

# PHY131H1F - Hour 27

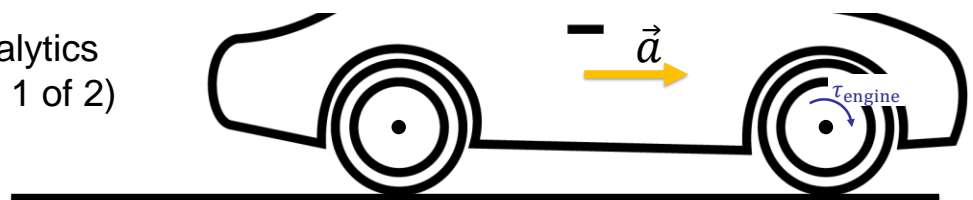


## Today:

9.3 Newton's Second Law for Rotational Motion

9.4 Rotational Momentum

### Learning Catalytics Question (part 1 of 2)



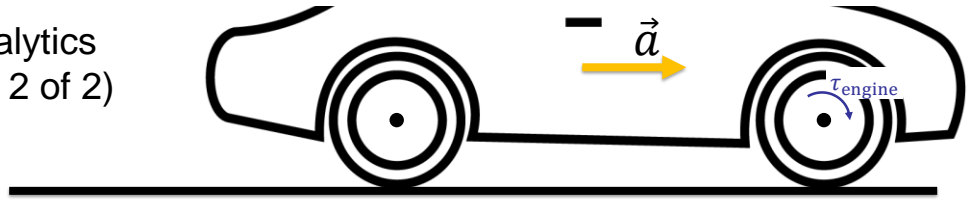
You are sitting in your car, and you step on the gas pedal. The car accelerates forward.

Your car has Front Wheel Drive (FWD). That means the front two wheels are connected to the engine, but the back two wheels just freely rotate on their axles.

As you accelerate, what is the direction of the force of static friction of the road upon the **front** wheels?

- A. Forward
- B. Backward
- C. The static friction force on the front wheels is zero

## Learning Catalytics Question (part 2 of 2)



You are sitting in your car, and you step on the gas pedal. The car accelerates forward.

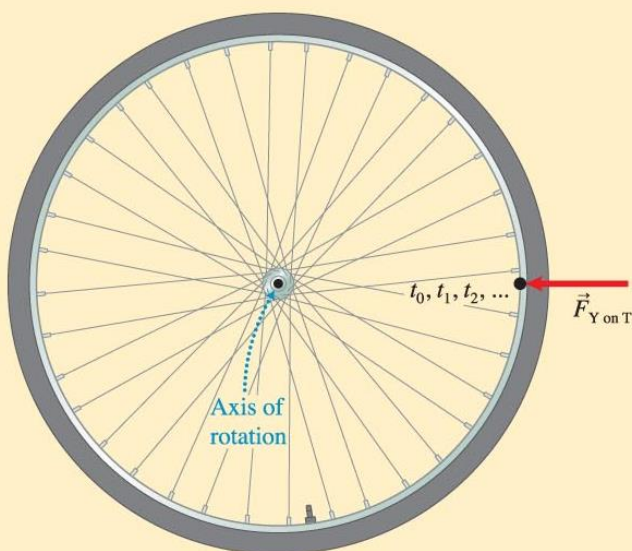
Your car has Front Wheel Drive (FWD). That means the front two wheels are connected to the engine, but the back two wheels just freely rotate on their axles.

As you accelerate, what is the direction of the force of static friction of the road upon the **rear** wheels?

- A. Forward
- B. Backward
- C. The static friction force on the rear wheels is zero

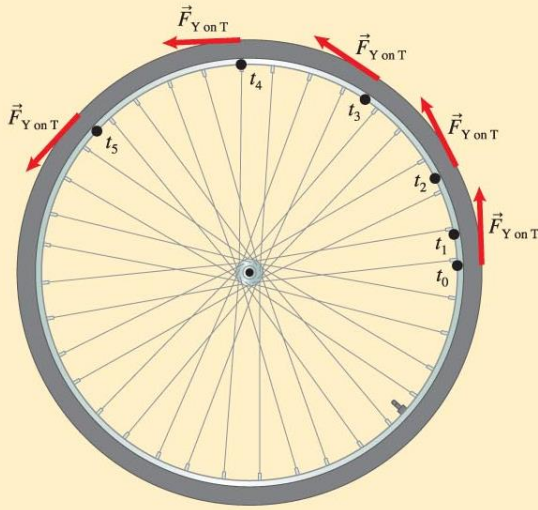
## Observational Experiment #1

**Experiment 1:** Your bike sits upside down. You push on the front tire toward the axle. The tire does not turn.



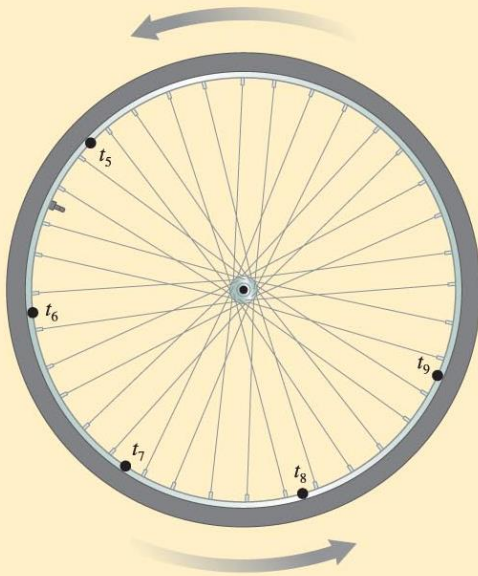
## Observational Experiment #2

**Experiment 2:** You push lightly and continuously on the outside of the tire in a counterclockwise (ccw) direction tangent to the tire. As you continue to push, the tire rotates ccw faster and faster.



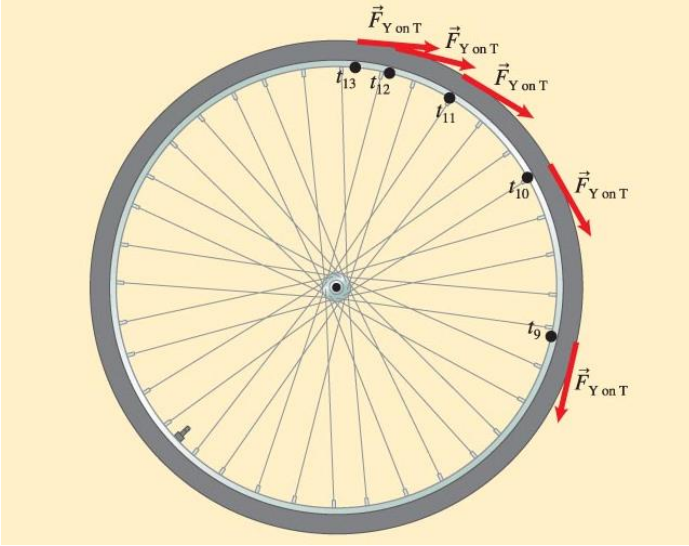
## Observational Experiment #3

**Experiment 3:** You release the spinning tire and watch it. The tire continues rotating ccw at a constant rate.



## Observational Experiment #4

**Experiment 4:** With the tire still rotating ccw fast, you gently and continuously push clockwise (cw) against the tire. The rotational speed decreases.



### Rotational form of Newton's Second Law

- One or more objects exert forces on a rigid body with rotational inertia  $I$  that can rotate about some axis.
- The sum of the torques  $\Sigma\tau$  (net torque) due to these forces about that axis causes the object to have a rotational acceleration  $\alpha$ :

$$\alpha = \frac{1}{I} \Sigma\tau$$

# Analogy chart

## Linear

- $x$
- $v_x$
- $a_x$

## Rotational Analogy

- $\theta$
- $\omega$
- $\alpha$

- 
- Force:  $F_x$
  - Mass:  $m$

- Torque:  $\tau$
- Rotational Inertia:  $I$

Newton's Second Law:

$$a_x = \frac{(F_{net})_x}{m}$$

$$\alpha = \frac{\tau_{net}}{I}$$

[Doc Cam Example]

The engine in a small airplane is specified to have a torque of 60.0 N m. This engine drives a propeller whose rotational inertia is 13.3 kg m<sup>2</sup>. On start-up, how long does it take the propeller to reach 200 rpm?

Sketch and translate

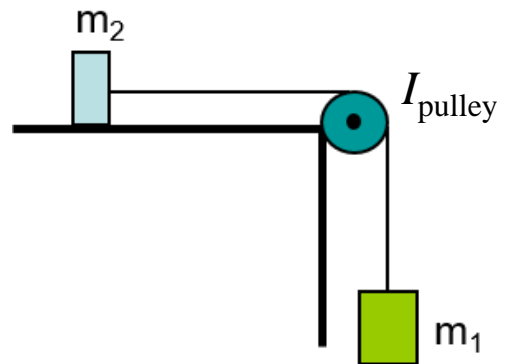
Represent mathematically

Simplify and diagram

Solve and Evaluate

# Massive Pulleys

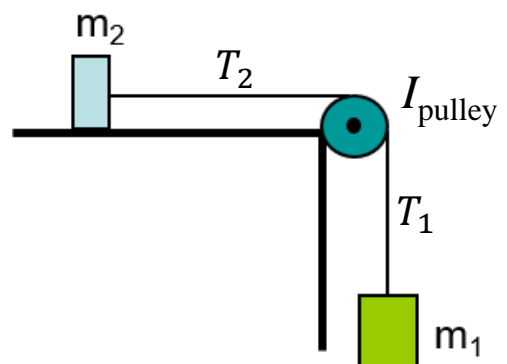
- A real pulley has some rotational inertia,  $I_{\text{pulley}}$ .
- This means that it will require a net torque in order to cause it to have rotational acceleration.
- For an accelerating system, the tension must **change** as it goes over the pulley.



## Learning Catalytics Question

- Mass  $m_2$  is on a frictionless table. It is attached to a cord that wraps around a massive pulley, and is attached to a hanging mass  $m_1$ .
- After the system is released, how does the magnitude of the tension in the rope attached to  $m_1$  compare to the magnitude of the force of gravity,  $m_1g$ , on  $m_1$ ?

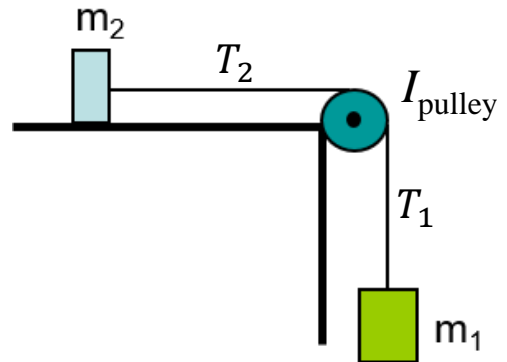
- A.  $T_1 > m_1g$
- B.  $T_1 < m_1g$
- C.  $T_1 = m_1g$



# Learning Catalytics Question

- Mass  $m_2$  is on a frictionless table. It is attached to a cord that wraps around a massive pulley, and is attached to a hanging mass  $m_1$ .
- After the system is released, how do the tensions in the two ropes compare?

- A.  $T_1 > T_2$
- B.  $T_1 < T_2$
- C.  $T_1 = T_2$

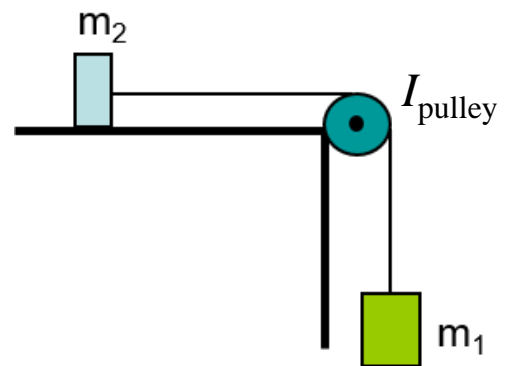


## The Massive Pulley Constraints

When an object is attached to a pulley, the distance the object travels is equal to the change in angular position of the pulley times the radius of the pulley.

- Distance  $m_2$  and  $m_1$  move is:  $s = \theta R$ , where  $\theta$  is the angle the pulley turns through.
- Speeds of  $m_2$  and  $m_1$  are related to the angular speed of the pulley by:  $v = \omega R$
- Accelerations of  $m_2$  and  $m_1$  are related to the angular acceleration of the pulley by:

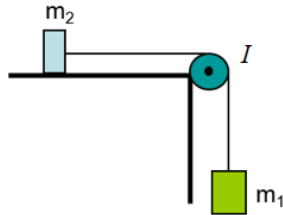
$$a = \alpha R$$



[Doc Cam Example]

- Mass  $m_2 = 2.5 \text{ kg}$  is on a frictionless table. It is attached to a cord that wraps around a massive pulley of radius  $R = 0.20 \text{ m}$ , and rotational inertia  $I = 0.050 \text{ kg m}^2$ . The same cord is attached to a hanging mass  $m_1 = 0.75 \text{ kg}$ . After the system is released, what is the acceleration of  $m_1$  as it falls?

Sketch and translate



Simplify and diagram

Represent mathematically

Solve and Evaluate



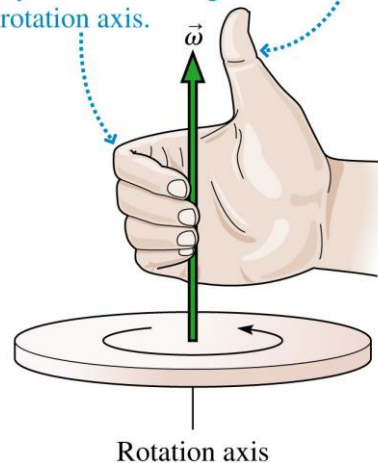
## The Vector Description of Rotational Motion

- One-dimensional motion uses a scalar velocity  $v$  and force  $F$ .
- A more general understanding of motion requires **vectors**  $\vec{v}$  and  $\vec{F}$ .
- Similarly, a more general description of rotational motion requires us to replace the scalars  $\omega$  and  $\tau$  with the vector quantities  $\vec{\omega}$  and  $\vec{\tau}$ .
- Doing so will lead us to the concept of *angular momentum*.

### “The Right Hand Rule”

- The magnitude of the angular velocity  $\vec{\omega}$  vector is  $\omega$ .
- The angular velocity vector points along the axis of rotation in the direction given by the right-hand rule as illustrated.

1. Using your right hand, curl your fingers in the direction of rotation with your thumb along the rotation axis.
2. Your thumb is then pointing in the direction of  $\vec{\omega}$ .



# Rotational Momentum



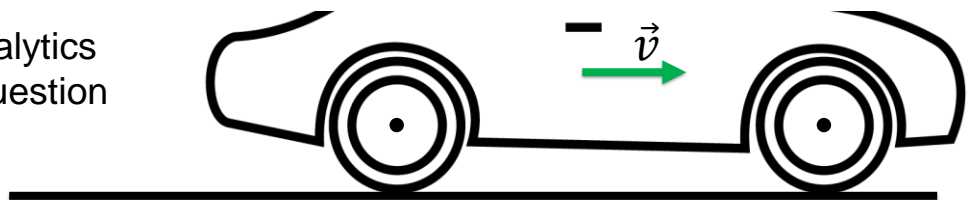
- Rotational momentum  
= rotational inertia  $\times$  rotational velocity

$$\vec{L} = I\vec{\omega}$$

- This is analogous to  
Linear momentum = mass  $\times$  velocity

$$\vec{p} = m\vec{v}$$

Learning Catalytics  
Discussion Question



You are driving your car towards the right.

What is the direction of the angular momentum,  $\vec{L}$ , of the wheels?

- A. Left
- B. Right
- C. Up
- D. Down
- E. Into the page
- F. Out of the page

## Conservation of Angular Momentum

An isolated system that experiences no net torque has

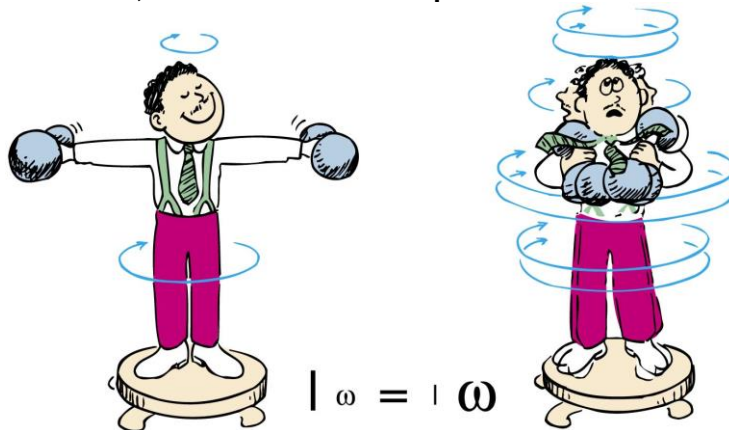
$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}} = \vec{0}$$

and thus the angular momentum vector  $\vec{L}$  is a constant.

## Conservation of Angular Momentum

Example:

- When the professor pulls the weights inward, his rotational speed increases!



## Nuclear Magnetic Resonance

- A proton in the nucleus of an atom is rotating like a little spinning top.
- When placed in a strong static magnetic field, the magnetic force produces a torque on the proton, which causes it to precess at a lower rate than the spin; similar to the hanging bicycle wheel demonstration.
- The precession frequency is in the radio-frequency range, which allows the proton to absorb and re-emit radio-waves.
- This allows doctors to image inside the human body using harmless radio waves.

