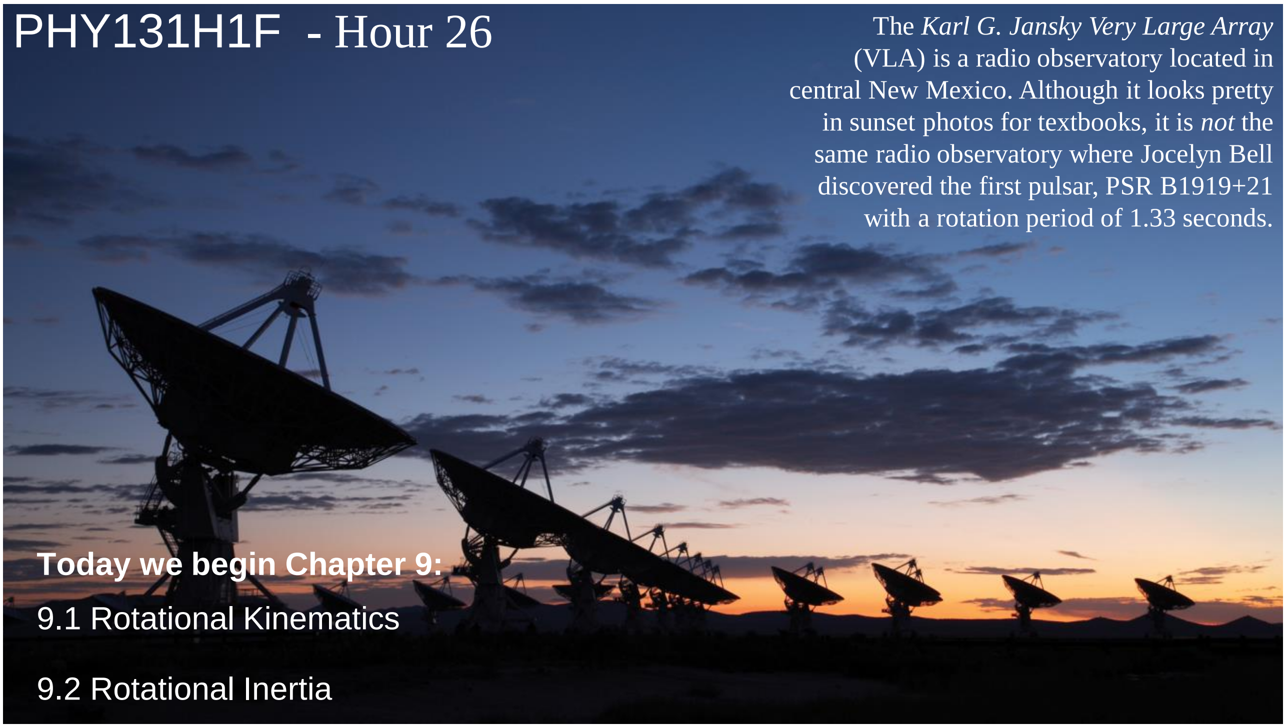
PHY131H1F - Hour 26



The *Karl G. Jansky Very Large Array* (VLA) is a radio observatory located in central New Mexico. Although it looks pretty in sunset photos for textbooks, it is *not* the same radio observatory where Jocelyn Bell discovered the first pulsar, PSR B1919+21 with a rotation period of 1.33 seconds.

Today we begin Chapter 9:

9.1 Rotational Kinematics

9.2 Rotational Inertia

Mastering Physics

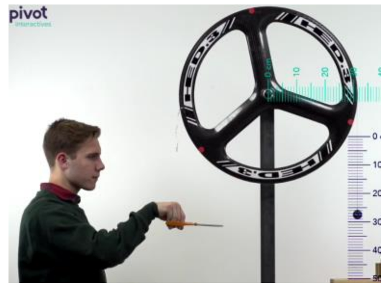
What's up on the MyLab and Mastering?

- Notice that Homework 9 has been posted on MasteringPhysics. It is due Friday Nov.23.
- Also, I have posted an optional item called "Ch.9 Videos – Optional" which I recommend you check out.

Mastering Physics

Chapter 9 Videos (Optional)

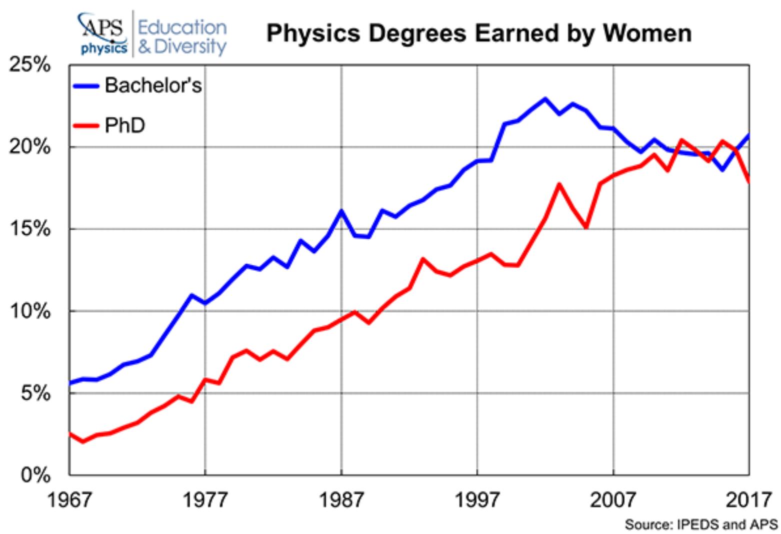
- Featuring, Buzzcut Guy gets really dizzy!



- Plus, yet another white guy!
- And two Khan-Academy-style videos about solving Ch.9 problems.

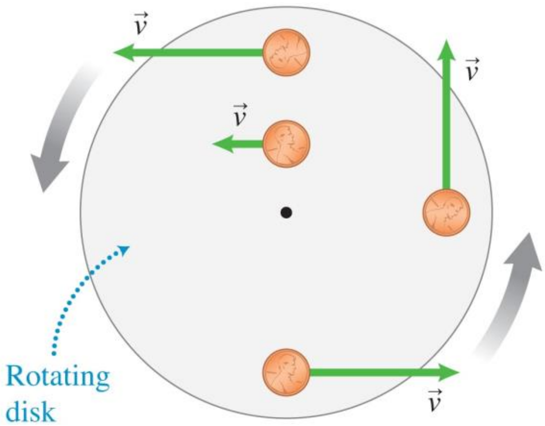
..from the Physics Inclusivity Committee

- Our department is committed to encouraging the recruitment, retention, and career development of women physicists at all levels.

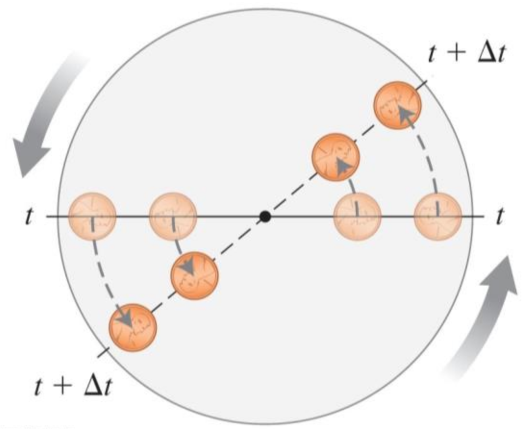


Suppose a horizontal disk is rotating on a lab bench and your are looking down on it. The rotation axis passes through the centre, and is perpendicular to the disk (out of page)

The direction of the velocity $\vec{v}$ for each coin changes continually.



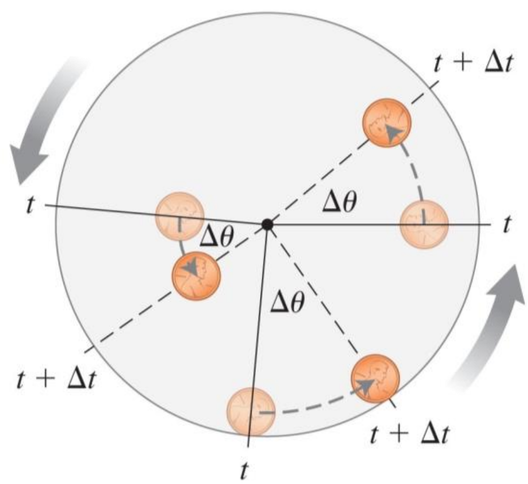
Coins at the edge travel farther during Δt than those near the center. The speed v will be greater for coins near the edge than for coins near the center.



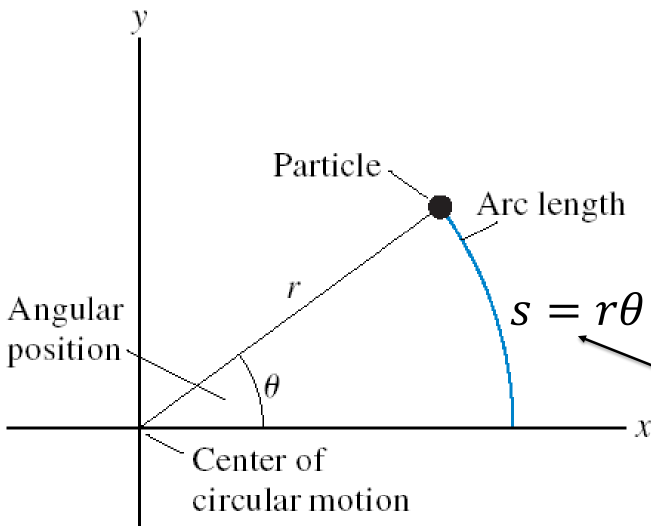
Rotational kinematics

- There are similarities between the motions of different points on a rotating rigid body.
  - During a particular time interval, all coins at the different points on the rotating disk turn through the same angle.
  - **Perhaps we should describe the rotational position of a rigid body using an angle.**

All coins turn through the same angle in Δt, regardless of their position on the disk.

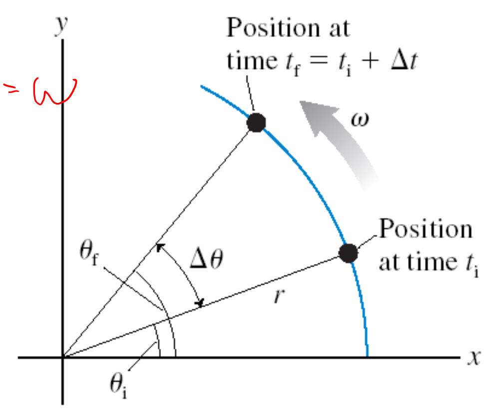


Rotational (Angular) Position



NOTE: This equation only works if θ is measured in **radians!**

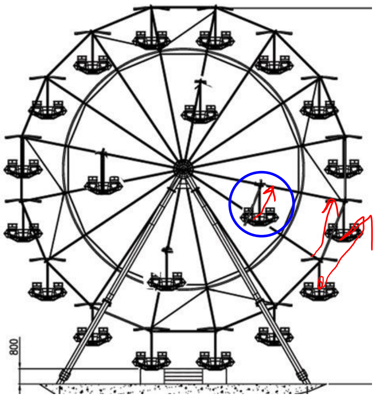
Angular Velocity



"omega"

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad \text{(angular velocity)}$$

$$\theta_f = \theta_i + \omega \Delta t \quad \text{(uniform circular motion)}$$

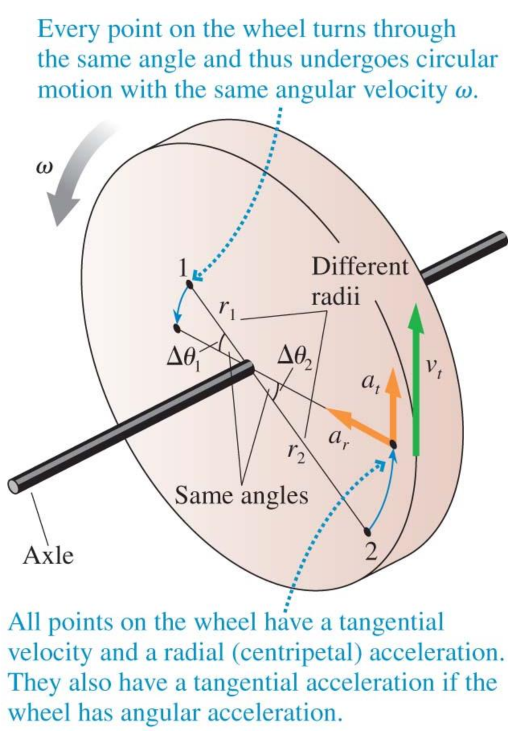


A carnival has a Ferris wheel where some seats are located halfway between the center and the outside rim. Compared with the seats on the outside rim, the inner cars have

A. Smaller angular speed and greater tangential speed
B. Greater angular speed and smaller tangential speed
C. The same angular speed and smaller tangential speed
D. Smaller angular speed and the same tangential speed
E. The same angular speed and the same tangential speed

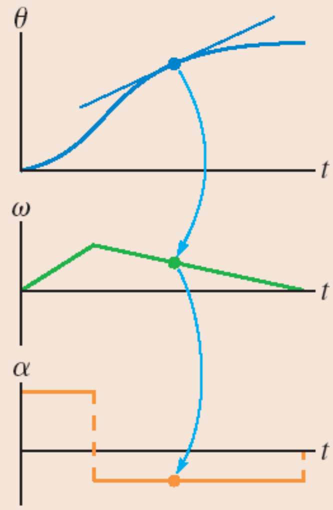
Rigid Body Rotation

- Angular velocity, ω, is the rate of change of angular position, θ.
- The units of ω are rad/s.
- If the rotation is speeding up or slowing down, then its angular acceleration, α, is the rate of change of angular velocity, ω.
- The units of α are rad/s^2.
- All points on a rotating rigid body have the same ω and the same α.



Angle, angular velocity, and angular acceleration are related graphically.

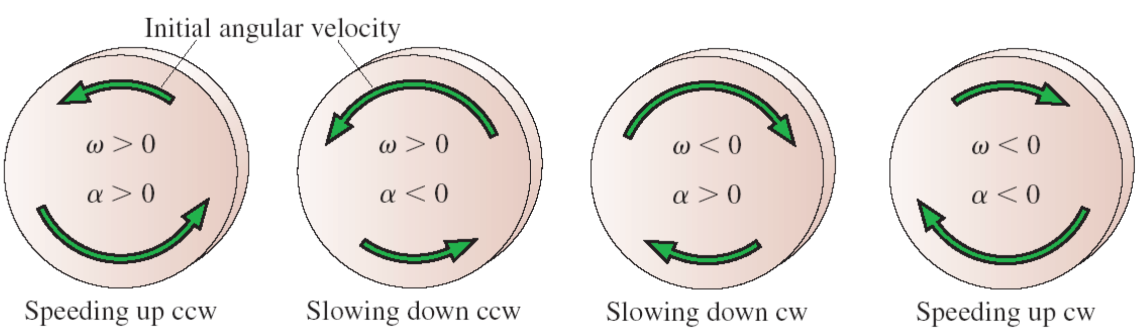
- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.



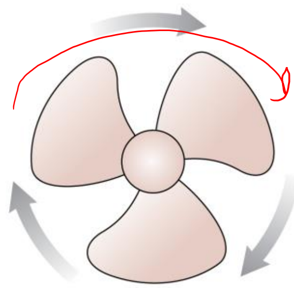
- Arc length: $s = \theta r$
- Tangential velocity: $v_t = \omega r$
- Tangential acceleration: $a_t = \alpha r$

Rotational Kinematics

The signs of angular velocity and angular acceleration.



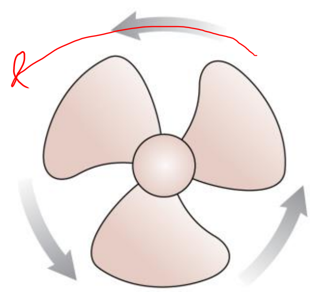
Historical Convention:
Define **positive** angular displacement to be **counter-clockwise**.



The fan blade is speeding up. What are the signs of ω and α?

A. ω is positive and α is positive.
B. ω is positive and α is negative.
C. ω is negative and α is positive.
D. ω is negative and α is negative.

Historical Convention:
Define **positive** angular displacement to be **counter-clockwise**.



The fan blade is slowing down. What are the signs of ω and α?

A. ω is positive and α is positive.
B. ω is positive and α is negative.
C. ω is negative and α is positive.
D. ω is negative and α is negative.

Rotational Kinematics

Linear	**Rotational Analogy**
• x specifies position. The S.I. Unit is metres.	• θ is **angular position**. The S.I. Unit is radians, where 2π radians = 360°.
• Velocity, v_x, is the slope of the x vs t graph. [m/s]	• **Angular velocity**, ω, is the slope of the θ vs t graph. [rad/s]
• Acceleration, a_x, is the slope of the v_x vs t graph. [m/s^2]	• **Angular Acceleration**, α, is the slope of the ω vs t graph. [rad/s^2]

Radians are the Magical Unit!

- Radians appear and disappear as they please in your equations!!!
- They are the only unit that is allowed to do this!
- Example: $v_t = \omega r$

Units: $\left[\frac{m}{s}\right] = \left[\frac{\text{rad}}{s}\right][m]$??

Rotational Kinematics

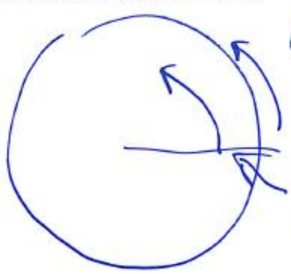
Table 9.1, Page 256

Translational motion	Rotational motion	
$v_x = v_{0x} + a_x t$	$\omega = \omega_0 + \alpha t$	(9.3)
$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$	(9.6)
$2a_x(x - x_0) = v_x^2 - v_{0x}^2$	$2\alpha(\theta - \theta_0) = \omega^2 - \omega_0^2$	(9.7)

[Doc Cam Example]

A bicycle wheel has an initial angular velocity of 1.50 rad/s, and a constant angular acceleration of 0.200 rad/s^2. Through what angle has the wheel turned between t = 0 and t = 2.50 s?

Sketch and translate



Simplify and diagram

$\omega_0 = 1.5 \ \frac{rad}{s}$ $\alpha = +0.2 \ \frac{rad}{s^2}$

$t = 0, \ \theta_0 = 0$ rad

Need θ at $t = 2.5$ s

Represent mathematically

Let's use:

From aid sheet: $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

Solve and Evaluate

$$\theta = 0 + 1.5(2.5) + \frac{1}{2}(0.2)(2.5)^2$$

$$= 3.75 + 0.625$$

$$\theta = 4.37 \text{ rad}$$

→ 6 radians is full circle.
This is less than that.

The "Rolling Without Skidding" Constraints

When a round object rolls without skidding, the distance the axis, or centre of mass, travels is equal to the change in angular position times the radius of the object.

$$s = \theta R$$

The speed of the centre of mass is

$$v = \omega R$$

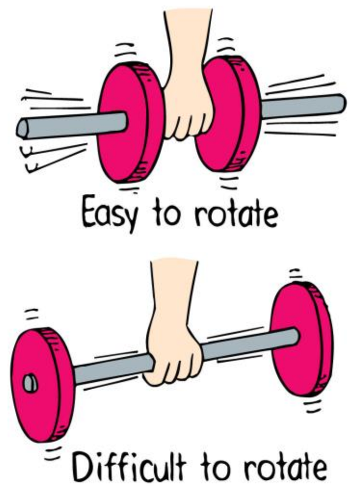
The acceleration of the centre of mass is

$$a = \alpha R$$

Rotational Inertia

Depends upon:

- mass of object.
- distribution of mass around axis of rotation.
  - The greater the distance between an object's mass concentration and the axis, the greater the rotational inertia.



Rotational Inertia

units: $(kg \cdot m^2)$ I

Consider a body made of N particles, each of mass m_i, where i = 1 to N. Each particle is located a distance r_i from the axis of rotation. For this body made of a countable number of particles, the rotational inertia is:

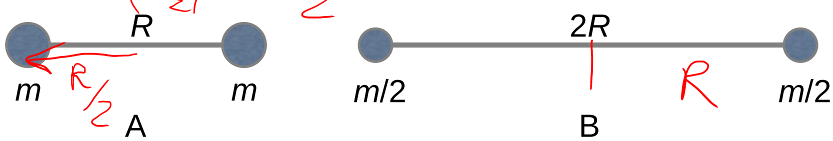
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2$$

The units of rotational inertia are kg m^2. An object's rotational inertia depends on the axis of rotation.

$I = m_1 r_1^2 + m_2 r_2^2$

Which dumbbell has the larger rotational inertia about the midpoint of the rod? The connecting rod is massless.

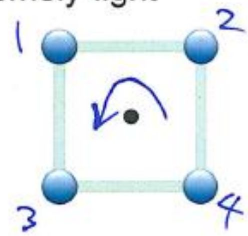
$I_A = 2m\left(\frac{R}{2}\right)^2 = \frac{mR^2}{2}$ $\qquad$ $I_B = 2mR^2$



A. Dumbbell A.
B. Dumbbell B
C. Their rotational inertias are the same.

[Doc Cam Example]

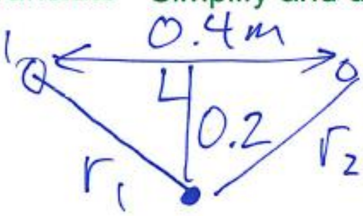
Four small metal spheres, each with mass 0.2 kg, are arranged in a square 0.40 m on a side and connected by extremely light rods.



Find the rotational inertia about an axis through the centre of the square, perpendicular to its plane.

Sketch and translate

Simplify and diagram



All 4 masses contribute same.

Represent mathematically

$I = m_1 r_1^2 + m_2 r_2^2 + \ldots$

$= 4mr^2$

Solve and Evaluate

Pythagorous: $r^2 = 0.2^2 + 0.2^2$

$r^2 = 0.08\ m^2$

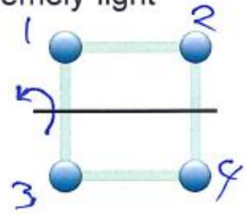
$I = 4(0.2)0.08$

$I = 0.064\ kgm^2$

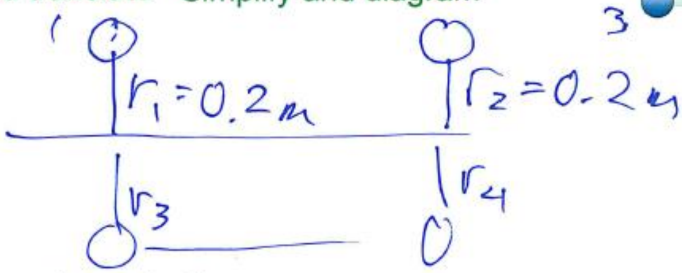
2

Four small metal spheres, each with mass 0.2 kg, are arranged in a square 0.40 m on a side and connected by extremely light rods.

Find the rotational inertia about an axis through the centre of the square, parallel to its plane.



Sketch and translate Simplify and diagram



Represent mathematically

All 4 masses contribute same.

$I = 4mr^2$

Solve and Evaluate

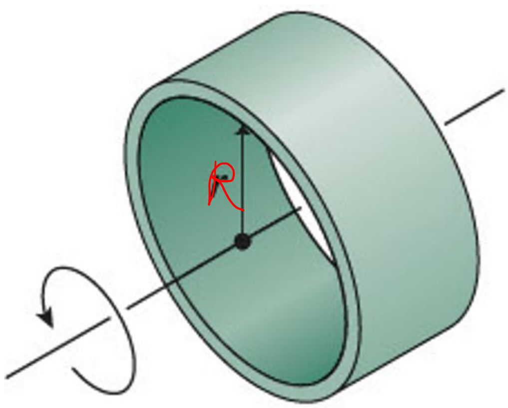
$I = 4(0.2)(0.2)^2$

$I = 0.032 \text{ kg m}^2$

→ half of I for perpendicular axis.

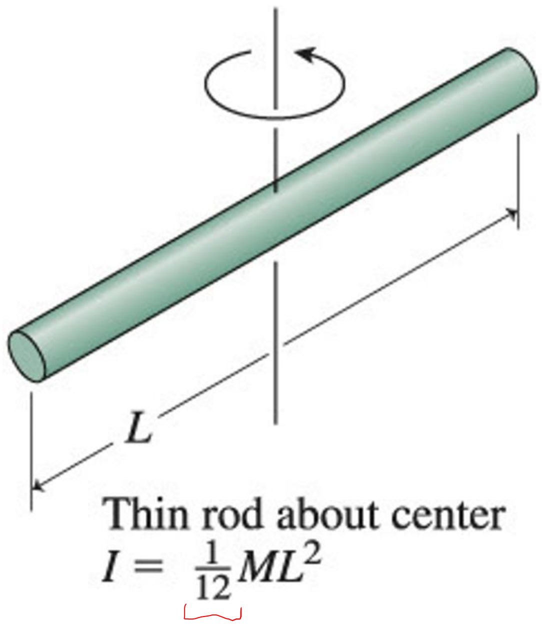
3

Rotational Inertias of Simple Objects

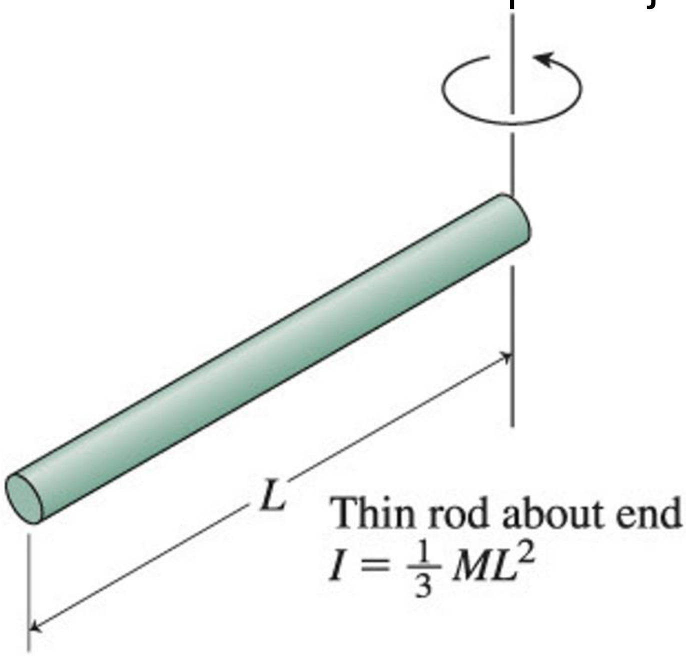


Thin ring or hollow cylinder about its axis
$I = MR^2$

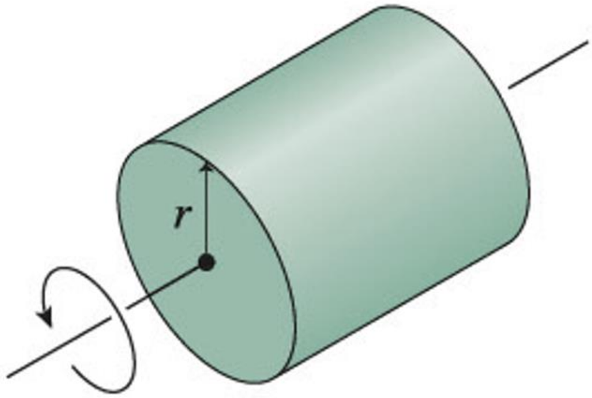
Rotational Inertias of Simple Objects



Rotational Inertias of Simple Objects



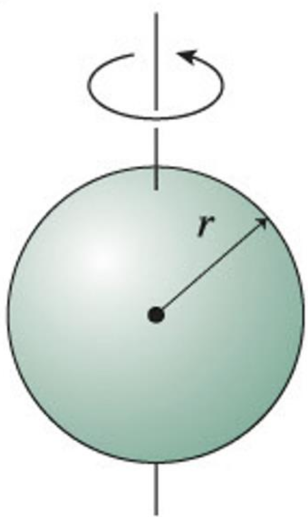
Rotational Inertias of Simple Objects



Disk or solid cylinder
about its axis
$I = \frac{1}{2} MR^2$

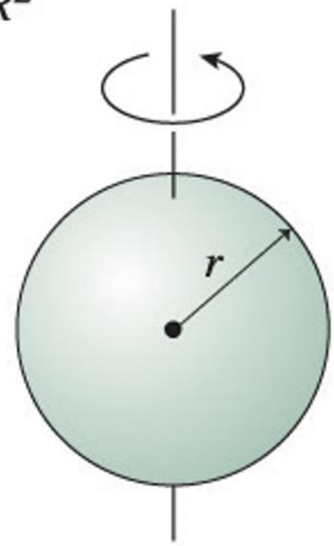
Rotational Inertias of Simple Objects

Solid sphere about diameter
$I = \frac{2}{5} MR^2$



Rotational Inertias of Simple Objects

Hollow spherical shell about diameter
$I = \frac{2}{3} MR^2$



Students want to know!

· Will the rotational inertia of simple objects' table be given on the test??
Harlow answer: Yes!

· Why does a hollow sphere (which is lighter than a filled sphere) have a higher rotational inertia?
Harlow answer: It doesn't!!

Solid sphere about diameter
$I = \frac{2}{5} MR^2$

Hollow spherical shell about diameter
$I = \frac{2}{3} MR^2$

- If you take the same material and just cut out the middle, the rotational inertia goes way down!! Remember, rotational inertia is just the sum of all the mr^2 of every particle in the object.
- But if the mass is the **same**, then the hollow shell must be much more dense, and it has a higher rotational inertia, since this mass has a higher average value of r.

Quick quiz..

***Rotational inertia, I,* is**

A. the rotational equivalent of kinetic energy.
B. the rotational equivalent of mass.
C. the rotational equivalent of momentum.
D. the tendency for anything that is rotating to continue rotating.

Next up: Rotational **Dynamics**…

Linear	**Rotational Analogy**
• x	• θ
• v_x	• ω
• a_x	• α
• Force: F_x	• Torque: τ
• Mass: m	• Rotational Inertia: I

Newton's Second Law:

$$a_x = \frac{(F_{net})_x}{m} \qquad \alpha = \frac{\tau_{net}}{I}$$