

# PHY131H1F - Hour 31



## Today:

10.6 Solving SHM Problems

10.7 Damped Vibrational Motion

10.8 Driven Vibrational Motion

From <http://www.cavatoyota.com/blog/what-are-shock-absorbers/> :  
To test your vehicle's shock absorbers, simply push down on the each corner of the vehicle and observe its bounce. The vehicle should bounce up and return to its center resting position. If it continues to bounce, the shock absorber **should be replaced**.

## Learning Catalytics Question

A grandfather clock, calibrated at sea level, is now operating in Calgary, Alberta, which is 1 km above sea level.

This clock runs

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Calgary:  $g$  is less,

$T$  is more

A. fast.

B. slow.

C. normally, as it does at sea level.

tick... tick... tick



Image from [https://www.1-800-4clocks.com/Bulova-Vickery-Wall-Chimes-Clock\\_C4329\\_CUV](https://www.1-800-4clocks.com/Bulova-Vickery-Wall-Chimes-Clock_C4329_CUV)

**Latest Comments** from Learning Catalytics: "Please enter at least one specific question or concern you would like me to address in class."

- *"What is the weight of each chapter be on final test?"*

**Harlow answer:** Chapters 2 to 7 make up about 40% of the final exam; each of these chapters is weighted equally at about 7% each. Chapters 8 to 11 make up about 60% of the final exam; each of these chapters is weighted equally at about 15% each. So it is a bit more heavily weighted toward what we did since the second midterm.

- *"does the exam follow the same format as the mid-terms?"*

**Harlow answer:** Yes. It is exactly double one midterm. So, there are 24 scratch-card multiple choice questions, and two written-answer questions that you have to do with the 4-step model.

**Latest Comments** from Learning Catalytics: "Please enter at least one specific question or concern you would like me to address in class."

- *Will we have an aid sheet for the final exam?*

**Harlow answer:** Yes. It can only be one 8.5"x11" sheet, double-sided, so you will have to consolidate somewhat.

- *"please make the final easy we are all suffering"*

**Harlow answer:** Too late! It's already been submitted to the Faculty!

- *"I'm not in class but I want participation marks."*

**Harlow answer:** I appreciate your honesty. But, really, you should come to class.

**Latest Comments** from Learning Catalytics: "Please enter at least one specific question or concern you would like me to address in class."

- *"im STRESSED"*

**Harlow answer:** I am sorry to hear that. Stress can be a serious issue at university. It's important to talk to people about it, and work with your friends and family network to maintain your mental health.

- *"How do you determine what the fraction for rotational inertia is do we have to memorize them for different shapes"*

**Harlow answer:** Those fractions in front of the  $MR^2$  are all determined by integrals. This is not a calculus-based course, so you don't have to determine these. We will include Table 9.5 from page 263 on the final exam, so you don't have to write these on your aid sheet.

**Latest Comments** from Learning Catalytics: "Please enter at least one specific question or concern you would like me to address in class."

- *"Since we skip 9.6, does that mean we don't have to know it for the final exam?"*

**Harlow answer:** That's correct! Here is a complete list of things that will **not** be on the final exam from chapters 2-11:

- Section 4.5 on two-dimensional projectile motion
- Kepler's Laws from Section 5.5, pages 136 and 137
- Section 9.6 on tides and Earth's day
- Section 11.10 on the Doppler effect

- *"Does all class lectures end on December 4<sup>th</sup>"*

**Harlow answer:** My last class will be on Wednesday December 5. Technically, Thursday Dec. 6 has been reserved for professors to do "make-up Monday" classes, but we have chosen not to do this in this course.

**Latest Comments** from Learning Catalytics: "Please enter at least one specific question or concern you would like me to address in class."

- "when do we get our tests back if we asked for a remark"

**Harlow answer:** If we made a mistake in marking the midterm, we will certainly correct it. For midterm 2, these be available after Dec.10.

- "is this week the last tutorial"

**Harlow answer:** Right. The final Practical week runs Nov.27-Dec.3.

- "Is the earth flat?"

**Harlow answer:** No. The Earth is a sphere. Gravity pulls all objects on the surface toward the centre of the sphere. This explains a whole lot of things, like time zones, for example.

[Doc Cam MasteringPhysics Question]

"Homework 9 question 4, please and thank you :)"

- Two disks are cut from the same uniform board. The radius of disk B is twice the radius of the disk A. The disks can rotate around vertical axes with negligible friction. Two very light battery-powered fans are attached to the edges of the disks, as shown. When switched on, the fans exert equal forces on the disks.

What is the angular acceleration of A in terms of  $\alpha_B$ , the angular acceleration of B?

Sketch and translate

$$F_A = F_B = F \leftarrow \text{same}$$

$$R_B = 2 R_A$$

Simplify and diagram

Newton's 2nd

Law for rotation.

$$\alpha = \frac{\sum \tau}{I}$$

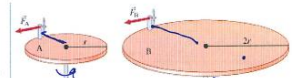
Represent mathematically

$$\alpha_A = \frac{F R_A \sin 90^\circ}{\frac{1}{2} M_A R_A^2} = \frac{2F}{M_A R_A}$$

$$\alpha_B = \frac{F R_B \sin 90^\circ}{\frac{1}{2} M_B R_B^2} = \frac{2F}{M_B R_B}$$

Solve and Evaluate

$$\alpha_B = \frac{2F}{(4M_A)2R_A} = \frac{1}{8} \left[ \frac{2F}{M_A R_A} \right]$$



"cut from the same uniform board" → same density, or mass per area.

$$\frac{M_A}{\text{Area}_A} = \frac{M_B}{\text{Area}_B}$$

$$\frac{M_A}{\pi R_A^2} = \frac{M_B}{\pi R_B^2}$$

$$M_B = \frac{R_B^2}{R_A^2} M_A$$

$$M_B = 4 M_A$$

$$\alpha_A = 8 \alpha_B$$

**Latest Comments** from Learning Catalytics: “Please enter at least one specific question or concern you would like me to address in class.”

- Gary writes: “Angela, I'm skipping today, pls lemme see ur notes & tell me what I missed thx”

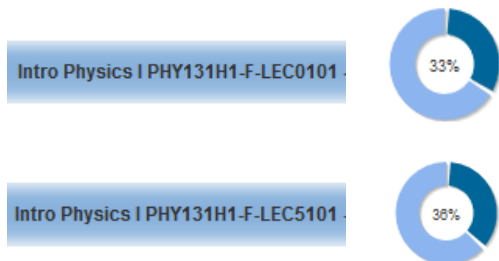
**Harlow answer:** Hahahah!

- “65% in EACH section?”

**Harlow answer:** Well, technically the entire class has to get up to 65%. And both sections are around equal numbers, so, the average has to be 65%. Go do your course evaluations!

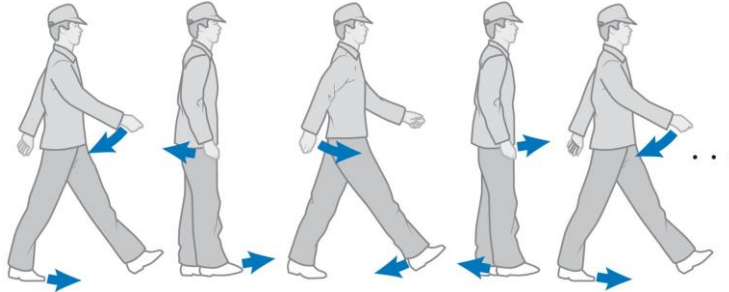
## **Bonus Point for Over 65% Course Evaluation Response Rate**

- The end of the evaluation period for this semester next Friday December 7 at 11:59PM.
- If, by the end of the course evaluation period, at least 65% of the students enrolled in *both* sections of this course have completed the course evaluations, then **every student in the course will have 1% bonus added to their final course mark.**
- Results so far:



# Leg swinging frequency

- When you walk, your arms and legs swing back and forth. These motions repeat themselves.



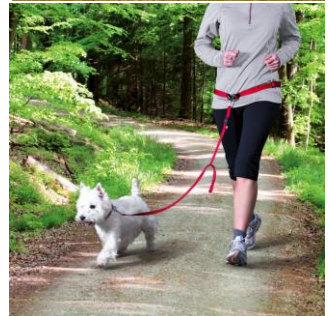
- The back-and-forth motion of an object that passes through the same positions is an important feature of vibrational motion.

# Leg swinging frequency

- Your leg can be modeled as a simple pendulum, with length equal to the distance between your hip joint (rotation axis) and the centre of mass,  $L = 0.5$  m.
- In this case, the frequency is:

$$f \approx \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.8}{0.5}} = 0.7 \text{ Hertz}$$

- Longer legs have lower swinging frequencies.
- Giraffes take fewer steps per second than humans because of their long legs.
- Small dogs take more steps per second than humans because of their short legs.



[Doc Cam Example]

## Learning Catalytics Question

Handwritten:  $k$   $\overline{m}$

A 1.00 kg mass is attached to a horizontal spring with a spring constant of 1.00 N/m. When the mass is at  $x = 0$ , the spring is in equilibrium. The mass is pulled to  $x = +1.00$  m, and then it is released from rest at time  $t = 0.00$  s.

$$T = 2\pi \sqrt{\frac{m}{k}}$$

At what time does the mass reach a position of  $x = -1.00$  m?

$$T = 2\pi \sqrt{\frac{1 \text{ kg}}{1 \text{ N/m}}}$$

$T = 2\pi$  seconds

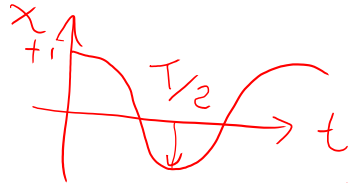
A.  $t = 1.00$  s

B.  $t = 2.00$  s

C.  $t = 3.14$  s

D.  $t = 4.00$  s

E.  $t = 6.28$  s



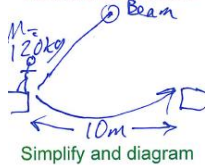
$$x = A \cos\left(\frac{2\pi}{T} t\right)$$

$t = \pi$        $A = 1 \text{ m}$

[Excel Spreadsheet]

Luke and Leia have a combined mass of 120 kg and both grasp a rope of length 30 m that is attached to a beam above them. The beam is half-way across a 10 m horizontal gap, and they want to swing across. If they start from rest and swing down and up, just reaching the other side, how long does this take?

Sketch and translate



Simplify and diagram

Assume S.H.M.,  
Simple pendulum,  
 $L = 30$  m, small angle  
oscillations.

Back & Forth: one period  
Across once:  $\frac{1}{2} T$ .

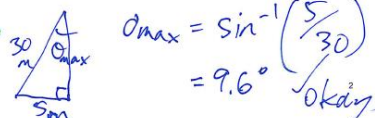
$$T = 2\pi \sqrt{\frac{30}{9.8}} = 10.99 \text{ s}$$

Represent mathematically

$$t = \frac{T}{2} = \boxed{5.5 \text{ s}}$$

check: is  $\theta_{\max} \leq 10^\circ$ ?

Solve and Evaluate



$$\theta_{\max} = \sin^{-1}\left(\frac{5}{30}\right)$$

$$= 9.6^\circ \text{ okay.}$$

## Simple Harmonic Motion (SHM)

- If the net force on an object is a linear restoring force (ie a mass on a spring, or a pendulum with small oscillations), then the position as a function of time is related to cosine:

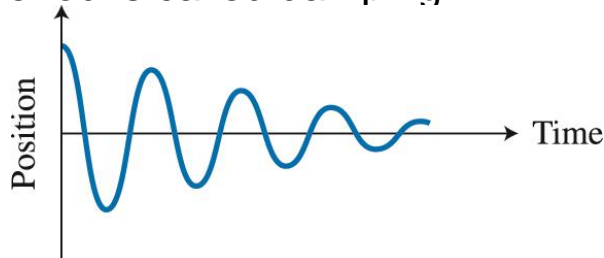
$$x = A \cos\left(\frac{2\pi}{T} t\right)$$



- Cosine is a function that goes forever, but in real life, due to friction or drag, all oscillations eventually slow down.

## Damping

- The phenomenon of decreasing vibration amplitude and increasing period is called damping.

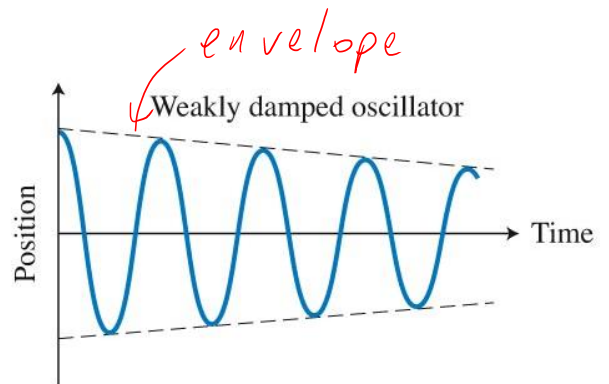


- Damping is a useful aspect of the design of vehicles and bridges.



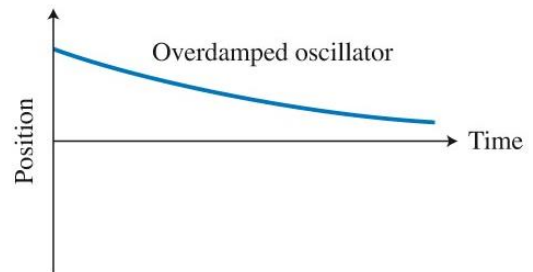
# Three Classes of Damping

1. A **weakly damped** system continues to vibrate for many periods.

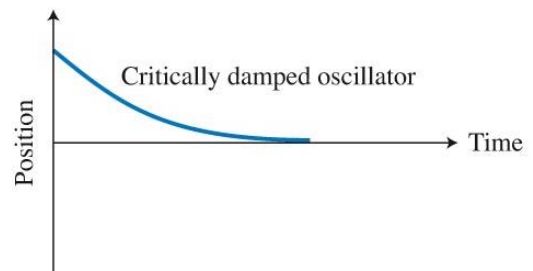


# Three Classes of Damping

2. In an **overdamped** system, the vibrating system takes a long time to return to the equilibrium position, if it ever does.



3. In a **critically damped** system, the vibrating object returns to equilibrium in the shortest time possible.

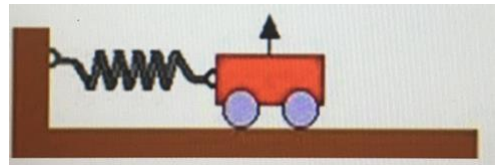


## Damped Vibrations

- A 0.5 kg cart is attached to a horizontal spring, the other end of which is attached to a fixed bumper. The spring constant is 150 N/m. The cart is pulled to the side and released from rest when the spring has been stretched by 0.035 m. How much mechanical energy is converted to internal energy before the cart stops oscillating?

- A. 0.09 Joules
- B. 0.11 Joules
- C. 0.9 Joules
- D. 1.1 Joules
- E. 9 Joules

Initial: Total  
 energy =  $\frac{1}{2}kA^2$   
 $A = 0.035\text{ m}$



Final:  $E = 0$ , Energy lost =  $0.5(150)(0.035)^2$

## Driven Oscillations and Resonance

- Consider an oscillating system that, when left to itself, oscillates at a frequency  $f_0$ . We call this the **natural frequency** of the oscillator.

$$i.e. f_0 = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

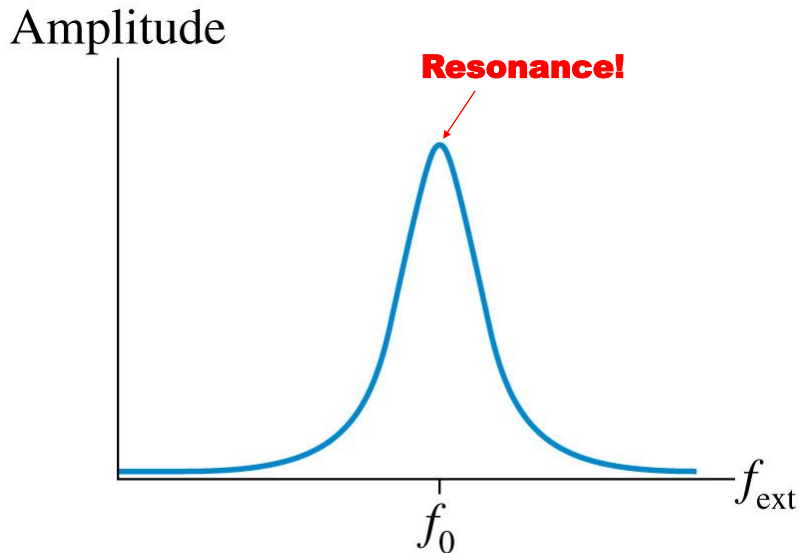
- Suppose that this system is subjected to a *periodic* external force of frequency  $f_{\text{ext}}$ . This frequency is called the **driving frequency**. Driven systems oscillate at  $f_{\text{ext}}$ .

- The amplitude of oscillations is generally not very high if  $f_{\text{ext}}$  differs much from  $f_0$ .

- As  $f_{\text{ext}}$  gets closer and closer to  $f_0$ , the amplitude of the oscillation rises dramatically.

Resonance.

## 14.8 Externally Driven Oscillations



### Feeling Road Vibrations in a Car

- If there are some equally spaced bumps on the road, every 10 m, and the natural frequency of the shock absorbers in your car is about 0.90 Hz, at what driving speed will you feel the bumps the most?

A. 0.9 m/s

B. 1.1 m/s

**C. 9 m/s**

D. 11 m/s

E. 90 m/s

$d = 10\text{m}$   $\rightarrow$   $v = \frac{d}{t}$

set  $t = T$  for resonance

$$v = d \left( \frac{1}{T} \right) = d f = (10\text{m})(0.9\text{Hz})$$

$$v = 9\text{m/s}$$



# Energy transfer through resonance

- Resonance caused the collapse of the Tacoma Narrows Bridge in Washington only four months after its completion.

(a)

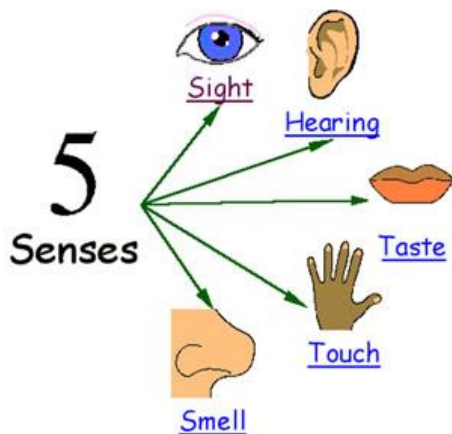


(b)



## Next up: Chapter 11 **Mechanical Waves**

- If you haven't done it, please check your utoronto email, respond to the course\_evaluations email and evaluate this course!



- Something to think about: Two of the five senses depend on **waves** in order to work: which two?

Image from <http://reger.weebly.com/the-five-senses.html>