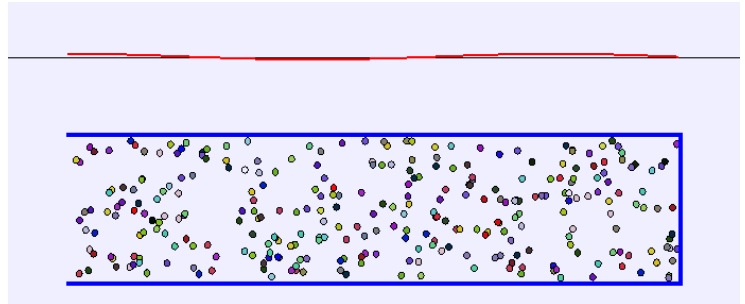


PHY131H1F - Hour 34 – Where did the semester go?



Today:

11.7 Sound Waves, Beats

11.8 Standing Waves on Strings

11.9 Standing Waves in Air Columns

This is a standing wave of sound in an open-closed tube.

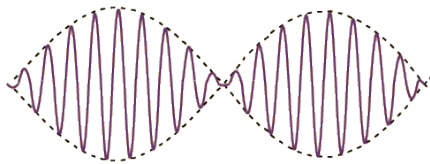
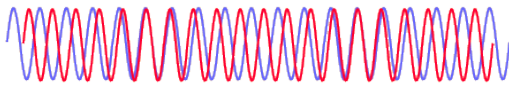
Latest Comments from Learning Catalytics: “Please enter at least one specific question or concern you would like me to address in class.”

- “How does the small angle approximation work?”
- **Harlow answer:** when θ is measured in radians, then:
- $\sin \theta \approx \theta$
- This helps in many cases, for example whenever you have a tall skinny isosceles triangle.

Angle [deg]	Angle [rad]	sine	% error	sig figs
0.1	0.00174533	0.00174533	0.0001%	6
1	0.01745	0.01745	0.01%	4
3	0.05236	0.05234	0.05%	3
5	0.08727	0.08716	0.13%	2
10	0.17453	0.17365	0.51%	2
15	0.26180	0.25882	1.15%	2
20	0.34907	0.34202	2.06%	1
25	0.43633	0.42262	3.25%	1

Beats

- Periodic variations in the loudness of sound due to interference
- Occur when two waves of similar, but not equal frequencies are superposed.
- Provide a comparison of frequencies
- Frequency of beats is equal to the **difference** between the frequencies of the two waves.



[image from <http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html>]

Beats



- Applications

- Piano tuning by listening to the disappearance of beats from a known frequency and a piano key
- Tuning instruments in an orchestra by listening for beats between instruments and piano tone

Beat and beat frequencies

A **beat** is a wave that results from the superposition of two waves of about the same frequency. The beat (the net wave) has a frequency equal to the average of the two frequencies and has variable amplitude. The frequency with which the amplitude of the net wave changes is called the **beat frequency** f_{beat} ; it equals the difference in the frequencies of the two waves:

$$f_{\text{beat}} = |f_1 - f_2| \quad (20.10)$$

Learning Catalytics Question

- If you combine the sounds of two pure tones, one with a frequency of 440 Hertz, and the other with a frequency of 220 Hertz, what do you get?
 - A. Beats with a frequency of 2 Hertz
 - B. Beats with a frequency of 220 Hertz
 - C. Beats with a frequency of 440 Hertz
 - D. A continuous sound which humans perceive to be two tones played at once

Learning Catalytics Question

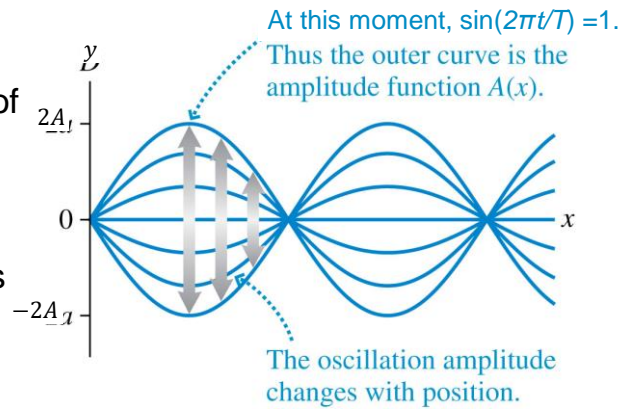
- You are tuning a piano and you want to make the frequency of an A key to be 440 Hertz, but you suspect it is out of tune.
 - You have a reference sound source that you know for sure makes a pure tone of 440 Hertz.
 - When you sound the reference at the same time as the piano A key, you hear 3 beats per second.
 - What is the frequency of your out-of-tune piano A key?
- A. 440 Hz
 - B. 443 Hz
 - C. 437 Hz
 - D. It's impossible to know with the information given

Learning Catalytics Question

- You are tuning a piano and you want to make the frequency of the A key to be 440 Hertz, but you suspect it is out of tune. You have a reference sound source that you know for sure makes a pure tone of 440 Hertz. When you sound the reference at the same time as your out-of-tune piano A key, you hear 3 beats per second.
 - You then tighten the string on the piano, which you know raises the frequency of the A key a bit. When you sound the reference at the same time now, you hear 7 beats per second! What is the new frequency of the out-of-tune piano A key?
- A. 440 Hz
 - B. 447 Hz
 - C. 433 Hz
 - D. It's impossible to know with the information given

The Mathematics of Standing Waves

- Shown is the graph of $y(x,t)$ at several instants of time.
- The nodes occur at $x_m = m\lambda/2$, where m is an integer.



$$y(x, t) = A(x) \sin\left(\frac{2\pi}{T} t\right) \quad A(x) = 2A \sin\left(\frac{2\pi}{\lambda} x\right)$$

Standing Waves on a String

For a string of fixed length L , the boundary conditions can be satisfied only if the wavelength has one of the values:

$$\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, 4, \dots$$

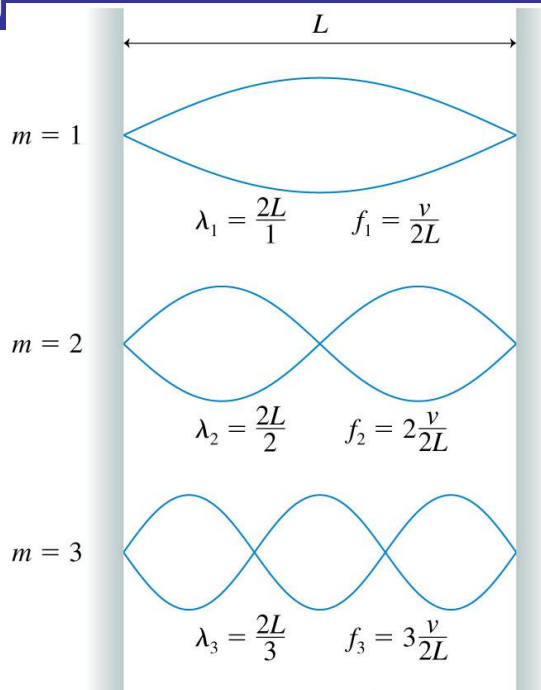
Because $\lambda f = v$ for a sinusoidal wave, the oscillation frequency corresponding to wavelength λ_m is:

$$f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = m \frac{v}{2L} \quad m = 1, 2, 3, 4, \dots$$

The lowest allowed frequency is called the **fundamental frequency**: $f_1 = v/2L$.

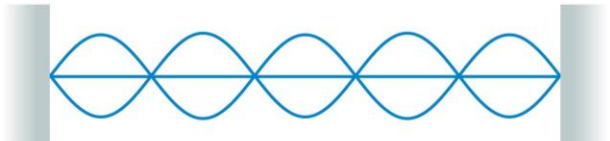
Standing Waves on a String

- Shown are various standing waves on a string of fixed length L .
- These possible standing waves are called the **modes** of the string, or sometimes the *normal modes*.
- Each mode, numbered by the integer m , has a unique wavelength and frequency.



Learning Catalytics Question

What is the mode number of this standing wave?



Standing Waves on a String

There are three things to note about the normal modes of a string:

1. m is the number of *antinodes* on the standing wave.
2. The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
3. The frequencies of the normal modes form a series: $f_1, 2f_1, 3f_1, \dots$. These are also called **harmonics**. $2f_1$ is the “second harmonic”, $3f_1$ is the “third harmonic”, etc.

Standing Waves on a String

- m is the number of *antinodes* on the standing wave.
- The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
- The frequencies of the normal modes form a series: $f_1, 2f_1, 3f_1, \dots$
- The fundamental frequency f_1 can be found as the *difference* between the frequencies of any two adjacent modes: $f_1 = \Delta f = f_{m+1} - f_m$.
- Below is a time-exposure photograph of the $m = 3$ standing wave on a string.



Learning Catalytics Discussion Question

The frequency of the third harmonic of a string is

- A. One-third the frequency of the fundamental.
- B. Equal to the frequency of the fundamental.
- C. Three times the frequency of the fundamental.
- D. Nine times the frequency of the fundamental.

Sound Waves

- Your ears are able to detect sinusoidal sound waves with frequencies between about 20 Hz and 20 kHz.

- Low frequencies are perceived as “low pitch” bass notes, while high frequencies are heard as “high pitch” treble notes.
- Sound waves with frequencies above 20 kHz are called *ultrasonic* frequencies.
- Oscillators vibrating at frequencies of many MHz generate the ultrasonic waves used in ultrasound medical imaging.



Image from <http://www.weblocal.ca/uc-baby-3d-ultrasound-brampton-on.html>

Standing Sound Waves

- A long, narrow column of air, such as the air in a tube or pipe, can support a longitudinal standing sound wave.
- An open end of a column of air must be a pressure node (always at ambient pressure), thus the boundary conditions—nodes at the ends—are the same as for a standing wave on a string.
- A closed end forces a pressure antinode.

Musical Instruments

- With a wind instrument, blowing into the mouthpiece creates a standing sound wave inside a tube of air.
- The player changes the notes by using her fingers to cover holes or open valves, changing the length of the tube and thus its fundamental frequency:

$$f_1 = \frac{v}{2L} \quad \text{for an open-open tube instrument,} \\ \text{such as a flute}$$

$$f_1 = \frac{v}{4L} \quad \text{for an open-closed tube} \\ \text{instrument, such as a clarinet}$$

- In both of these equations, v is the speed of sound in the air *inside* the tube.
- Overblowing wind instruments can sometimes produce higher harmonics such as $f_2 = 2f_1$ and $f_3 = 3f_1$.

$$\begin{cases} \lambda_m = \frac{2L}{m} \\ f_m = m \frac{v}{2L} = mf_1 \end{cases} \quad \begin{array}{l} m = 1, 2, 3, 4, \dots \\ \text{(open-open or closed-closed tube)} \end{array}$$

$$\begin{cases} \lambda_m = \frac{4L}{m} \\ f_m = m \frac{v}{4L} = mf_1 \end{cases} \quad \begin{array}{l} m = 1, 3, 5, 7, \dots \\ \text{(open-closed tube)} \end{array}$$

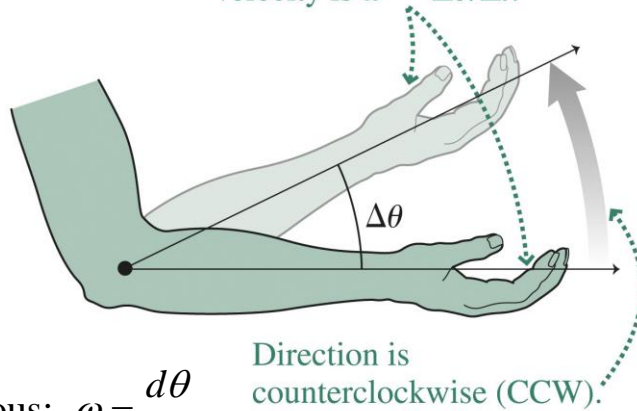
Review: Instantaneous Velocity

- The instantaneous velocity at time t is the average velocity during a time interval Δt centered on t , as Δt approaches zero
- In calculus, this is called *the derivative of x with respect to t*
- Graphically, $\Delta x / \Delta t$ is the slope of a straight line
- In the limit $\Delta t \rightarrow 0$, the straight line is **tangent** to the curve
- The instantaneous velocity at time t is the slope of the line that is tangent to the position-versus-time graph at time t

$v =$ the slope of the position-versus-time graph at t

Review: Angular Velocity

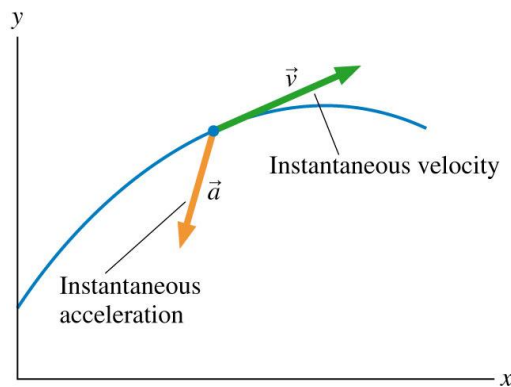
The arm rotates through the angle $\Delta\theta$ in time Δt , so its average angular velocity is $\bar{\omega} = \Delta\theta/\Delta t$.



Instantaneous: $\omega = \frac{d\theta}{dt}$

Review: Acceleration

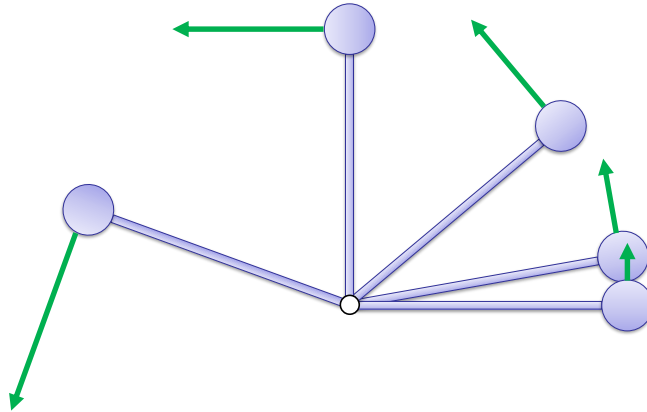
- By definition, \vec{a} is the rate at which \vec{v} is changing at that instant.



Review: Angular Acceleration

- Angular acceleration α is the rate of change of angular velocity.

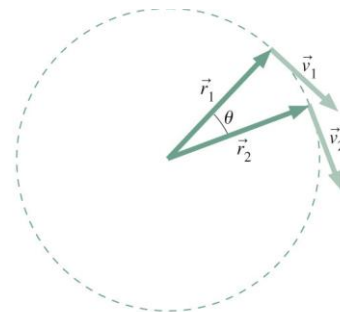
Average: $\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$ Instantaneous: $\alpha = \frac{d\omega}{dt}$



Review: Centripetal Acceleration

- The figure shows the velocity \vec{v}_1 at one instant and the velocity \vec{v}_2 an infinitesimal amount of time dt later
- By definition, $\vec{a} = d\vec{v}/dt$
- By analyzing the isosceles triangle of velocity vectors, we can show that:

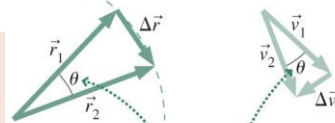
$$\vec{a} = \left(\frac{v^2}{r}, \text{toward center of circle} \right)$$



(a)

$\Delta\vec{r}$ is the difference $\vec{r}_2 - \vec{r}_1, \dots$

\dots and $\Delta\vec{v}$ is the difference $\vec{v}_2 - \vec{v}_1$.



(b)

(c)

These angles are the same, so the triangles are similar.

Review: Static Friction

- A shoe pushes on a wooden floor but does not slip.
- On a microscopic scale, both surfaces are “rough” and high features on the two surfaces touch and adhere.
- This produces force *parallel* to the surface, called the **static friction** force.
- With increased normal force, the shapes ‘lock-together’ better, there’s more contact area, hence the maximum friction force increases.

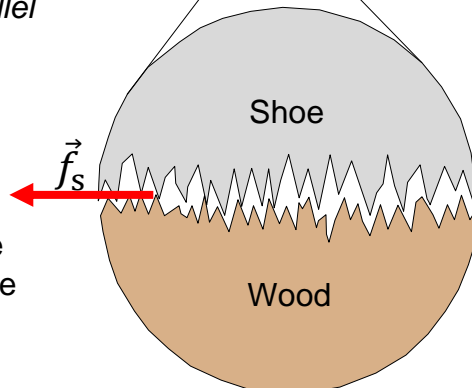


Image from <http://www.which.co.uk/home-and-garden/leisure/reviews/walking-shoes/best-buy/abau/>

Gravitational Potential Energy

- It's convenient to take the zero of gravitational potential energy at infinity. Then the gravitational potential energy becomes

$$U(r) = -\frac{GMm}{r}$$

- When $r = R + y$, with $y \ll R$, we can redefine the zero-point of gravitational potential energy to be at $r = R$.
- Then we have an approximate equation:

$$U \approx \frac{GMm}{R^2} y = mgy$$

- Where $g = GM/R^2$ is the acceleration due to gravity at $r = R$.

Review: Conservation of Mechanical Energy

$$K_1 + U_1 = K_2 + U_2$$

$$K_1 = 0$$
$$U_1 = 10,000 \text{ J}$$



$$K_2 = 2,500 \text{ J}$$
$$U_2 = 7,500 \text{ J}$$



$$K_3 = 7,500 \text{ J}$$
$$U_3 = 2,500 \text{ J}$$



$$K_4 = 10,000 \text{ J}$$
$$U_4 = 0$$



Law of conservation of momentum:

In the absence of an external force, the momentum of a system remains unchanged.

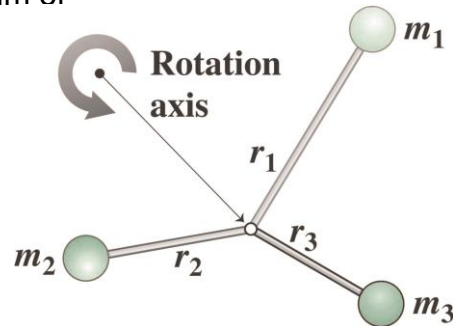
This is usually applied during brief collisions or explosions, in which internal forces are much much greater than any external forces for a short time.



Review: Rotational Inertia

- For a system of discrete masses, the rotational inertia is the sum of the rotational inertias of the individual masses:

$$I = \sum m_i r_i^2$$



The Final Exam!

PHY131H1F	A - LEE	TUE 11 DEC	EV 7:00 - 10:00	EX 100
PHY131H1F	LEN - YO	TUE 11 DEC	EV 7:00 - 10:00	EX 200
PHY131H1F	YU - Z	TUE 11 DEC	EV 7:00 - 10:00	EX 300

- EX is Central Exams Facility, 255 McCaul St. (just south of College St.)

What to expect

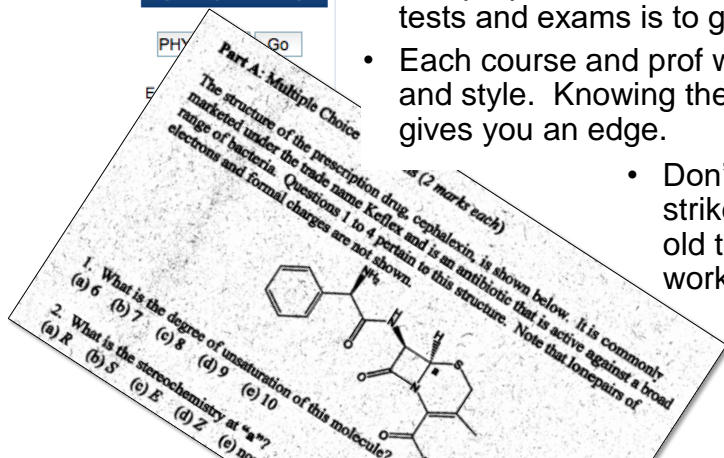
- 3 hours.
- 24 multiple choice questions worth 2 points each, which you do by scratching one of those cards very carefully (48 points total)
- 2 long-answer problems worth 8 points each for which you must show your work using the four-step method from the textbook (16 points total).
- Final exam is out of 64 points.

Study Groups – working with Peers

- Find student (students) in class that you work well with on MasteringPhysics, end-of-chapter suggested problems, and past tests.



- ***The best way to learn is to teach!*** If you can't explain to someone else what you have done, you haven't really understood it! (This is harder than you think!)

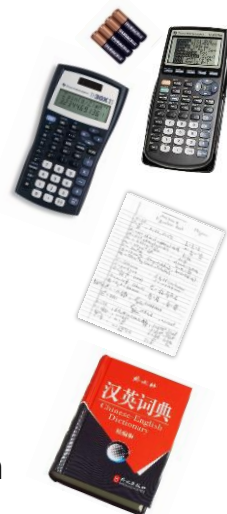


Past Tests and Exams

- The purpose of obtaining and going through old tests and exams is to get to know “the system”.
- Each course and prof will have a certain pattern and style. Knowing the pattern in advance gives you an edge.
- Don’t count on lightning to strike twice – memorizing old test questions rarely works!

Aids Allowed on the Final Exam

- Any calculator without communication capability.
- Aid sheet: one single, original, handwritten 8 1/2 x 11 inch sheet of paper, which may be written on both sides.
- A ruler.
- A **paper** copy of an English translation dictionary.
- Also:



During the Exam

- Exam begins at **7:00pm SHARP!!!**
Seating will begin at 6:50pm, pens hit paper at 7:00.
- This exam is run by the faculty, not the physics department, so be extra careful about the rules.
- Skim over the entire exam from front to back **before** you begin. Look for problems that you have confidence to solve first.
- If you start a problem but can't finish it, leave it, make a mark on the edge of the paper beside it, and come back to it after you have solved all the easy problems.
- Quite snacks or drinks are allowed, and recommended by me.



Monday Dec. 10 after 10:00pm, you **must**:

Relax, maybe watch Netflix, go to bed early.



- The evening before a test is NOT the best time to study (it is just the most popular)
- Don't worry – you have been studying since the 1st week of classes!
- You need to **relax** and get your mind **physically** ready to focus on Tuesday

See you at the final!

- If you haven't done it, please check your utoronto email, respond to the course_evaluations email and evaluate me! The deadline is tomorrow!
- The faculty runs a final exam for this course on Thursday Dec.11 at 7:00pm. See you there!
- Please email me (jharlow@physics.utoronto.ca) with any questions. Keep in touch! It's been a really fun course for me!