PHY131 F Fall 2020
Class 14

Today:

- Let's review last night's Midterm Assessment questions
- Radial Acceleration (a.k.a. "centripetal")
- Vertical Circular Motion
- Banked Curves

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Midterm Assessment 2

- 796 students wrote the second midterm last night and were able to successfully upload their images to the Quercus. Thank you!!
- I have emailed my TAs who will be working on marking your questions this week.
- My hope is they will be done by Friday, at which point I will post your marks.

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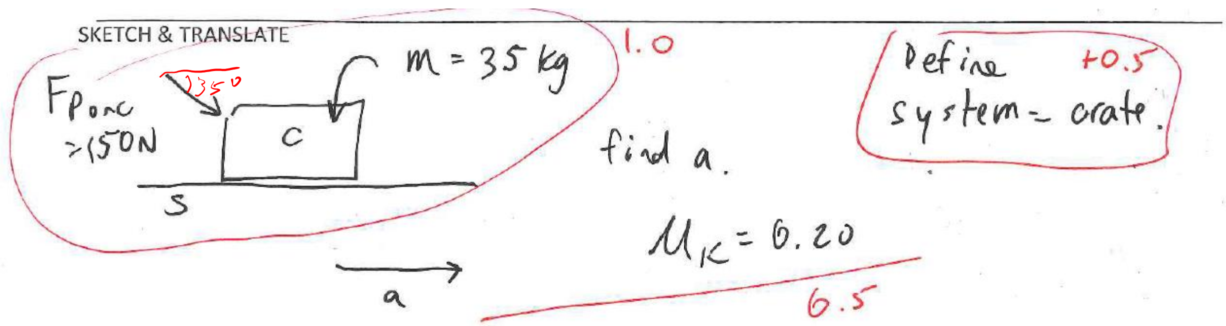
Midterm Assessment: Crate Question Solution #1

A crate with a mass of 35 kg rests on a horizontal surface. The coefficients of friction between the crate and the surface are μ_s = 0.50 and μ_k = 0.20. A person pushes on the crate with a force of magnitude of 150 N at an angle of θ = 35° below the horizontal. What is the magnitude of the acceleration of the crate?

SKETCH & TRANSLATE



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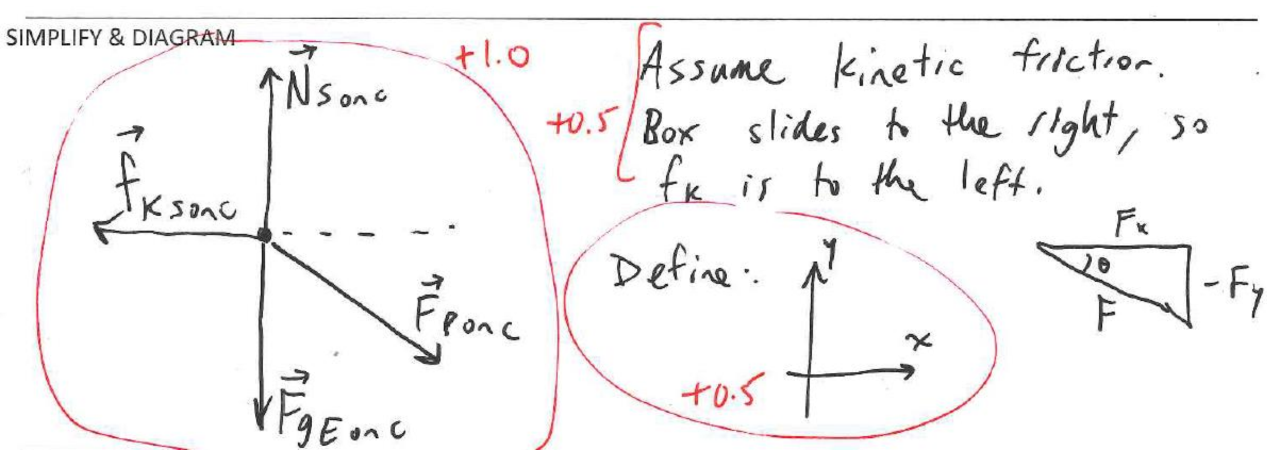
SIMPLIFY & DIAGRAM



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REPRESENT MATHEMATICALLY

$$a_y = 0 \Rightarrow \Sigma F_y = 0 = N - mg - F\sin\theta$$

$$\Rightarrow \boxed{N = mg + F\sin\theta}$$

$$a_x = \frac{\Sigma F_x}{m} = \frac{1}{m}\left[F\cos\theta - f_k\right] = \frac{1}{m}\left[F\cos\theta - \mu_k N\right]$$

$$a = \frac{1}{m}\left[F\cos\theta - \mu_k mg - \mu_k F\sin\theta\right]$$

2/2

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Midterm Assessment: Crate Question Solution #1

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SOLVE & EVALUATE

$$a = \frac{1}{35}\left[150\left(\cos 35 - 0.2\sin 35\right) - 0.2(35)(9.8)\right]$$

2. sig digs

$$\boxed{a = +1.1\ m/s^2}$$

← Sliding to the right with positive acceleration ⇒ speeding up.

2/2

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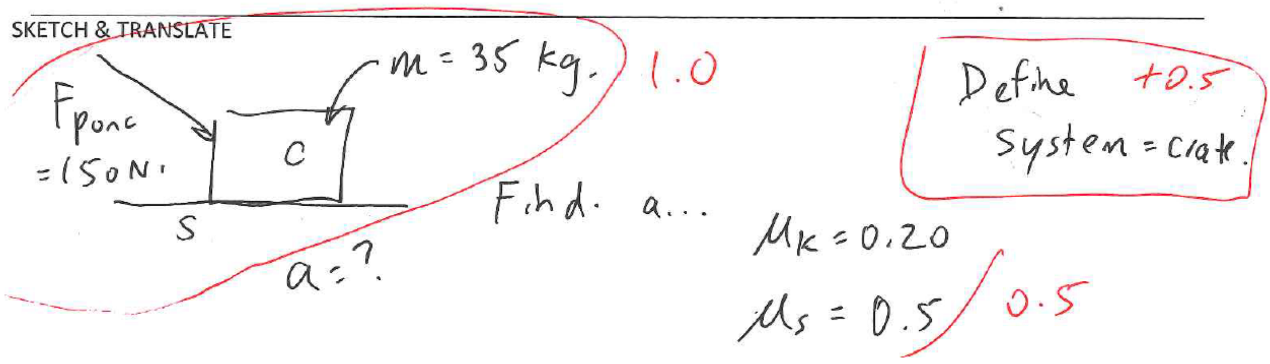
Midterm Assessment: Crate Question Solution #2

A crate with a mass of 35 kg rests on a horizontal surface. The coefficients of friction between the crate and the surface are μ_s = 0.50 and μ_k = 0.20. A person pushes on the crate with a force of magnitude of 150 N at an angle of θ = 35° below the horizontal. What is the magnitude of the acceleration of the crate?

SKETCH & TRANSLATE



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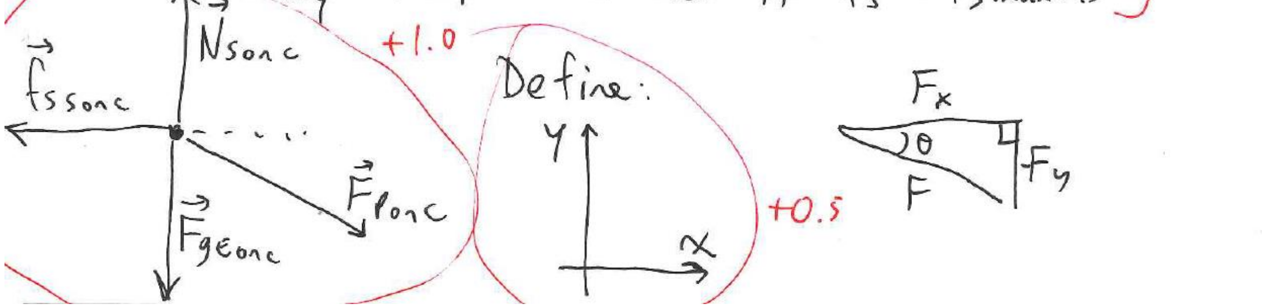
SIMPLIFY & DIAGRAM

Assume crate starts at rest. Static friction may keep it at rest if $f_s < f_{s\,max}$... +0.5



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REPRESENT MATHEMATICALLY

$$a_y = 0 \Rightarrow \Sigma F_y = 0 = N - mg - F\sin\theta$$

$$\Rightarrow N = mg + F\sin\theta$$

$$a_x = 0 \Rightarrow \Sigma F_x = 0 = -f_s + F\cos\theta \Rightarrow f_s = F\cos\theta$$

$$\text{But } f_{s\,max} = \mu_s N = \mu_s(mg + F\sin\theta)$$

Compare f_s to $f_{s\,max}$: which is bigger?

2/2

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Midterm Assessment: Crate Question Solution #2

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SOLVE & EVALUATE

$$f_s = 150 \cdot \cos 35 = 122.9 \text{ N}$$

$$f_{s\,max} = 0.5(35 \times 9.8 + 150 \sin 35)$$

$$f_{s\,max} = 214.5 \text{ N}.$$

$f_s < f_{s\,max}$, so crate remains at rest.

2/2.

$$a = 0$$

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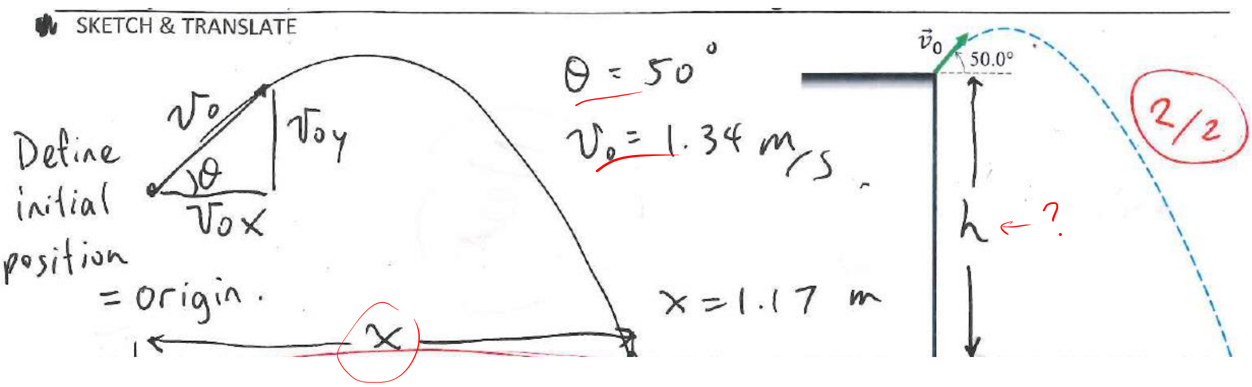
Midterm Assessment Grasshopper Question

A grasshopper leaps into the air from the edge of a vertical cliff, as shown. The grasshopper's initial velocity is 1.34 m/s, 50.0° above the horizontal. The grasshopper lands on the flat ground a horizontal distance of 1.17 m from the base of the cliff. What is the height of the cliff, *h*?



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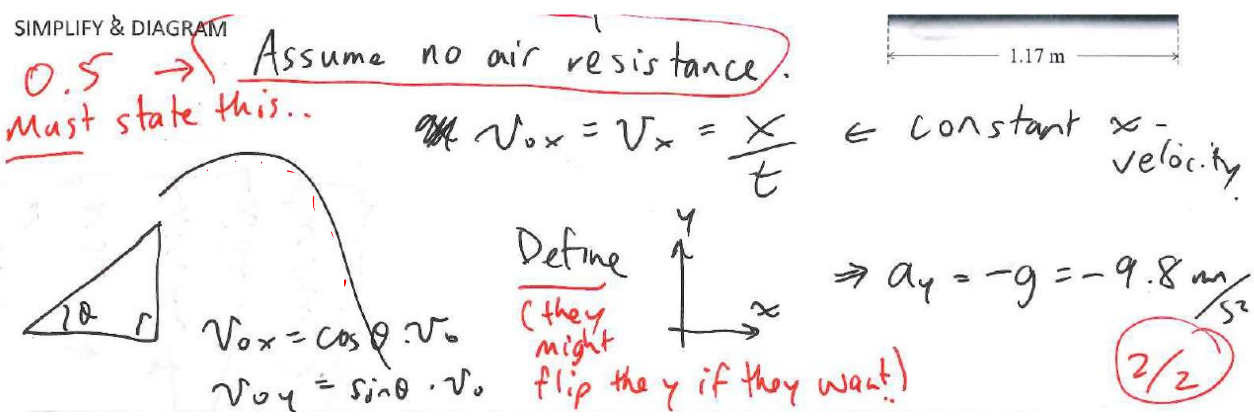

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REPRESENT MATHEMATICALLY

$$v_{0x} = v_0 \cos\theta = \frac{x}{t} \Rightarrow t = \frac{x}{v_0 \cos\theta}$$

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \qquad y_0 = 0$$

$y - y_0$ is the displacement

$$y = v_0 \sin\theta \left[\frac{x}{v_0 \cos\theta}\right] - \frac{g}{2}\left[\frac{x}{v_0 \cos\theta}\right]^2$$

$$y = x\tan\theta - \frac{g x^2}{2 v_0^2 \cos^2\theta}$$

2/2

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Midterm Assessment Grasshopper Question

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SOLVE & EVALUATE

$$y = 1.17 \cdot \tan 50 - \frac{9.8(1.17)^2}{2(1.34)^2(\cos 50°)^2}$$

3 sig. digs

$$y = -7.65 \text{ m}$$

← good that this is negative.

Hard to "evaluate" really...

Height of cliff = 7.65 m

2/2

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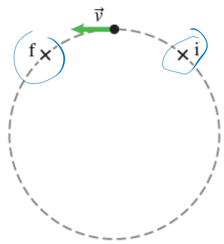
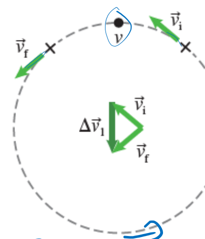
Uniform Circular Motion

Dependence of acceleration on speed

Observational experiment	Analysis
Experiment 1. An object moves in a circle at constant speed.	The acceleration is toward the center of the circular path.





$$a_1 = \frac{\Delta v_1}{\Delta t_1}$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

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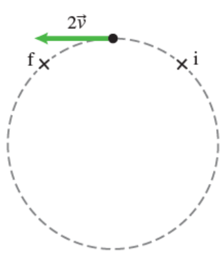
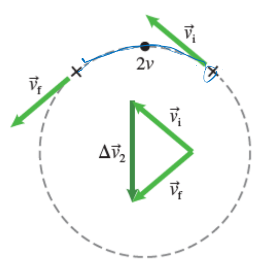
Uniform Circular Motion

Dependence of acceleration on speed

Experiment 2. An object moves in the same circle at a constant speed that is twice as fast as in Experiment 1.



When the object moves twice as fast between the same two points on the circle, the velocity change doubles. In addition, the velocity change occurs in one-half the time interval since it is moving twice as fast. Hence, the acceleration increases by a factor of 4.



$$a_2 = \frac{\Delta v_2}{\Delta t_2} = \frac{2\Delta v_1}{\Delta t_1/2} = 4\frac{\Delta v_1}{\Delta t_1}$$

Δv is 4× larger.

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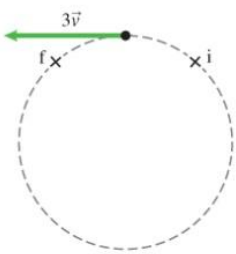
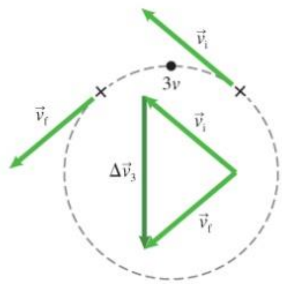
Uniform Circular Motion

Dependence of acceleration on speed

Observational experiment	Analysis
Experiment 3. An object moves in the same circle at a constant speed that is three times as fast as in Experiment 1.	Tripling the speed triples the velocity change and reduces to one-third the time interval needed to travel between the points. The acceleration increases by a factor of 9.
	

$$a_3 = \frac{\Delta v_3}{\Delta t_3} = \frac{3\Delta v_1}{\Delta t_1 / 3} = 9\,\frac{\Delta v_1}{\Delta t_1}$$

Δv is 9 times larger.

From the Observational experiment, it is concluded that the magnitude of radial acceleration is proportional to the speed squared. $a_r \propto v^2$ (5.1)

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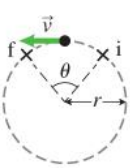
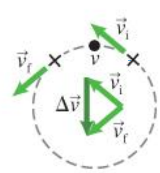
Uniform Circular Motion

Dependence of acceleration on radius

Observational experiment	Analysis
Experiment 1. An object moves in a circle of radius *r* at speed *v*. Choose two points on the circle to examine the velocity change from the initial to the final location.	
	

$$a_1 = \frac{\Delta v}{\Delta t_1}$$

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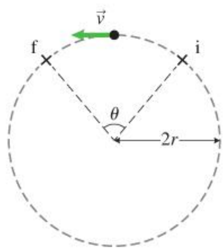
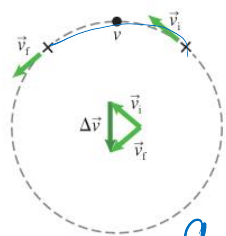
Uniform Circular Motion

Dependence of acceleration on radius

Observational experiment	Analysis
Experiment 2. An object moves in a circle of radius $2r$ at speed v. Choose two points so that the velocity change is the same as in Experiment 1. This occurs if the radii drawn to the location of the object at the initial position and the final position make the same angle as in Experiment 1.	To have the same velocity change as in Experiment 1, the object has to travel twice the distance because the radius is twice as long. Since the speed of the object is the same as in the first experiment, it takes the object twice as long to travel that distance. Hence, the magnitude of the acceleration is half that in Experiment 1.





$$a_2 = \frac{\Delta v}{\Delta t_2} = \frac{\Delta v}{2\Delta t_1} = \frac{1}{2}a_1$$

a_r is $\frac{1}{2}$ of before

Uniform Circular Motion

Dependence of acceleration on radius

Repeating the experiment with different radii, similar results are obtained. It is concluded that the magnitude of radial acceleration is inversely proportional to the radius of the circle.

$$a_r \propto \frac{1}{r} \qquad (5.2)$$

By combining the two proportionalities: $a_r \propto \frac{v^2}{r}$

With the constant of proportionality being 1, a mathematical equation is derived.

Radial acceleration For an object moving at constant speed *v* on a circular path of radius *r*, the magnitude of the radial acceleration is:

$$a_r = \frac{v^2}{r} \qquad (5.3)$$

Direction of $\vec{a}_r$ is toward centre of circular path.

Poll Question

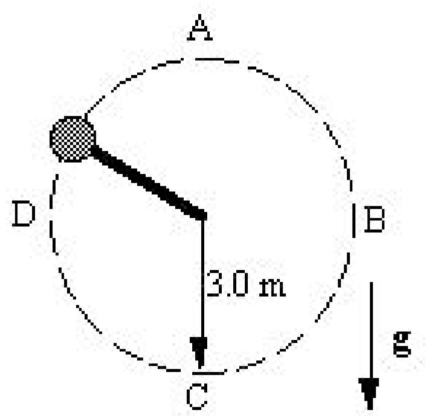
- A ball is whirled on a string in a vertical circle. As it is going around, the tension in the string is

A. greatest at the top of the motion

B. constant.

C. greatest at the bottom of the motion

D. greatest somewhere in between the top and bottom.



Poll Question

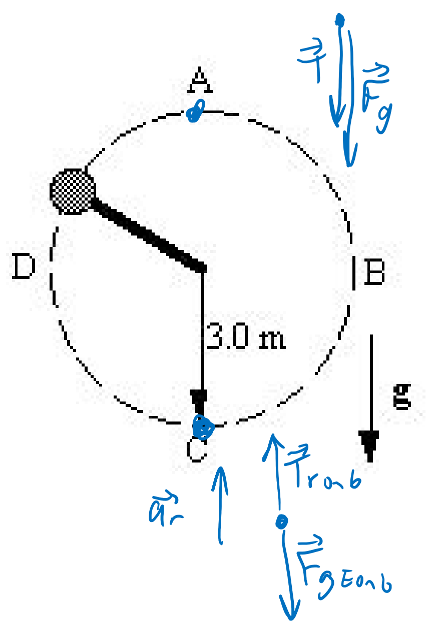
- A ball is whirled on a string in a vertical circle. As it is going around, the tension in the string is

C. greatest at the bottom of the motion

As the ball goes around the circle, it must be accelerating *toward the centre.* A combination of gravity and tension must provide the inward radial force necessary to give it the necessary radial acceleration, v^2/r.

At the bottom of the motion, gravity acts in exactly the wrong direction, so tension must be maximum in order to counter it.

Also, the ball is probably moving fastest at the bottom of the motion, increasing v^2/r.



"Centrifugal Force" (a fictitious force)

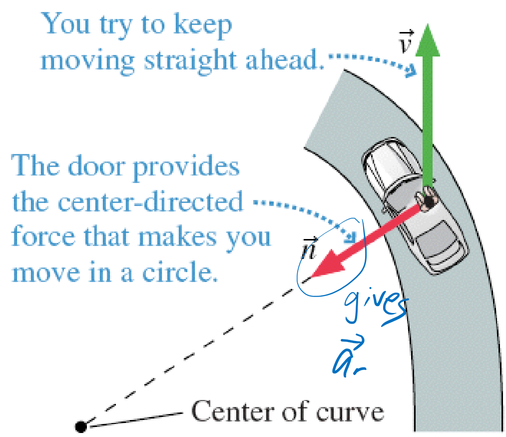
• If the car you are in turns a corner quickly, you feel "thrown" against the door.

• The fictitious "force" that seems to push an object to the outside of a circle is called the "*centrifugal force"*.

• It helps describe your experience *relative to a noninertial reference frame*.

• In the inertial frame of the ground, the only real force is toward the centre not away.

Reality:



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Why Does the water stay in the upside down bucket?

• Watch Harlow swing a bucket of water over his head. If he swings the bucket quickly, the water stays in.

•The minimum speed of the water up at the top of the vertical path is that at which gravity alone is sufficient to cause circular motion at the top.

$$v_{min} = \sqrt{gr}$$

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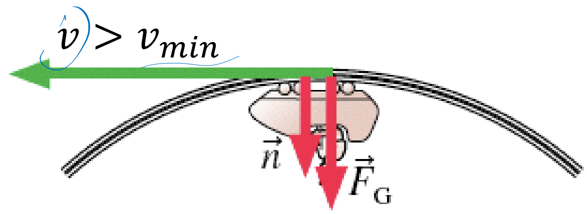
More than enough angular speed

The normal force adds to gravity to make a large enough force for the car to turn the circle.

$a_r = \frac{v^2}{r}$ ← requires $\Sigma F_r = m a_r$

$F_g + n = \frac{m v^2}{r}$



The point is: Normal force must always be *away* from the surface. It can never be *toward* the surface (unless the surface is covered with glue!)

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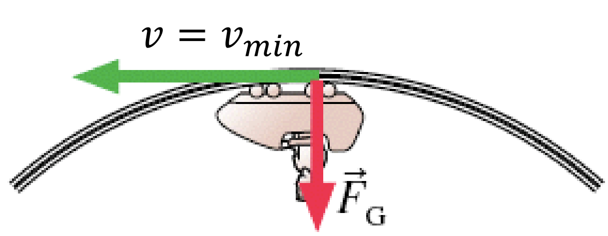
Just enough angular speed

gravity alone is enough force for the car to turn the circle. $\vec{n} = \vec{0}$ at the top point.



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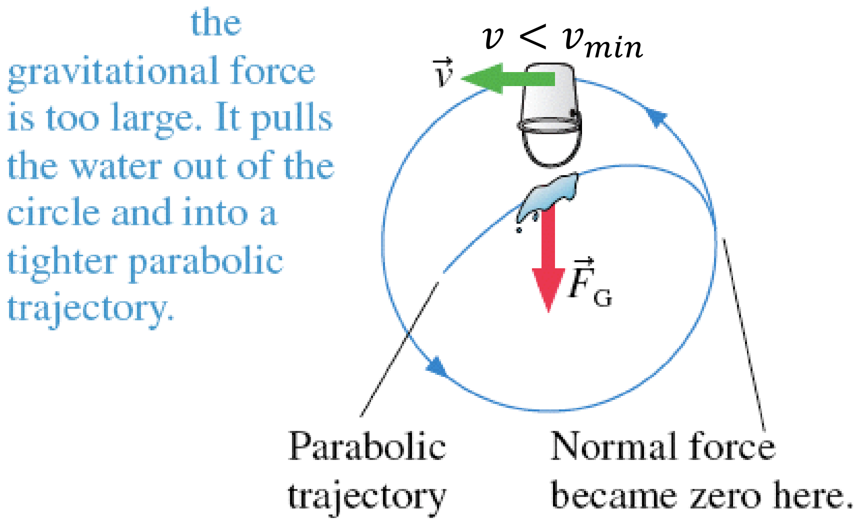
Not enough angular speed



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Self-adjusting forces

- The force of gravity, F_G, has an equation for it which predicts the correct magnitude (it's always mg here on Earth). Also Kinetic Friction has an equation, $f_k = \mu_k N$.
- Remember: Normal force, Tension and Static friction are all self-adjusting forces: ***there are no equations for these!!***
- **Normal force** is whatever is needed to keep the object from crashing through the surface.
- **Tension** is whatever is needed to keep the string or rope from breaking.
- **Static friction** is whatever is needed to keep the object from slipping along the surface.
- In all these cases, you must draw a ~~free-body~~ force diagram and figure out by using equilibrium and Newton's 2nd law what the needed force is.

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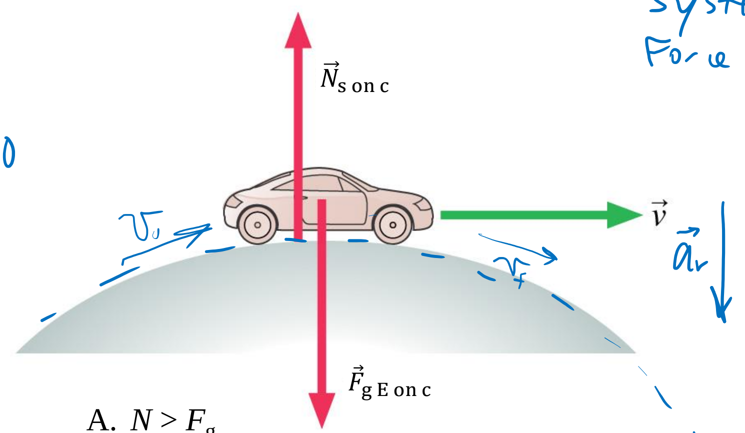
Poll Question

A car is rolling over the top of a hill at speed *v*. At this instant,



A. $N > F_g$
B. $N < F_g$
C. $N = F_g$
D. We can't tell about N without knowing v.

system = car.
Force diagram

a_r: $\Sigma F_r > 0$
$F_g - N > 0$
$F_g > N$
$N < F_g$

Define +r ↓

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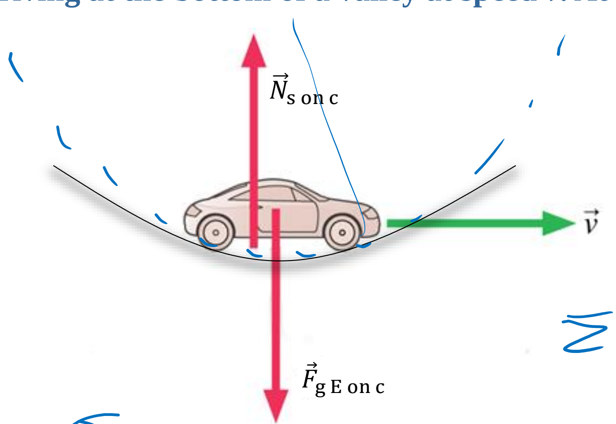
Poll Question

A car is driving at the bottom of a valley at speed *v*. At this instant,



A. $N > F_g$
B. $N < F_g$
C. $N = F_g$
D. We can't tell about N without knowing v.

Define +r ↑

$\Sigma F_r > 0 = N - F_g$
$N - F_g > 0$
$N > F_g$

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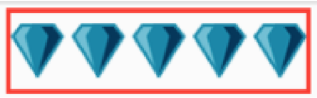

Friday’s TeamUp Quiz

- For 10 minutes during Friday’s class (around 11:30) every student should go on Microsoft Teams and someone (most recent Facilitator) should place a video call to all 3 or 4 members of your Pod-Chat.
- As with last Friday, there will be three multiple choice questions to work on together, each worth a possible maximum of 5 homework credits.

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Before Class 15 on Friday

- Finish reading Chapter 5
- Plan to meet up with your Practical Pod during Friday’s class – you should be able to turn on your microphone in order to participate in the TeamUp Quiz Module 3 Ch.5.
- If you cannot do the TeamUp quiz during class, it can be done either with your pod or on your own at any time over the weekend.

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