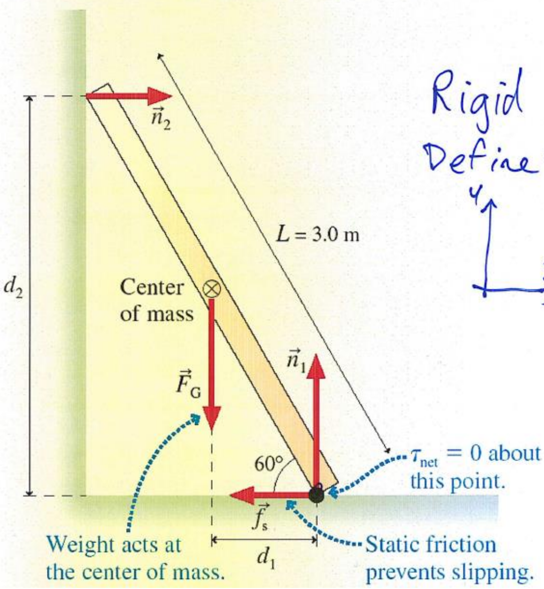
A. only one force is acting on it.

B. two or more forces are acting on it.

C. only one torque is acting on it.

[Doc Cam examples]

A 3.0-m-long ladder leans against a frictionless wall at an angle of 60°. What is the minimum value of μ_s, the coefficient of static friction with the ground, that prevents the ladder from slipping?



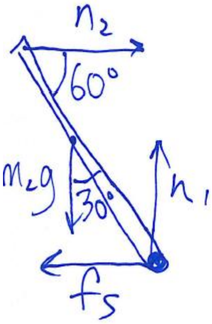
Sketch and translate

Static Rigid Body Equilibrium

Define:

y, x

Rotation axis: Foot of ladder



minimum μ_s will be for "just about to slip"

$f_s = f_{s,max}$

$f_s = \mu_s n_1$

Simplify and diagram

$\sum F_x = n_2 - f_s = 0 \Rightarrow n_2 = f_s$ (1)

$\sum F_y = -m_L g + n_1 = 0 \Rightarrow n_1 = m_L g$ (2)

$\sum \tau = -n_2 L \sin 60^\circ + m_L g \left(\frac{L}{2}\right) \sin 30^\circ = 0$ (3)

Represent mathematically

Also, $f_s = \mu_s n_1$ (4)

Unknowns: n_1, n_2, f_s, μ_s Find μ_s

(4) $\mu_s = \frac{f_s}{n_1}$ ← (2) $\mu_s = \frac{f_s}{m_L g}$ ← (1) $\mu_s = \frac{n_2}{m_L g}$

(3) $\Rightarrow n_2 \cancel{L} \sin 60 = m_L g \left(\frac{\cancel{L}}{2}\right) \sin 30$

Solve and Evaluate

$n_2 = m_L g \frac{\sin 30}{2 \sin 60}$

$\mu_s = \frac{\cancel{m_L g} \sin 30}{\cancel{m_L g}\, 2 \sin 60} = \frac{\sin 30}{2 \sin 60} = \boxed{0.29}$

Evaluate: as $\theta = 60$ ↑, $\mu_{s\,min}$ goes down.

⇒ harder to slip.

A uniform steel beam of length L and mass m_1 is attached via a hinge to the side of a building. The beam is supported by a steel cable attached to the end of the beam at an angle θ, as shown. Through the hinge, the wall exerts an unknown force, $\vec{F}$, on the beam. A workman of mass m_2 sits eating lunch a distance d from the building.

a) Find T, the tension in the cable.

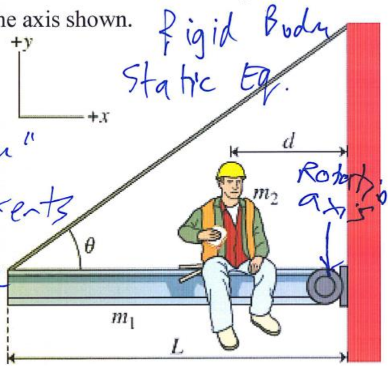
b) Find F_x, the x-component of the force exerted by the wall on the beam ($\vec{F}$), using the axis shown.

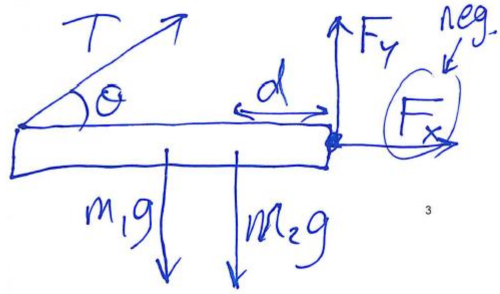
Rigid Body Static Eq.

Sketch and translate

Define System = "Beam"

Workman exerts downward force on beam $F = m_2 g$





3

Simplify and diagram

$$\sum F_x = T\cos\theta + F_x = 0 \Rightarrow F_x = -T\cos\theta \quad (1)$$

$$\sum F_y = T\sin\theta - m_1 g - m_2 g + F_y = 0 \quad (2)$$

$$\sum \tau = -TL\sin\theta + m_1 g\left(\frac{L}{2}\right)\sin 90^\circ + m_2 g d \sin 90^\circ = 0 \quad (3)$$

Represent mathematically

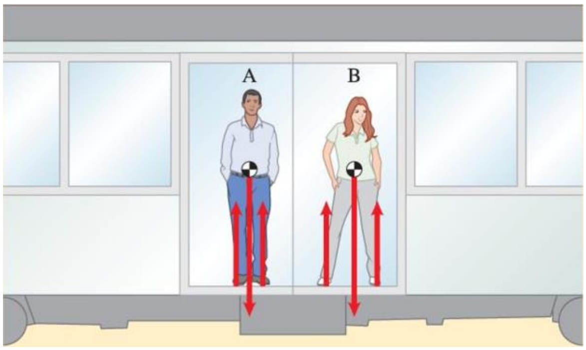
3 eqs, 3 unknowns:

T, F_x, F_y

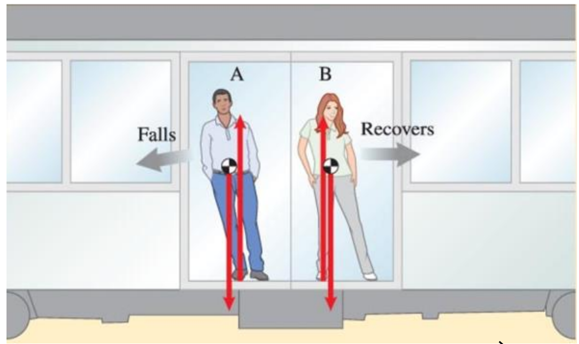
↑ normal force

Equilibrium and tipping objects

- You have probably observed that it is easier to balance and avoid falling while standing in a moving bus or subway train if you spread your feet apart in the direction of motion.
- By assuming this stance, you increase the **area of support**—the area of contact between an object and the surface it is supported by.



Train At Rest

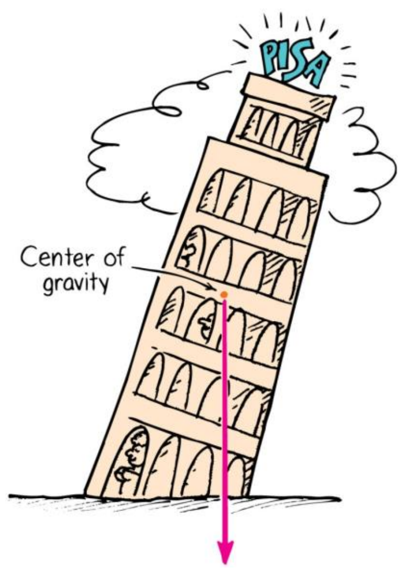


Accelerating to the Right $\vec{a}$

Centre of Gravity—Stability

The location of the centre of gravity is important for stability.

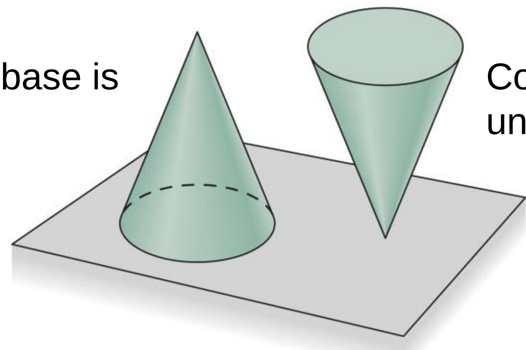
- If we draw a line straight down from the centre of gravity and it falls inside the base of the object, it is in stable **equilibrium;** it will balance.
- If it falls outside the base, it is unstable.



Stability

- An equilibrium is stable if a slight disturbance from equilibrium results in forces and/or torques that tend to restore the equilibrium.
- An equilibrium is unstable if a slight disturbance causes the system to move away from the original equilibrium.

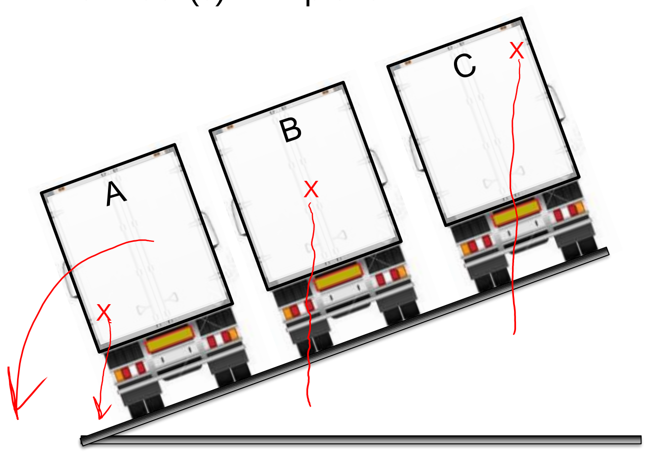
Cone on its base is stable



Cone on its tip is unstable

Learning Catalytics Question

The centres of gravity of the three trucks parked on a hill are shown by the Xs. Which truck(s) will tip over?



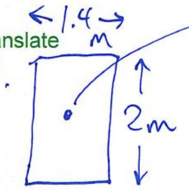
D. All three of the trucks will tip over.
E. None of the three will tip over.

[Doc Cam example]

A refrigerator is 2.0 m high, and 1.4 m wide. On a flat floor, by what maximum angle can it tip sideways and still not fall over on its side?

Sketch and translate

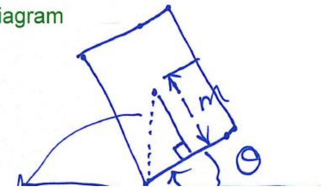
Initial.



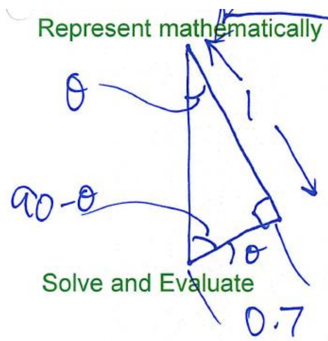
Assume center of mass is at geometric centre of fridge

Simplify and diagram

Final:



Represent mathematically



$\frac{1.4}{2} = 0.7\ m$

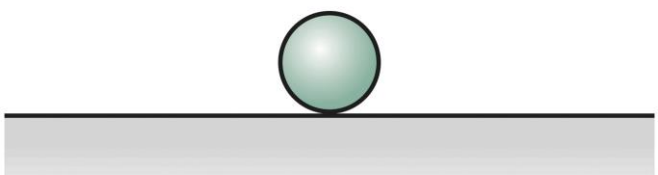
Solve and Evaluate

$\tan\theta = \frac{0.7}{1}$

$\Rightarrow \theta = \tan^{-1}(0.7/1)$

$\theta = 35°$

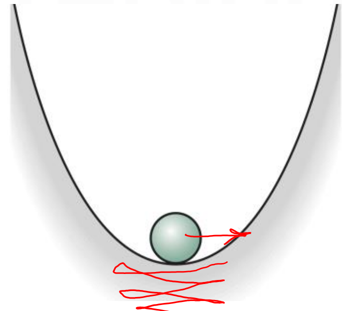
STABILITY JEOPARDY!



It could look like this.

A. What is “Stable Equilibrium”?
B. What is “Neutrally Stable Equilibrium”?
C. What is “Unstable Equilibrium”?
D. What is “Metastable Equilibrium”?

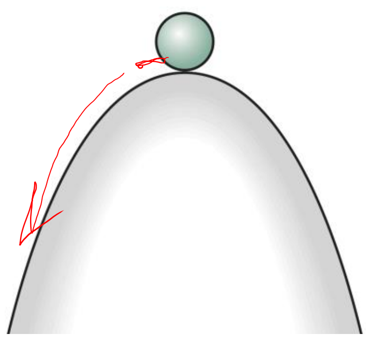
STABILITY JEOPARDY!



It could look like this.

A. What is “Stable Equilibrium”?
B. What is “Neutrally Stable Equilibrium”?
C. What is “Unstable Equilibrium”?
D. What is “Metastable Equilibrium”?

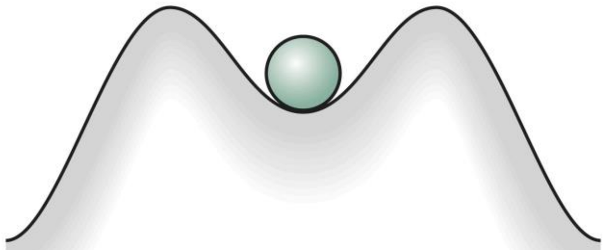
STABILITY JEOPARDY!



It could look like this.

A. What is “Stable Equilibrium”?
B. What is “Neutrally Stable Equilibrium”?
C. What is “Unstable Equilibrium”?
D. What is “Metastable Equilibrium”?

STABILITY JEOPARDY!

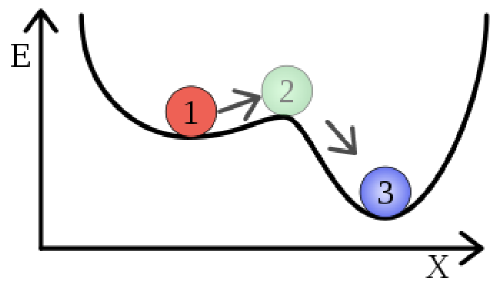


It could look like this.

A. What is “Stable Equilibrium”?
B. What is “Neutrally Stable Equilibrium”?
C. What is “Unstable Equilibrium”?
D. What is “Metastable Equilibrium”?

https://en.wikipedia.org/wiki/Metastability

Metastability



A metastable state of weaker bond (1), a transitional 'saddle' configuration (2) and a stable state of stronger bond (3).

- **Examples of Metastability:**
- A ball resting in a hollow on a slope. If the ball is only slightly pushed, it will settle back into its hollow, but a stronger push may start the ball rolling down the slope.
- Bowling pins. They may either merely wobble for a moment, or tip over completely.
- Isomerisation. Higher energy isomers are long lived as they are prevented from rearranging to their preferred ground state by small barriers in the potential energy.