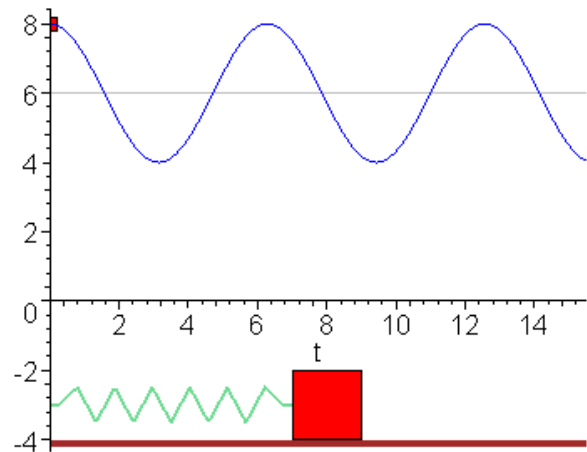# PHY131H1F - Hour 29

Today:

10.1 Period and Frequency

10.2 Simple Harmonic Motion

Animation from http://www.uni-saarland.de/fak7/knorr/homepages/patrick/theorex/MapleScripts/oscillations/forced_oscillations1.html

Mastering Physics

What's up on the MyLab and Mastering?

- Notice that Homework 10 has been posted on MasteringPhysics. It is due Friday Nov.30.
- Also, I have posted an optional item called "Ch.10 Videos – Optional" which I recommend you check out.

Chapter 10 Videos (Optional)

- There are 4 nice videos this week, all of which are animations (no Buzzcut Guy this week.. ☹)
- **Mass on a Spring:** Conceptual + Quantitative
- **Pendulum:** Conceptual + Quantitative
- I really recommend you check these out!
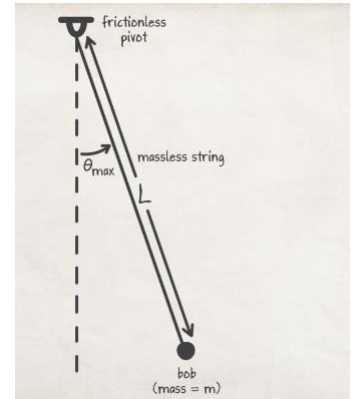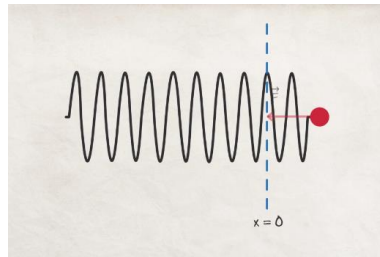
What's new in this chapter

- We have studied linear motion—objects moving in straight lines at either constant velocity or constant acceleration.
- We have also studied objects moving at constant speed in a circle.
- In this chapter we encounter a new type of motion, in which both direction and speed change.

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Some vibrations are *not* sinusoidal:

Position

T

t

Position

T

t

One cycle
takes time T.

T

t
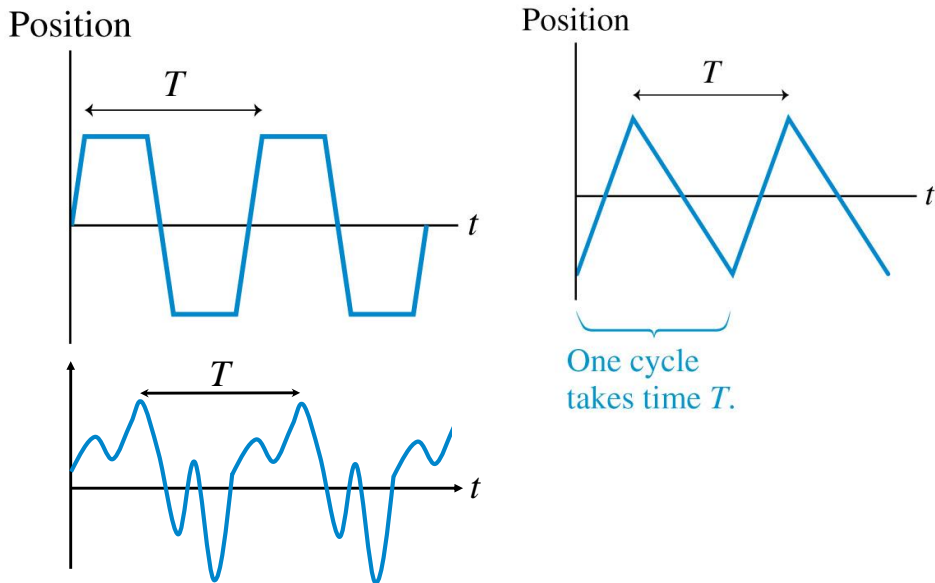
Period and Frequency

• The time to complete one full cycle, or one oscillation, is called the period, T.
• The frequency, f, is the number of cycles per second. Frequency and period are related by

$$f = \frac{1}{T} \qquad \text{or} \qquad T = \frac{1}{f}$$

• The oscillation frequency f is measured in cycles per second, or Hertz.

Learning Catalytics question

If we turn up the water flow on a Japanese tipping fountain, and **double the frequency**, which of the following statements about that system is true?

A. The period is doubled.
B. The period is reduced to one-half of what it was.
C. The period is unchanged, but the fountain tips twice as far.
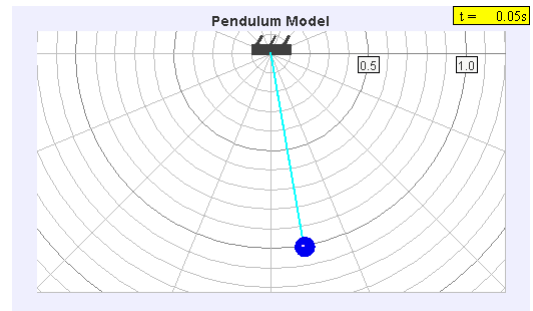D. The period is unchanged, but the fountain tips half as far.

$$T_1 = \frac{1}{f_1}$$

$$f_2 = 2f_1$$

$$T_2 = \frac{1}{2f_1} = \frac{1}{2}T_1$$

Learning Catalytics question

A pendulum has a period of 2.0 seconds (this is how long it takes to swing all the way back and forth, and back to where it started). What is its frequency?

A. 0.50 Hertz
B. 1.0 Hertz
C. 2.0 Hertz
D. 4.0 Hertz

$$f = \frac{1}{2} = 0.5 \frac{cyc}{s}$$

Pendulum Model

t = 0.05s

0.5   1.0

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Thought Experiment

- A spring with a mass attached to it is stretched and released. When the spring returns to equilibrium, will the mass be moving?

- Answer: Yes! It is not **accelerating**, but it is **moving** at that moment. Inertia then carries it past equilibrium to the other side. After passing equilibrium, the acceleration is opposite the velocity, so it slows down, eventually turning around.
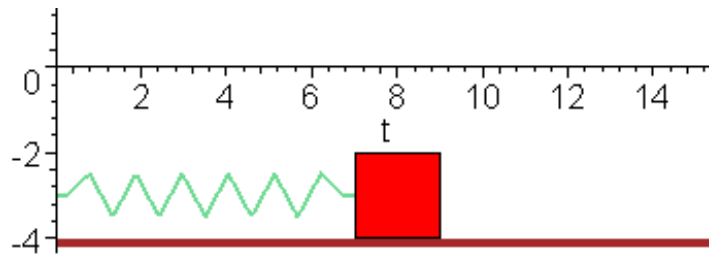

Vocabulary: "Equilibrium position"

Equilibrium position (or just **equilibrium**) The position at which a vibrating object resides when not disturbed. When resting at this position, the sum of the forces that other objects exert on it is zero. During vibrational motion the object passes back and forth through this position from two opposite directions.

Vocabulary: "Restoring force"

Restoring force When an object is displaced from equilibrium, some other object exerts a force with a component that always points opposite the direction of the vibrating object's displacement from equilibrium. This force tends to restore the vibrating object back toward equilibrium.

"Mass on a Horizontal Spring"
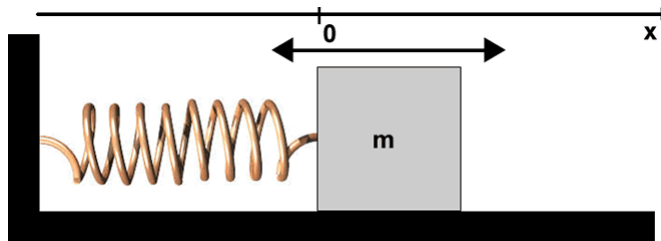
The restoring force exerted on the mass by the spring:

$$F = -k\,x \qquad \text{(Hooke's Law)}$$
$$F = m\,a \qquad \text{(Newton's Second Law)}$$

Combine and solve for acceleration: $a = -\dfrac{k}{m}x$
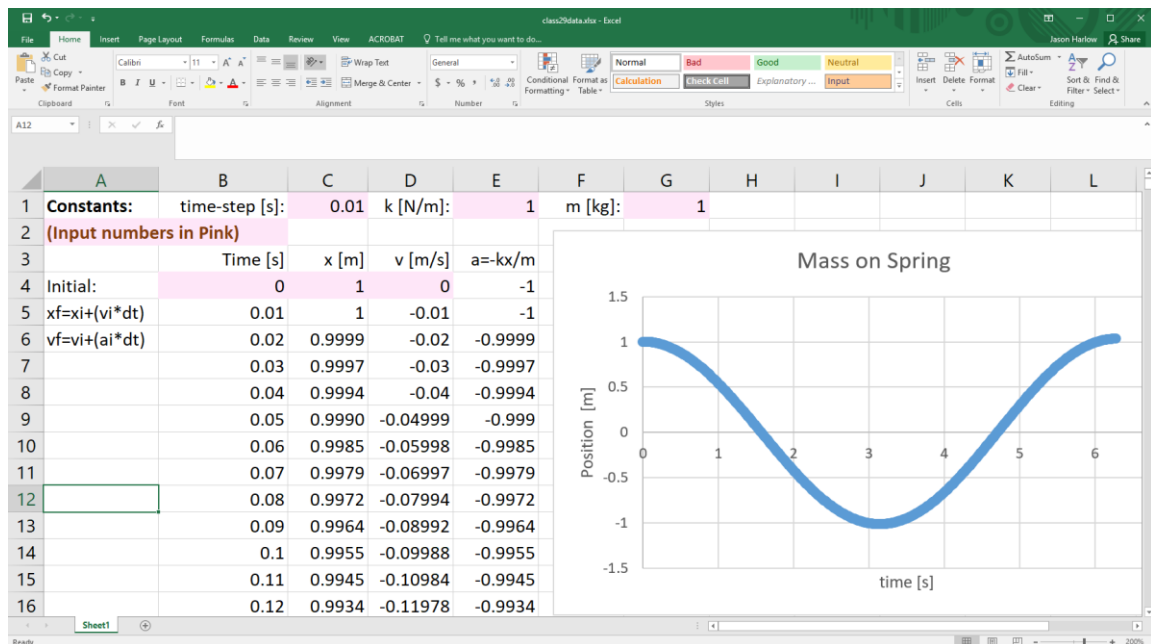
"Mass on a Horizontal Spring"

$$a = -\frac{k}{m}x$$

- What is acceleration? It is the rate of change of velocity.

- What is velocity? It is the rate of change of x.

- So, this equation is saying to us:

    "The slope of the slope of x equals a negative constant times x."

- This is a very weird thing!

- It is certainly NOT constant acceleration, so everything you know about kinematics **won't work** for a mass on a spring.

Kinematics won't work….   or will it?

- Let's try **numerical integration**. This is a trick where you split up the motion into many segments, each over a small time Δt in which things that are not constant are *almost* constant.

- If acceleration is *almost* constant of a small time, Δt, then you can estimate the new velocity after that small time with the kinematics equation: $v_\text{f} \approx v_\text{i} + a\,\Delta t$.

- If velocity is *almost* constant of a small time, Δt, then you can estimate the new position after that small time with the constant velocity equation: $x_\text{f} \approx x_\text{i} + v\,\Delta t$.

- Now that you have a new value of x, compute a new value of a, etc.

- This can be done with an Excel spreadsheet.

[Go to Excel Spreadsheet]

Please download and try it yourself: https://www.physics.utoronto.ca/%7Ejharlow/teaching/phy131f18/class29data.xlsx

"Mass on a Horizontal Spring"

$$a = -\frac{k}{m}x \qquad\qquad x = A\cos\left(\frac{2\pi}{T}t\right)$$

- I'm here to tell you that a lot of very brainy calculus people tried to solve the equation above, but in the end you just have to guess! The guess (greatly inspired by that Excel spreadsheet) is:

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

- Notice that $x = +A$ at $t = 0$. If an object is at $x = 0$ at $t = 0$, you can either adjust the $\cos$ function by adding $-(\pi/2)$ or use the sine function:
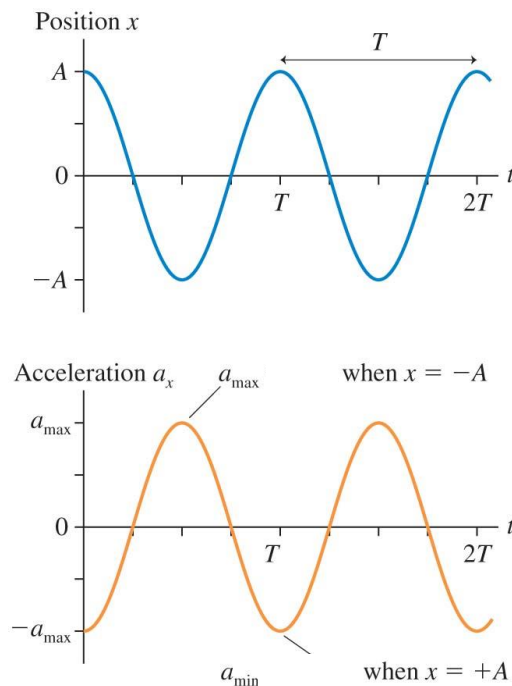
$$x = A\sin\left(\frac{2\pi}{T}t\right)$$

Vocabulary: "Simple harmonic motion"

- Simple harmonic motion (SHM) is motion that can be described by the following equation:

$$x = A \cos\left(\frac{2\pi}{T}t\right)$$
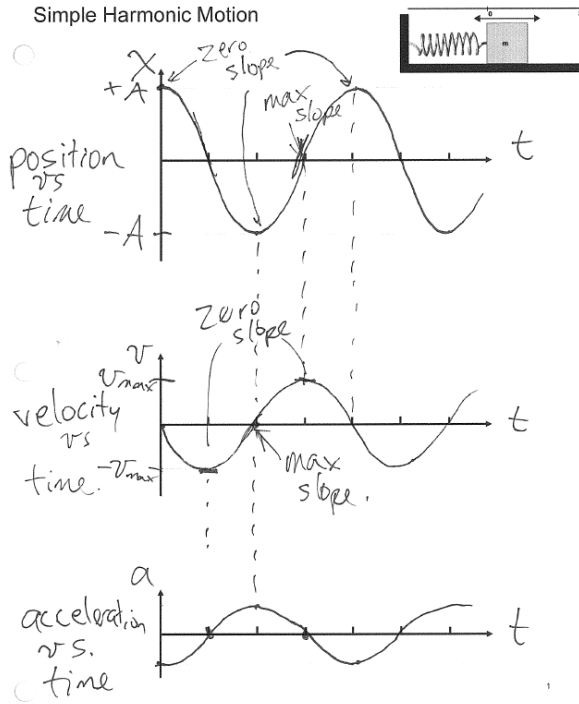
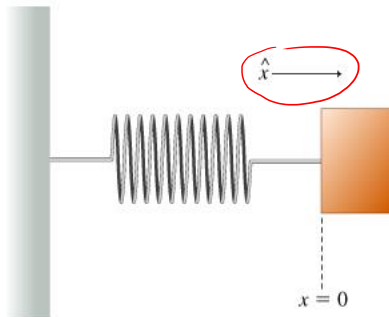- It is a mathematical model of motion.

$$F_s = -kx$$

$$a = \frac{F_{Net}}{m}$$
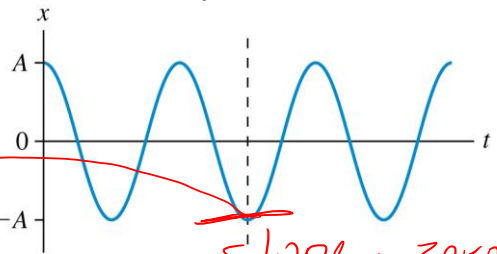
$$a_x = -\frac{k}{m}x$$

Simple Harmonic Motion

position
vs
time

+A

X

zero slope

max slope

-A

t

velocity
vs
time

V

V_max

-V_max

zero slope

max slope

t

acceleration
vs.
time

a

t

x̂

This is the position graph of a mass on a spring. What can you say about the velocity and the force at the instant indicated by the dotted line?

x = 0

x

A

0

-A

t

X < 0
Compressed Spring
F > 0

slope = zero
V = 0

A. Velocity is positive; force is zero.
B. Velocity is negative; force is zero.
C. Velocity is negative; force is to the right.
D. Velocity is zero; force is to the right.
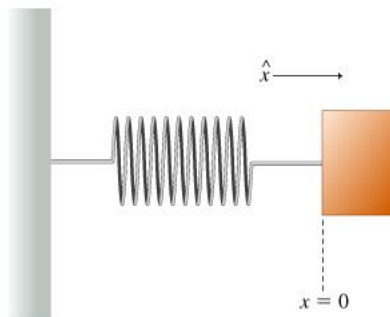E. Velocity is zero; force is to the left.

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This is the position graph of a mass on a spring. What can you say about the velocity and the force at the instant indicated by the dotted line?
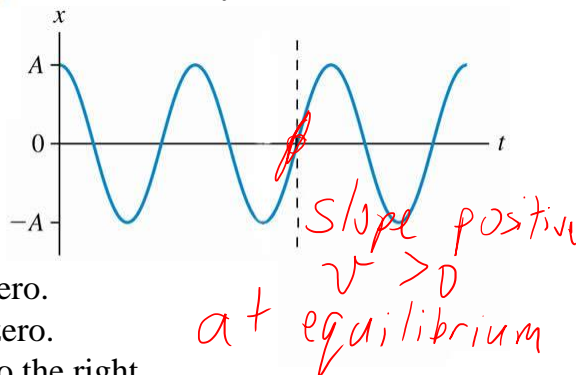
A. Velocity is positive; force is zero.
B. Velocity is negative; force is zero.
C. Velocity is negative; force is to the right.
D. Velocity is zero; force is to the right.
E. Velocity is zero; force is to the left.

Slope positive
v > 0
at equilibrium

S.H.M. notes.

- The period, T, is set by the properties of the system. In the case of a mass m attached to a spring of spring-constant k, the period is always

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- A is set by the initial conditions: x_0 (initial position) and v_0 (initial velocity).

S.H.M.

$$x = A \cos\left(\frac{2\pi}{T}t\right)$$
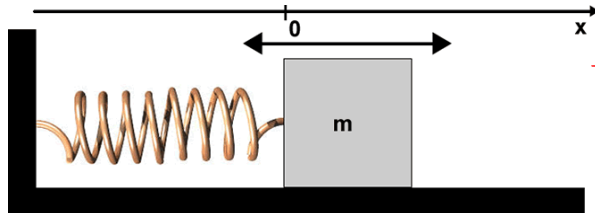
variables: t

x

$\pi = 3.14$

Which of the two constants (*A* and *T*) in the description of simple harmonic motion is determined by the initial position and velocity of the mass?

$$T = 2\pi\sqrt{\frac{m}{k}}$$

A. only the amplitude, *A*
B. only the period, *T*
C. both
D. neither

$$T = 2\pi\sqrt{\frac{m}{k}}$$

A mass is oscillating on a spring in S.H.M. When it passes through its equilibrium point, an external "kick" suddenly **decreases** its speed, but then it continues to oscillate.
As a result of this slowing, the period of the oscillation
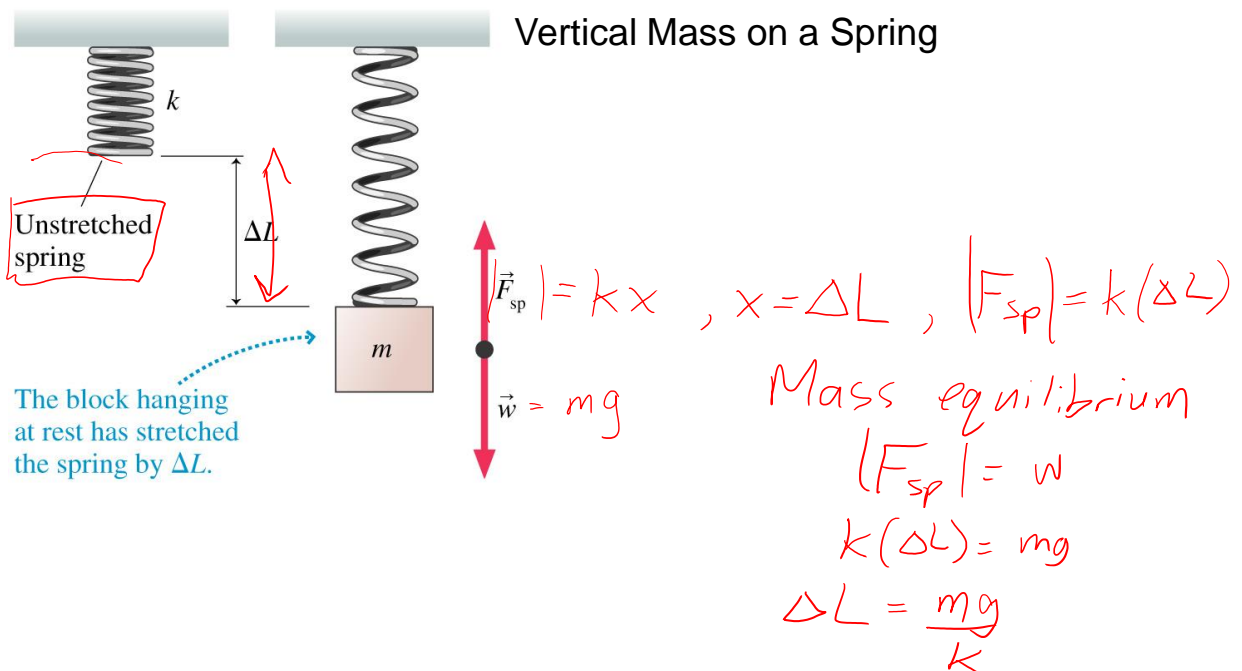
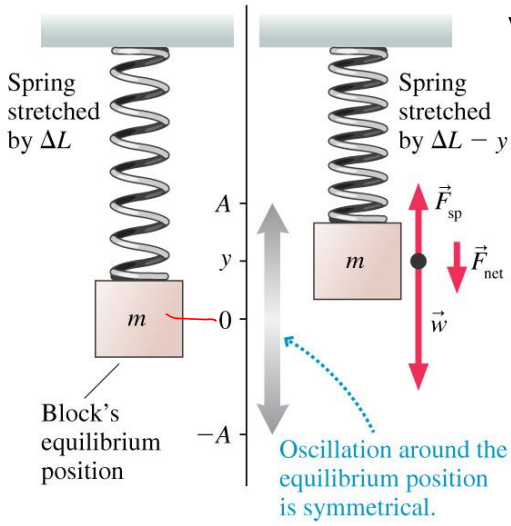also decrease amplitude, A.

A. goes up
B. goes down
C. stays the same

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Remember:

- The period, T, is set by the **properties of the system**. In the case of a mass m attached to a spring of spring-constant k, the period is always

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- A is set by the initial conditions: x_0 (initial position) and v_0 (initial velocity).

Vertical Mass on a Spring

k

Unstretched spring

ΔL

The block hanging at rest has stretched the spring by ΔL.

m

$|\vec{F}_{sp}| = kx$

$\vec{w} = mg$

$|\vec{F}_{sp}| = kx \quad , \quad x = \Delta L \quad , \quad |F_{sp}| = k(\Delta L)$

Mass equilibrium
$|F_{sp}| = w$
$k(\Delta L) = mg$
$\Delta L = \dfrac{mg}{k}$

Spring
stretched
by ΔL

Block's
equilibrium
position

A
y
0
$-A$

Spring
stretched
by $\Delta L - y$

m

$\vec{F}_{sp}$
$\vec{F}_{net}$
$\vec{w}$

Oscillation around the
equilibrium position
is symmetrical.

Vertical Mass on a Spring: Also S.H.M.!

Set $y = 0$ to be
when mass is at new
equilibrium.

$$y = A \cos\left(\frac{2\pi}{T} t\right)$$

where $T = 2\pi \sqrt{\frac{m}{k}}$