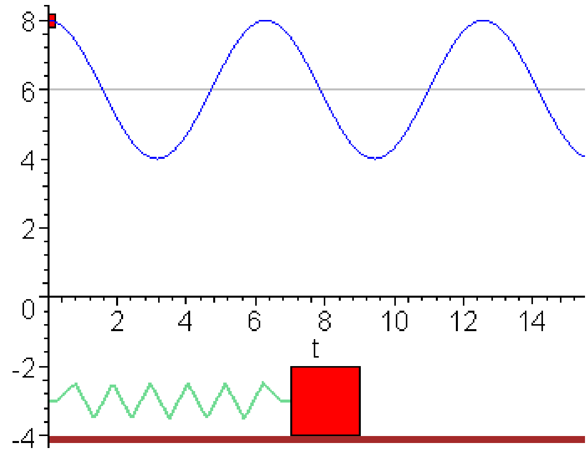
PHY131H1F - Hour 29



Today:

10.1 Period and Frequency

10.2 Simple Harmonic Motion

Animation from http://www.uni-saarland.de/fak7/knorr/homepages/patrick/theorex/MapleScripts/oscillations/forced_oscillations1.html

Mastering Physics

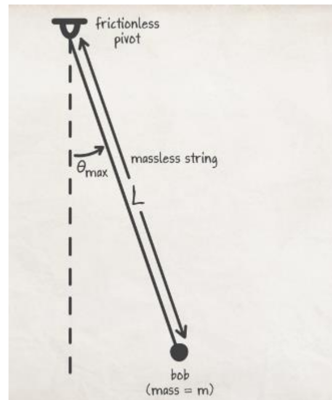
What's up on the MyLab and Mastering?

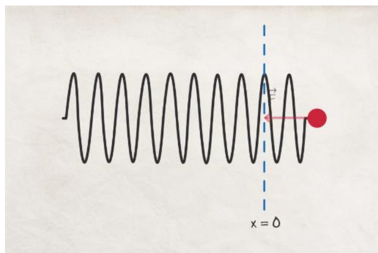
- Notice that Homework 10 has been posted on MasteringPhysics. It is due Friday Nov.30.
- Also, I have posted an optional item called "Ch.10 Videos – Optional" which I recommend you check out.

Mastering Physics

Chapter 10 Videos (Optional)

- There are 4 nice videos this week, all of which are animations (no Buzzcut Guy this week.. ☹)
- **Mass on a Spring:** Conceptual + Quantitative
- **Pendulum:** Conceptual + Quantitative
- I really recommend you check these out!

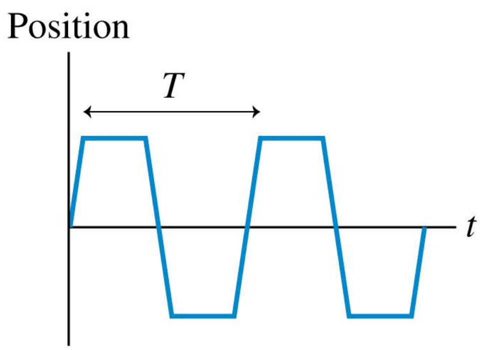


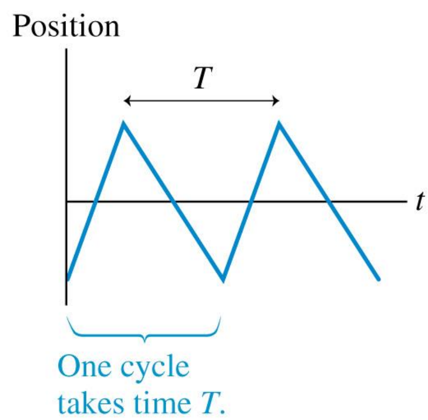


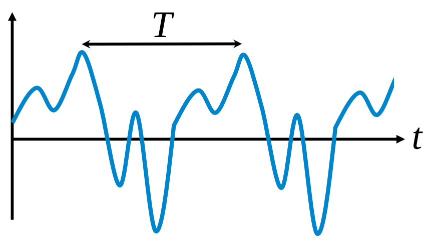
What's new in this chapter

- We have studied linear motion—objects moving in straight lines at either constant velocity or constant acceleration.
- We have also studied objects moving at constant speed in a circle.
- In this chapter we encounter a new type of motion, in which both direction and speed change.

Some vibrations are ***not*** sinusoidal:







Period and Frequency

• The time to complete one full cycle, or one oscillation, is called the period, T.
• The frequency, f, is the number of cycles per second. Frequency and period are related by

$$f = \frac{1}{T} \qquad \text{or} \qquad T = \frac{1}{f}$$

• The oscillation frequency f is measured in cycles per second, or Hertz.

Learning Catalytics question

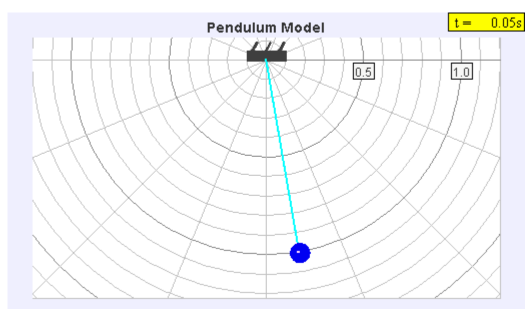
If we turn up the water flow on a Japanese tipping fountain, and **double the frequency**, which of the following statements about that system is true?

A. The period is doubled.
B. The period is reduced to one-half of what it was.
C. The period is unchanged, but the fountain tips twice as far.
D. The period is unchanged, but the fountain tips half as far.

Learning Catalytics question

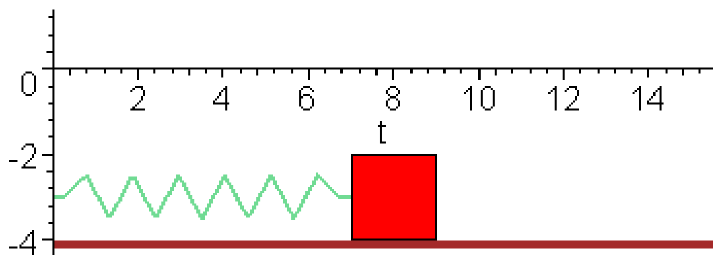
A pendulum has a period of 2.0 seconds (this is how long it takes to swing all the way back and forth, and back to where it started). What is its frequency?

A. 0.50 Hertz
B. 1.0 Hertz
C. 2.0 Hertz
D. 4.0 Hertz



Thought Experiment

- A spring with a mass attached to it is stretched and released. When the spring returns to equilibrium, will the mass be moving?
- Answer: Yes! It is not **accelerating**, but it is **moving** at that moment. Inertia then carries it past equilibrium to the other side. After passing equilibrium, the acceleration is opposite the velocity, so it slows down, eventually turning around.



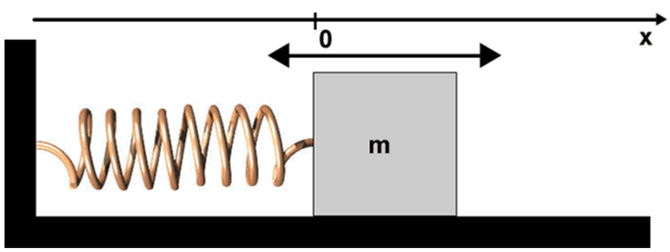
Vocabulary: "Equilibrium position"

Equilibrium position (or just **equilibrium)** The position at which a vibrating object resides when not disturbed. When resting at this position, the sum of the forces that other objects exert on it is zero. During vibrational motion the object passes back and forth through this position from two opposite directions.

Vocabulary: “Restoring force”

Restoring force When an object is displaced from equilibrium, some other object exerts a force with a component that always points opposite the direction of the vibrating object’s displacement from equilibrium. This force tends to restore the vibrating object back toward equilibrium.

“Mass on a Horizontal Spring”



The restoring force exerted on the mass by the spring:

$F = -k\,x$ (Hooke’s Law)

$F = m\,a$ (Newton’s Second Law)

Combine and solve for acceleration: $a = -\frac{k}{m}x$

"Mass on a Horizontal Spring"

$$a = -\frac{k}{m}x$$

$$a = -\frac{k}{m}x$$

- What is acceleration? It is the rate of change of velocity.
- What is velocity? It is the rate of change of x.
- So, this equation is saying to us:

  "The slope of the slope of x equals a negative constant times x."
- This is a very weird thing!
- It is certainly NOT constant acceleration, so everything you know about kinematics won't work for a mass on a spring.

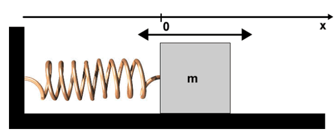
"Mass on a Horizontal Spring"

- Let's try **numerical integration**. This is a trick where you use the fact that, over small enough time intervals, even things that are not constant are *almost* constant.
- If acceleration is *almost* constant of a small time, Δt, then you can estimate the new velocity after that small time with the kinematics equation: $v_f \approx v_i + a\,\Delta t$.
- If velocity is *almost* constant of a small time, Δt, then you can estimate the new position after that small time with the constant velocity equation: $x_f \approx x_i + v\,\Delta t$.
- Now that you have a new value of x, compute a new value of a, etc.
- This can be done with an Excel spreadsheet.

[Go to Excel Spreadsheet]

Constants:	time-step [s]:	0.01	k [N/m]:	1	m [kg]:	1
(Input numbers in Pink)						
	Time [s]	x [m]	v [m/s]	a=-kx/m		
Initial:	0	1	0	-1		
xf=xi+(vi*dt)	0.01	1	-0.01	-1		
vf=vi+(ai*dt)	0.02	0.9999	-0.02	-0.9999		
	0.03	0.9997	-0.03	-0.9997		
	0.04	0.9994	-0.04	-0.9994		
	0.05	0.9990	-0.04999	-0.999		
	0.06	0.9985	-0.05998	-0.9985		
	0.07	0.9979	-0.06997	-0.9979		
	0.08	0.9972	-0.07994	-0.9972		

"Mass on a Horizontal Spring"



$$a = -\frac{k}{m}x$$

- I'm here to tell you that a lot of very brainy calculus people tried to solve the equation above, but in the end you just have to guess. The guess (greatly inspired by that Excel spreadsheet) is:

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

- Notice that $x = +A$ at $t = 0$. If an object is at $x = 0$ at $t = 0$, you can either adjust the $\cos$ function by adding $-(\pi/2)$ or use the sine function:

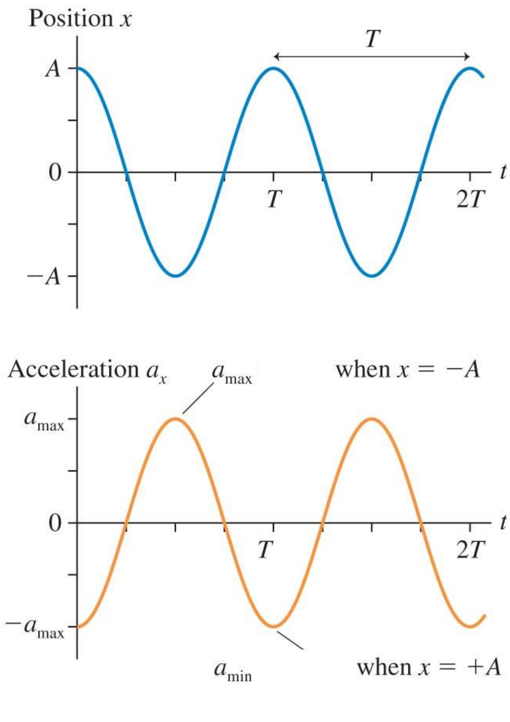
$$x = A\sin\left(\frac{2\pi}{T}t\right)$$

Vocabulary: “Simple harmonic motion”

- Simple harmonic motion (SHM) is motion that can be described by the following equation:

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

- It is a mathematical model of motion.



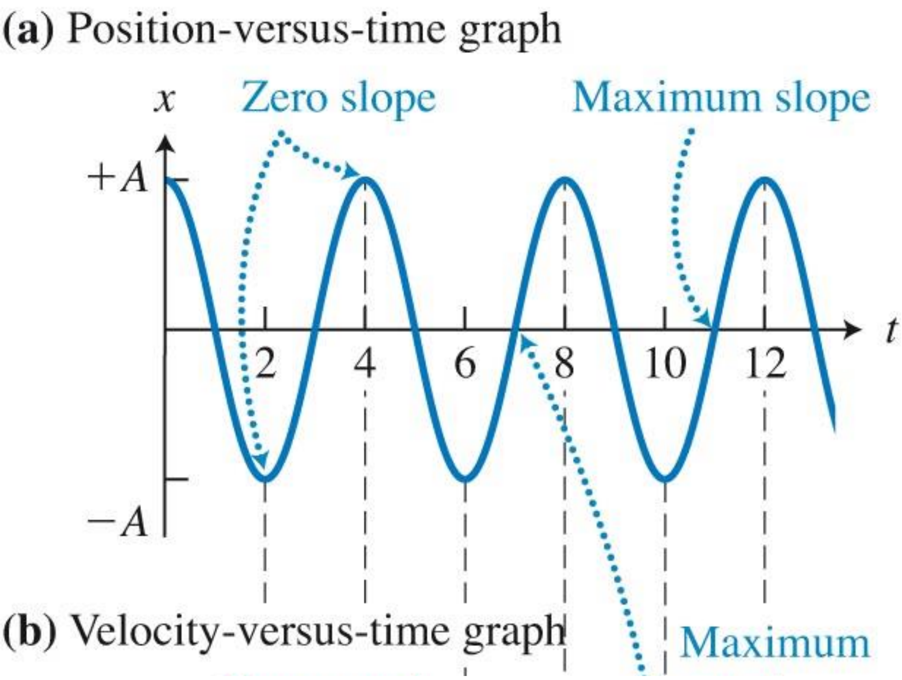
$$F_s = -kx$$

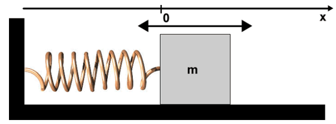
↓

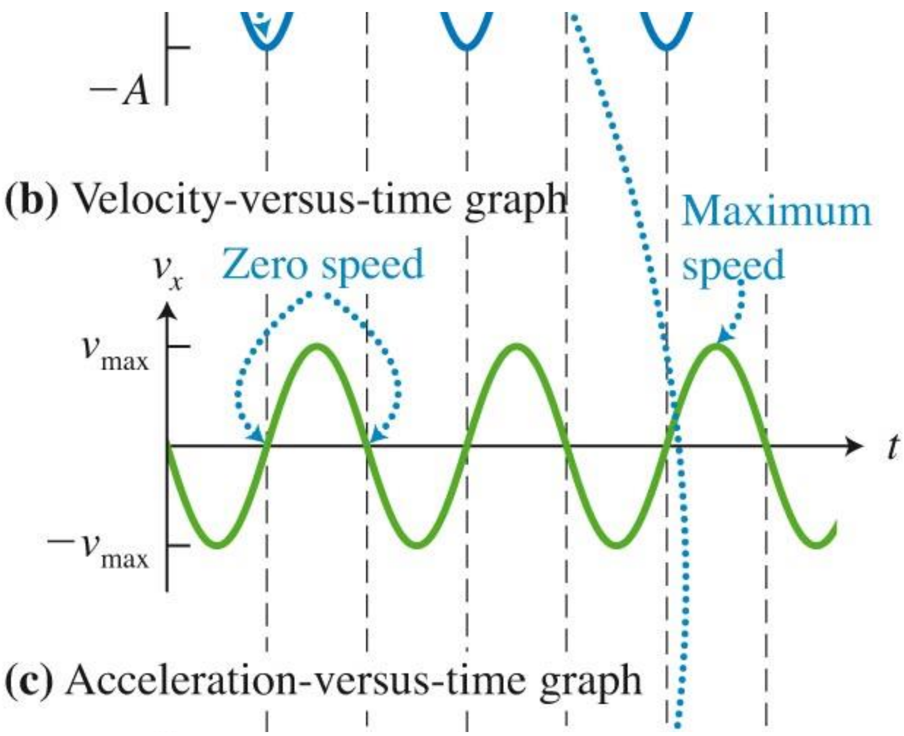
$$a = \frac{F_{Net}}{m}$$

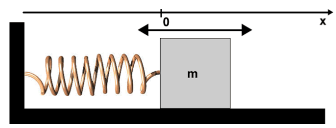
↓

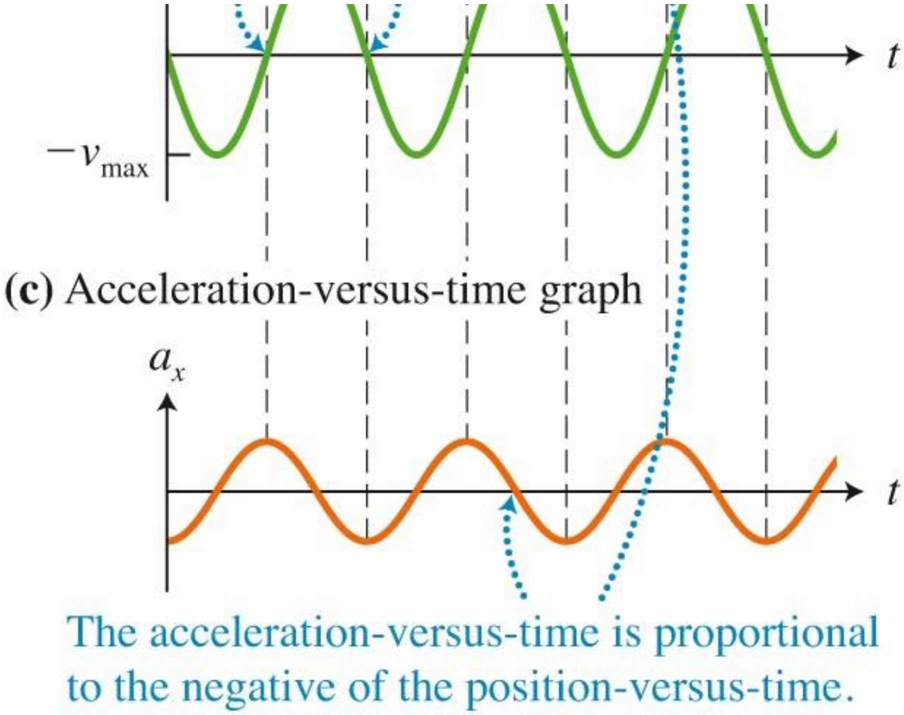
$$a_x = -\frac{k}{m}x$$

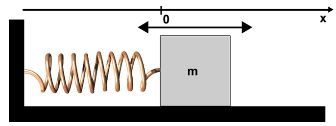

(a) Position-versus-time graph
x
Zero slope
Maximum slope
+A
−A
t
2
4
6
8
10
12
(b) Velocity-versus-time graph
Maximum


0
x
m


−A
(b) Velocity-versus-time graph
Maximum
speed
Zero speed
v_x
v_{max}
$-v_{max}$
t
(c) Acceleration-versus-time graph

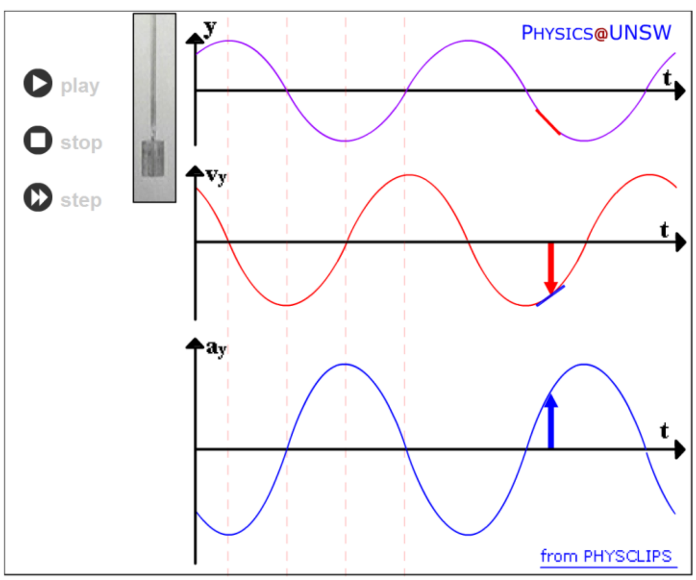

0
x
m



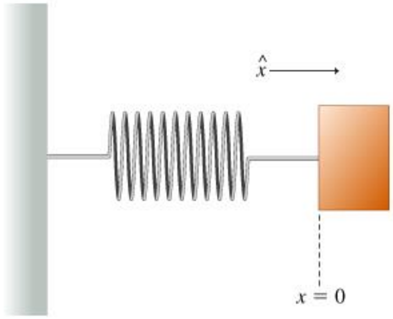


Quantities in Simple Harmonic Motion

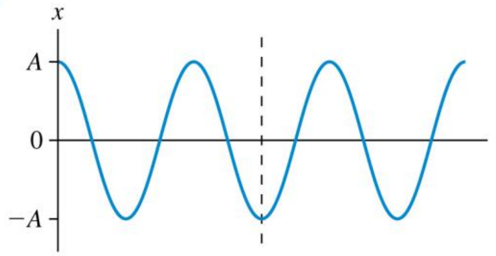
http://www.animations.physics.unsw.edu.au/jw/SHM.htm



Learning Catalytics Question

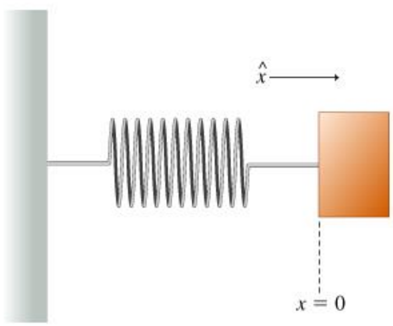


This is the position graph of a mass on a spring. What can you say about the velocity and the force at the instant indicated by the dotted line?



A. Velocity is positive; force is zero.
B. Velocity is negative; force is zero.
C. Velocity is negative; force is to the right.
D. Velocity is zero; force is to the right.
E. Velocity is zero; force is to the left.

Learning Catalytics Question



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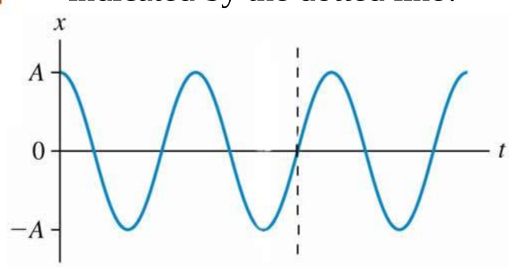

A. Velocity is positive; force is zero.
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S.H.M. notes.

- The frequency, f, is set by the properties of the system. In the case of a mass m attached to a spring of spring-constant k, the frequency is always

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

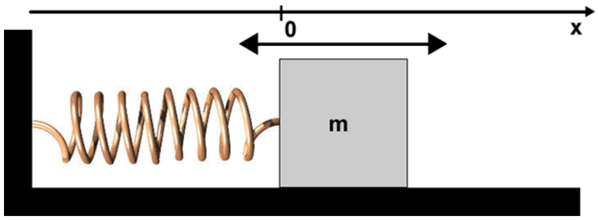
- A is set by the initial conditions: x_0 (initial position) and v_0 (initial velocity).

Learning Catalytics Question

Which of the following quantities in the description of **simple harmonic motion** is *not* determined by the initial position and velocity of the mass?

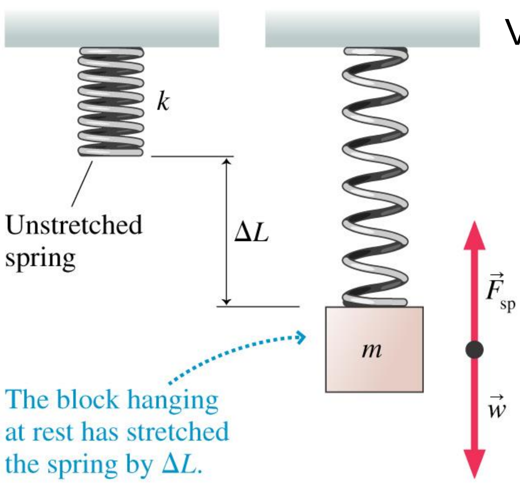
A. the amplitude, A
B. the period, T

Learning Catalytics Question

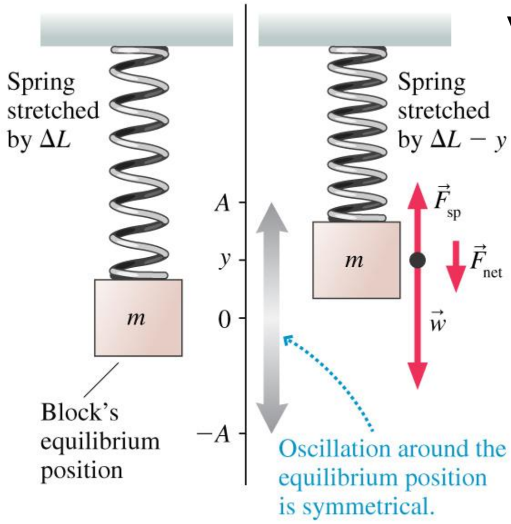


A mass is oscillating on a spring in S.H.M. When it passes through its equilibrium point, an external "kick" suddenly decreases its speed, but then it continues to oscillate. As a result of this slowing, the frequency of the oscillation

A. goes up
B. goes down
C. stays the same



Vertical Mass on a Spring



Vertical Mass on a Spring