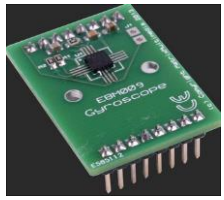
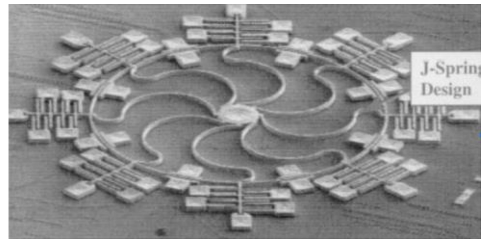
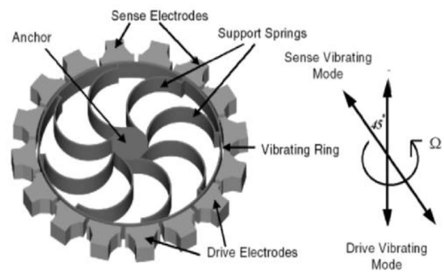
PHY131H1F - Hour 28







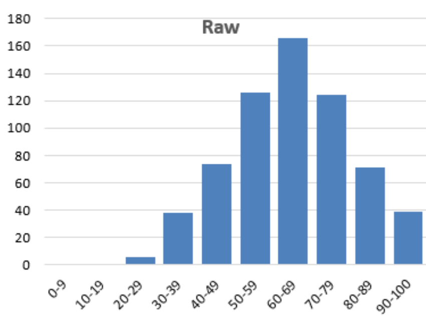
Today:

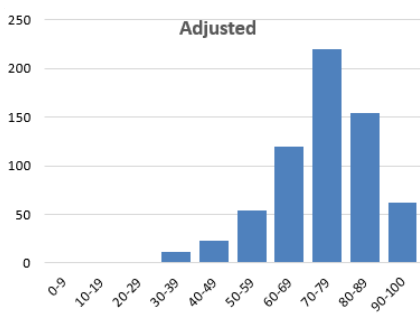
We finish up Chapter 9!

9.5 Rotational Kinetic Energy

(skip 9.6 on Tides and Earth's day)

Test 2 is Marked!





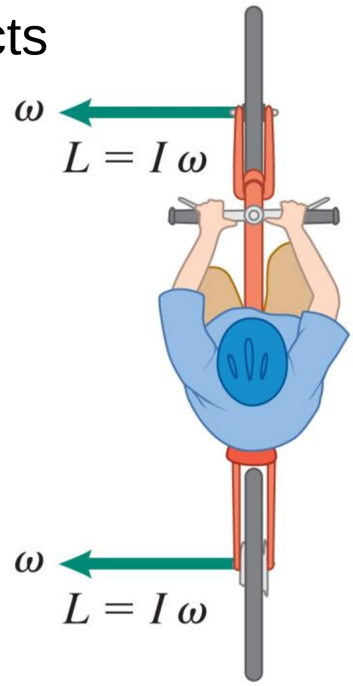
Breakpoint adjustment based on percentage out of 32:

From	To
0	0
50	65
100	100
Raw Avg:	64.4
Adj Avg:	74.1

- 2 students got 32/32.
- 6 students (including these 2) turned in perfect scratch-cards.
- 62 students (including these 6) had an adjusted mark above 29 points, or A+.
- Your test will be returned to you in Practicals. Have a look.
- If you find a mistake in the marking, let us know by Dec.3, and we'll fix the mistake.

Stability of rotating objects

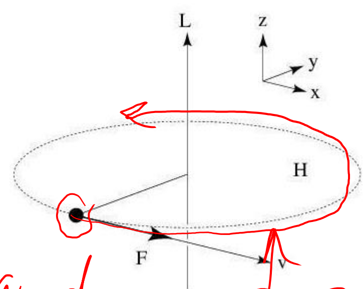
- If the rider's balance shifts a bit, the bike + rider system will tilt and the gravitational force exerted on it will produce a torque.
  - The rotational momentum of the system is large, so torque does not change its direction by much.
  - The faster the person is riding the bike, the greater the rotational momentum of the system and the more easily the person can keep the system balanced.



Learning Catalytics Question

A person spins a tennis ball on a string in a horizontal circle (so that the axis of rotation is vertical). At the point indicated below, the ball is given a sharp blow in the forward direction. This causes a change in angular momentum dL in the

A. x-direction
B. y-direction
C. z-direction



Right Hand Rule

Rotational momentum of an isolated system is constant

- If the net torque that external objects exert on a turning object is zero, or if the torques add to zero, then the rotational momentum L of the turning object remains constant:

$$L_f = L_i \quad \text{or} \quad I_f\,\omega_f = I_i\,\omega_i$$

(Eq. 9.13 from Etkina, pg.268)

TIP Rotational momentum is sometimes called angular momentum.

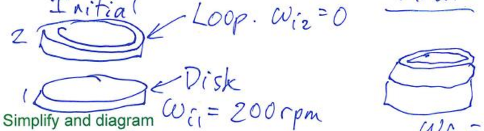
[Doc Cam example]

$R_1 = R_2 = R = 10\text{cm}$

A 20-cm-diameter, 2.0 kg solid disk is rotating at 200 rpm. A 20-cm-diameter, 1.0 kg circular loop is dropped straight down onto the rotating disk. Friction causes the loop to accelerate until it is "riding" on the disk. What is the final angular speed of the combined system?

Sketch and translate

Initial — Final



Simplify and diagram

Assume totally inelastic collision. → Rotational Momentum conserved.

$\boxed{L_f = L_i}$ Use $L = I\omega$, $I = \frac{1}{2}MR^2$ ← disk

$I = MR^2$ ← loop

Represent mathematically

$L_f = L_i$

$\frac{1}{2}M_1R_1^2\omega_f + M_2R_2^2\omega_f = \frac{1}{2}M_1R_1^2\omega_{i1} + 0$

$R_1 = R_2 = R$ ← divide both sides by R^2

$M_1 = 2\,\text{kg}$ $M_2 = 1\,\text{kg}$ $\frac{M_1\omega_f}{2} + M_2\omega_f = \frac{M_1\omega_{i1}}{2}$

Solve and Evaluate

$(1+1)\omega_f = \frac{2}{2}\omega_f + 1\omega_f = \frac{2}{2}\omega_{i1}$

$\omega_f = \frac{\omega_{i1}}{2} = \frac{200\,\text{rpm}}{2} = \boxed{100\,\text{rpm}}$ ← No need to convert to SI.

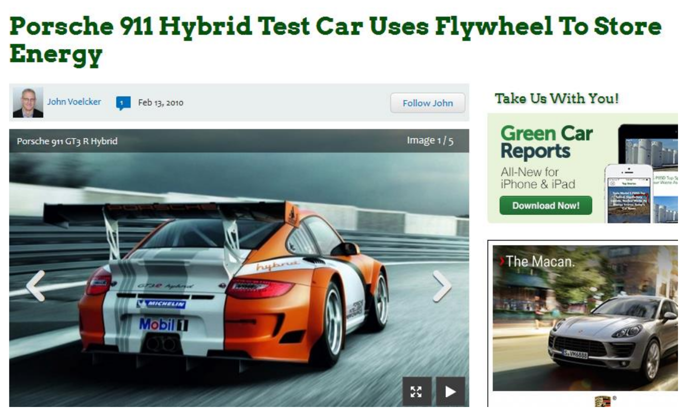
Rotational Kinetic Energy

A rotating rigid body has kinetic energy because all atoms in the object are in motion. The kinetic energy due to rotation is called **rotational kinetic energy**.

$$K_{\text{rot}} = \frac{1}{2} I\omega^2$$

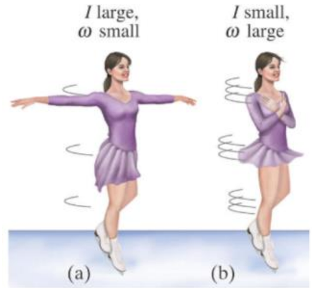
Flywheels for storing and providing energy

- In a car with a flywheel, instead of rubbing a brake pad against the wheel and slowing it down, the braking system converts the car's translational kinetic energy into the rotational kinetic energy of the flywheel.
- As the car's translational speed decreases, the flywheel's rotational speed increases. This rotational kinetic energy could then be used later to help the car start moving again.



https://www.greencarreports.com/news/1042570_porsche-911-hybrid-test-car-uses-flywheel-to-store-energy

Learning Catalytics Question



A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended.

When she pulls in her arms, she reduces her rotational inertia and her angular speed increases.

Compared to her initial rotational kinetic energy, her rotational kinetic energy after she has pulled in her arms must be:

A. the same because no work is done on her.

B. larger because she's rotating faster.

C. smaller because her rotational inertia is smaller.

$I_i \omega_i = I_f \omega_f$

$K = \frac{1}{2} I \omega^2 = \frac{1}{2}(I\omega)\omega$

$L = I\omega$ ← constant

$K = \frac{1}{2} L \omega$

$\omega \uparrow$, $K \uparrow$

Complete Linear / Rotational Analogy Chart

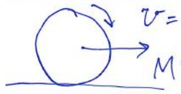
Linear	**Rotational Analogy**
• $\vec{s}$, $\vec{v}$, $\vec{a}$	• θ, ω, α
• Force: $\vec{F}$	• Torque: τ
• Mass: m	• Rotational Inertia: I

- Newton's 2nd law: $\vec{a} = \dfrac{\vec{F}_{net}}{m}$ $\qquad$ $\alpha = \dfrac{\tau_{net}}{I}$
- Kinetic energy: $K_{\text{tran}} = \dfrac{1}{2} m v^2$ $\qquad$ $K_{\text{rot}} = \dfrac{1}{2} I \omega^2$
- Momentum: $\vec{p} = m\vec{v}$ $\qquad$ $\vec{L} = I\vec{\omega}$

[Doc Cam example]

A 0.50 kg basketball rolls along the ground at 1.0 m/s. What is its *total* kinetic energy (translational plus rotational)? [Note that the rotational inertia of a hollow sphere is $I = 2/3\ MR^2$.] Assume rolling without skidding.

Sketch and translate



$v = 1.0$ m/s $\Rightarrow \boxed{v = \omega R}$

$M = 0.5$ kg Find $K = K_{tran} + K_{rot}$

Simplify and diagram

Use $\boxed{I = \frac{2}{3} MR^2}$

$K = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$ $\boxed{\omega = \frac{v}{R}}$

Represent mathematically

$$K = \frac{1}{2} Mv^2 + \frac{1}{2}\left(\frac{2}{3} MR^2\right)\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2} Mv^2 + \frac{2}{6} \frac{M\cancel{R^2}v^2}{\cancel{R^2}}$$ yay!

Solve and Evaluate

$$K = \frac{1}{2} M v^2 + \frac{2}{6} M v^2$$

$$K = \left(\frac{3}{6} + \frac{2}{6}\right) Mv^2 = \frac{5}{6} Mv^2 = \frac{5}{6}(0.5)\,1^2$$

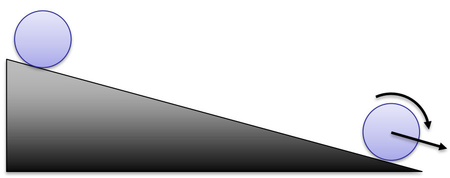
$$\boxed{K = 0.42\ \text{J}}$$

Summary of some Different Types of Energy:

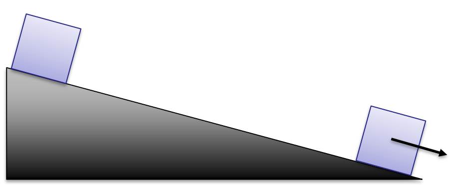
- Kinetic Energy due to bulk motion of centre of mass: $K = ½\, mv^2$ (Sometimes called Translational Kinetic Energy K_{tran})
- Gravitational Potential Energy $U_g = mgy$
- Spring Potential Energy: $U_s = ½\, kx^2$
- Rotational Kinetic Energy: $K_{rot} = ½\, I\omega^2$
- Internal Thermal Energy: ΔU_{int} (often created by the work of kinetic friction $\Delta U_{int} = |\, f_k d\, |$)
- A system can possess any or all of the above.
- One way of transferring energy in or out of a system is **work**:
- Work done by a constant force: $W = Fd\cos\theta$

- Learning Catalytics Question
- A hoop and a disk are both released from rest at the top of an incline. They both roll without slipping. Which reaches the bottom first? Shall we vote?
- A: hoop wins 42%
- B: disk wins 39%
- C: tie 17%

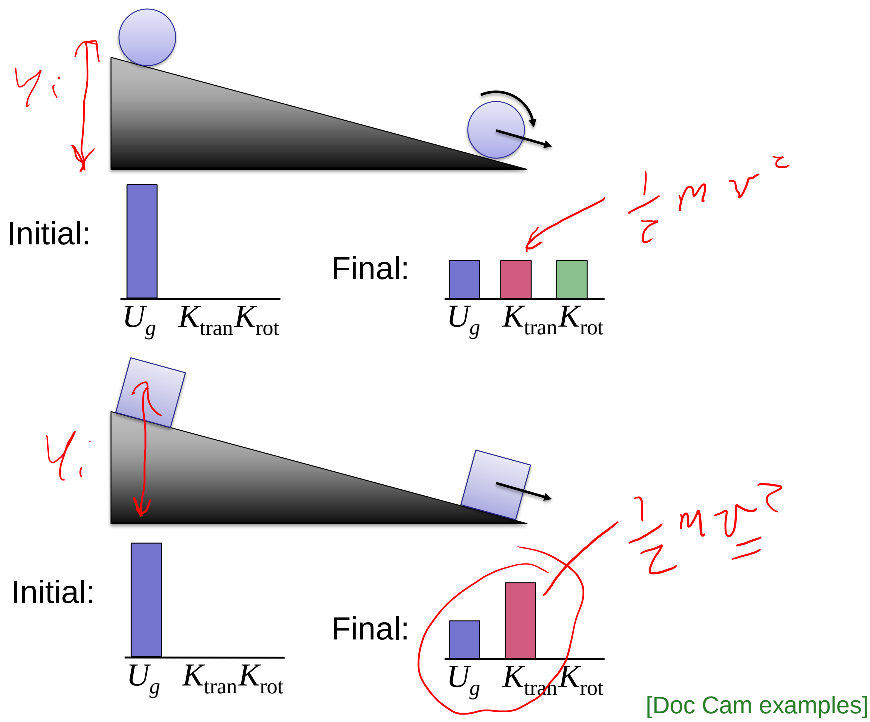
Don’t forget: Nature is not a democracy!



- A solid disk is released from rest and rolls without slipping down an incline. A box is released from rest and slides down a frictionless incline of the same angle. Which reaches the bottom first?
- A: disk wins
- B: box wins
- C: tie



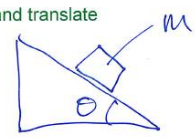
- Think about conservation of energy.
- A rolling object has two forms of kinetic energy which must be **shared**



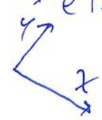
[Doc Cam examples]

1. What is the acceleration of a sliding object down a ramp inclined at angle θ? [assume no friction]

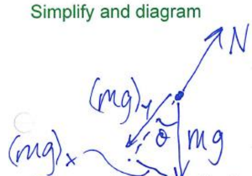
Sketch and translate



Define



Simplify and diagram



$(F_{Net})_y = 0 = N - mg\cos\theta$

Represent mathematically

$(F_{Net})_x = \cancel{m}a_x = \cancel{m}g\sin\theta$

$\boxed{a = g\sin\theta}$

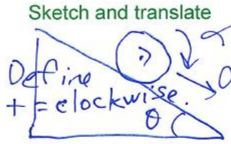
Less than g.

Solve and Evaluate

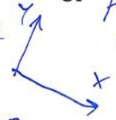
as $\theta \to 0$, $a \to 0$

2. What is the acceleration of a **solid disk of** rolling down a ramp inclined at angle θ? [assume rolling without skidding]

Sketch and translate



Define

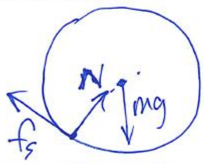


axes

Assume rolling without skidding

rotation axis = centre of disk.

Simplify and diagram



Torque of N & mg are zero.

$\sum \tau = I\alpha = +f_s R \sin 90°$

$\boxed{I\alpha = f_s R}$ (1)

Represent mathematically

$(\sum F)_x = \boxed{ma = mg\sin\theta - f_s}$ (2)

Unknowns: α, f_s, a, R

Use: $\boxed{a = \alpha R}$ (3)

(1) & (3) → $I\left(\frac{a}{R}\right) = f_s R$

$f_s = \frac{Ia}{R^2}$ plug into (2)

$ma = mg\sin\theta - \left(\frac{Ia}{R^2}\right)$ solve for a.

$a\left(m + \frac{I}{R^2}\right) = mg\sin\theta$

$a = \frac{g\sin\theta}{1 + I/MR^2}$

Solid disk: $I = \frac{1}{2}MR^2$

$a = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}\cancel{MR^2}}{\cancel{MR^2}}} = \frac{g\sin\theta}{(3/2)}$

Solve and Evaluate

$\boxed{a = \frac{2}{3} g\sin\theta}$

slower than sliding box.

5

Compare and Contrast Soup Cans

- About same mass
- About same radius and shape
- Thick paste, so when this can is rolling, the contents rotate along with the can as one solid object, like a solid cylinder
- Low viscosity liquid, so the can itself rolls while the liquid may just "slide" along.

Learning Catalytics

- Two soup cans begin at the top of an incline, are released from rest, and allowed to roll without slipping down to the bottom. Which will win?

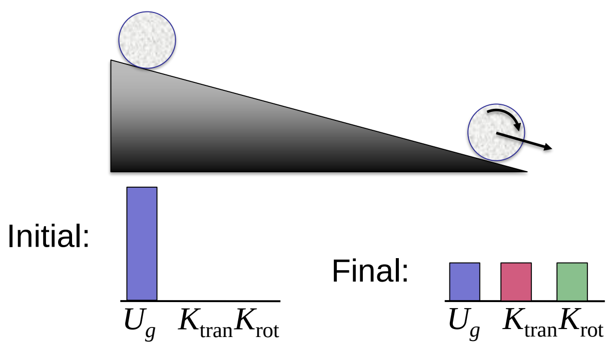
  Predict:

A. Cream of Mushroom will win

B. Chicken Broth will win

C. Both will reach the bottom at about the same time.

- Cream of Mushroom soup must **rotate**, like a solid disk.



- Chicken broth can slide down **without rotating** while the can rotates around it.