PHY131H1F - Class 25

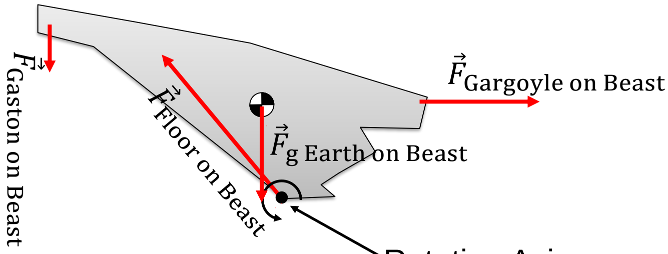
- Welcome back from break!
- What was I doing on my break?
- I did get outdoors a lot… I did also check out a few things on Disney+ with my kids!

Define Beast = “system”

Today:

8.5 Static Equilibrium Problems

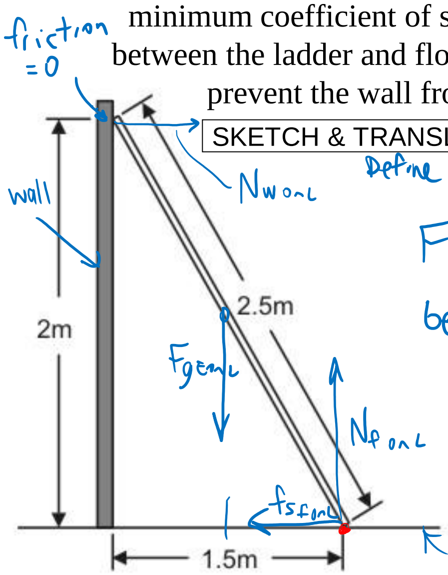
8.6 Stability



Rotation Axis

Example

A uniform ladder leans against a wall, as shown. The wall is frictionless. What is the minimum coefficient of static friction between the ladder and floor needed to prevent the wall from slipping?

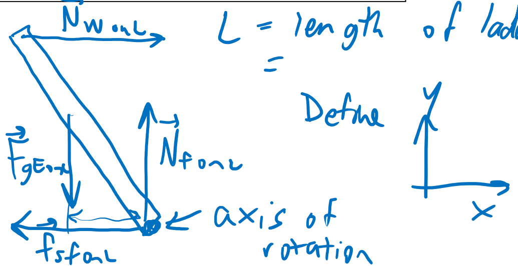


SKETCH & TRANSLATE.

Define system = ladder.

Find μ_s, between floor & ladder

SIMPLIFY & DIAGRAM



L = length of ladder
=

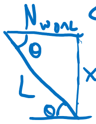
REPRESENT MATHEMATICALLY

Assume static equilibrium.

$$\Sigma F_x = 0 = N_{w\,on\,L} - f_s$$

$$\Sigma F_y = 0 = N_{f\,on\,L} - mg$$

$$\Sigma \tau = 0 = \tau_g + \tau_{N\,wall}$$

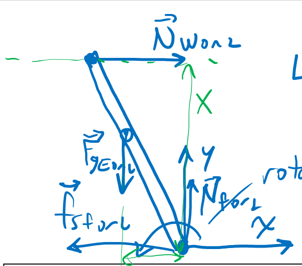


$x=2m$ $\tau_{N\,wall} = -N_{w\,on\,L}(2m)$

$\tau_g = +mg\left(\frac{1.5}{2}\right)$

SIMPLIFY & DIAGRAM

$\tau = Nx$



L = length of ladder = 2.5 m

rotation axis, counter-clockwise = +

REPRESENT MATHEMATICALLY

Equilibrium: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma \tau = 0$.

$$\Sigma F_x = N_{w\,on\,L} - f_s = 0 \qquad ①$$

$$\Sigma F_y = N_{f\,on\,L} - mg = 0 \qquad ②$$

$$\Sigma \tau = -N_{w\,on\,L}(2m) + mg(0.75m) = 0 \qquad ③$$

$$\underline{f_s = f_{s\,max} = \mu_s N_{f\,on\,L}} \quad \text{"just about to slip"} \qquad ④$$

$$④ \Rightarrow \mu_s = \frac{f_s}{N_{f\,on\,L}}$$

① $f_s = N_{w\,on\,L}$

② $\Rightarrow N_{f\,on\,L} = mg$

$$\mu_s = \frac{N_{w\,on\,L}}{mg}.$$

$$③ \Rightarrow \quad 2 N_{w\,on\,L} = 0.75\, mg$$

$$N_{w\,on\,L} = \frac{0.75}{2} mg$$

SOLVE & EVALUATE

$$\mu_s = \frac{0.75(\cancel{mg})}{2\cancel{mg}} = \frac{0.75}{2}$$

$$\boxed{\mu_s = 0.38}$$ ← minimum coefficient before slipping.

Seems reasonable < 1 . . .

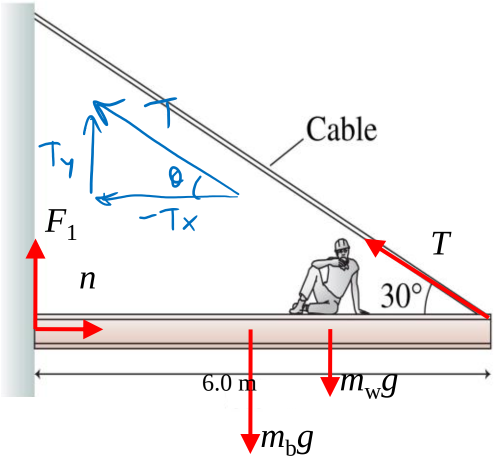
You Try!! Poll Question

- A construction worker of mass m_w sits 2.0 m from the end of a steel beam of mass m_b, as shown.
- The tension in the Cable is T
- The wall exerts a normal force, n on the beam, and an upward force, F_1.
- Define $+x$ = to the right, $+y$ = up, and the ~~pivot~~ is the point where the beam touches the wall. axis of rotation.
- What is the normal force, n?

Hint: balance x – forces !

$$\Sigma F_x = 0 = n - T\cos 30°$$

$$n = T\cos 30$$



A. $(m_b + m_w)g$

B. $(m_b + m_w)g - T\cos(30°)$

C. $(m_b + m_w)g - T\sin(30°)$

D. $T\sin(30°)$

[✓ E. $T\cos(30°)$]

You Try!! Poll Question

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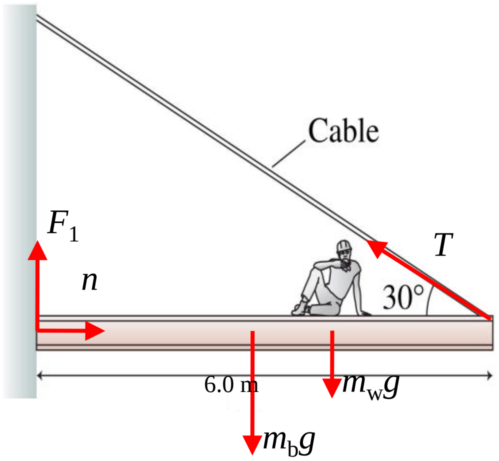
- The tension in the Cable is T
- The wall exerts a normal force, n on the beam, and an upward force, F_1.
- Define $+x$ = to the right, $+y$ = up, and the pivot is the point where the beam touches the wall.
- What is the force, F_1?



Hint: balance y-forces

$$\Sigma F_y = 0 = F_1 - m_b g - m_w g + T\sin 30$$

$$F_1 = m_b g + m_w g - T\sin 30$$

A. $(m_b + m_w)g$

B. $(m_b + m_w)g - T\cos(30°)$

[✓ C.] $(m_b + m_w)g - T\sin(30°)$

D. $T\sin(30°)$

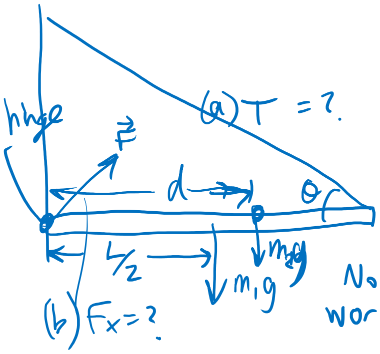
E. $T\cos(30°)$

A uniform steel beam of length L and mass m_1 is attached via a hinge to the side of a building. The beam is supported by a steel cable attached to the end of the beam at an angle θ, as shown. Through the hinge, the wall exerts an unknown force, $\vec{F}$, on the beam. A workman of mass m_2 sits eating lunch a distance d from the building.

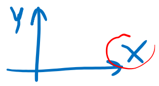
a) Find T, the tension in the cable.

b) Find F_x, the x-component of the force exerted by the wall on the beam ($\vec{F}$), using the axis shown.

SKETCH & TRANSLATE.



Define



System = beam.

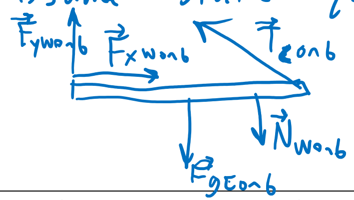
Normal force of workman on beam = $m_2 g$.

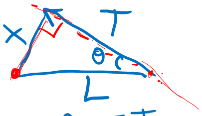
SIMPLIFY & DIAGRAM

Hinge = axis of rotation

Assume static equilibrium

c = cable.





$\tau_T = T x$

$\tau_T = T L \sin\theta$

REPRESENT MATHEMATICALLY

$$\text{(a)} \quad \Sigma \tau = 0 = T L \sin\theta - m_2 g d - m_1 g L/2$$

$$T L \sin\theta = m_2 g d + m_1 g L/2$$

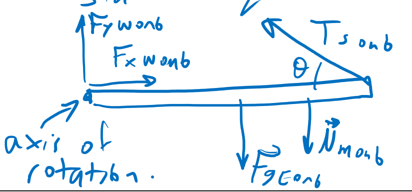
$$T = \frac{m_2 g d + m_1 g L/2}{L\sin\theta}$$

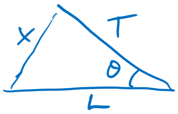
$$\text{(b)} \quad \Sigma F_x = 0 = F_x - T\cos\theta$$

$$F_x = T\cos\theta$$

SIMPLIFY & DIAGRAM

hinge = rotation axis. Assume static eq.





$\tau_T = Tx$

$\tau_T = TL\sin\theta$

REPRESENT MATHEMATICALLY

(a) $\sum \tau = 0 = TL\sin\theta - m_2 g d - m_1 g L/2$

$T L\sin\theta = m_2 g d + m_1 g L/2$

$$T = \frac{m_2 g d + m_1 g L/2}{L\sin\theta}$$

(b) $\sum F_x = 0 = F_x - T\cos\theta$

$F_x = T\cos\theta$

$$F_x = \frac{\cos\theta}{L\sin\theta}\left(m_2 g d + m_1 g L/2\right)$$

SOLVE & EVALUATE

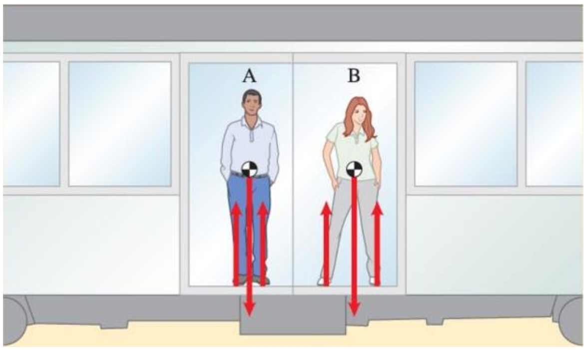
No numbers to plug in.

as $\theta \to 0$, $T \to \infty$, $F_x \to \infty$ } makes sense.

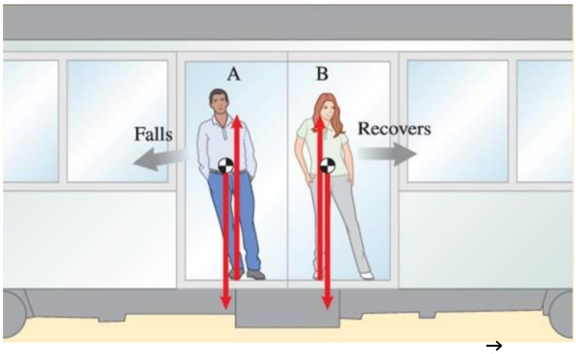
as m_1, m_2 increase, T and F_x increase...

Equilibrium and tipping objects

- You have probably observed that it is easier to balance and avoid falling while standing in a moving bus or subway train if you spread your feet apart in the direction of motion.
- By assuming this stance, you increase the **area of support**—the area of contact between an object and the surface it is supported by.



Train At Rest

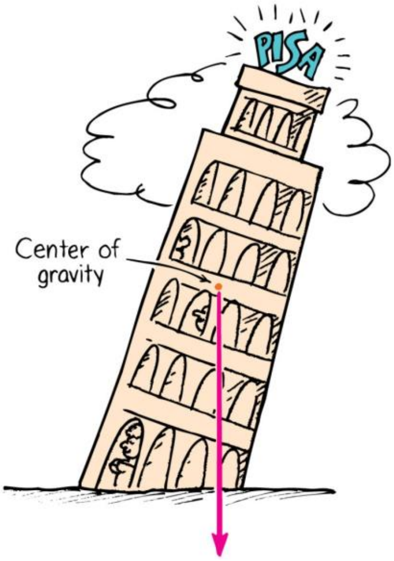


Accelerating to the Right $\vec{a}$

Centre of Gravity—Stability

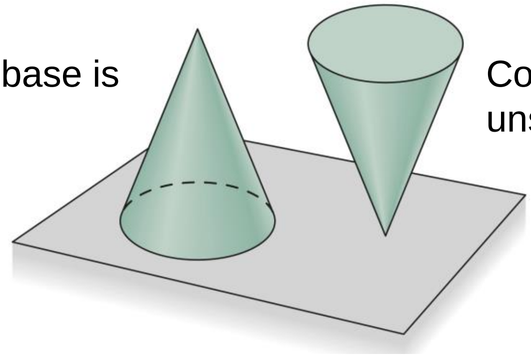
The location of the centre of gravity is important for stability.

- If we draw a line straight down from the centre of gravity and it falls inside the base of the object, it is in stable **equilibrium;** it will balance.
- If it falls outside the base, it is unstable.



Stability

- An equilibrium is stable if a slight disturbance from equilibrium results in forces and/or torques that tend to restore the equilibrium.
- An equilibrium is unstable if a slight disturbance causes the system to move away from the original equilibrium.

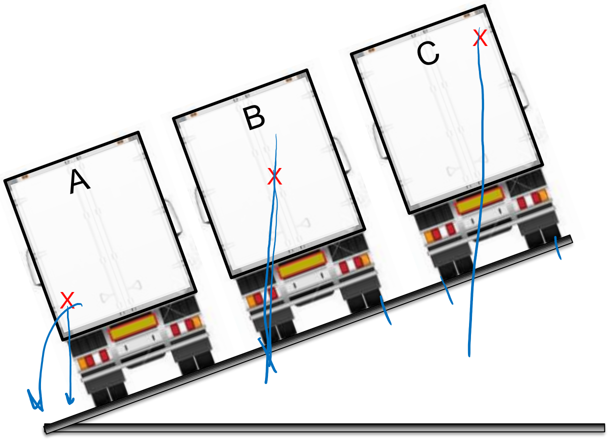


Cone on its base is stable

Cone on its tip is unstable

Poll Question

The centres of gravity of the three trucks parked on a hill are shown by the Xs. Which truck(s) will tip over?

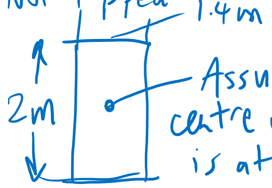


D. All three of the trucks will tip over.
E. None of the three will tip over.

A refrigerator is 2.0 m high, and 1.4 m wide. On a flat floor, by what maximum angle can it tip sideways and still not fall over on its side?

SKETCH & TRANSLATE.

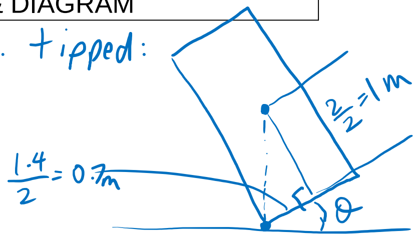
Not tipped:



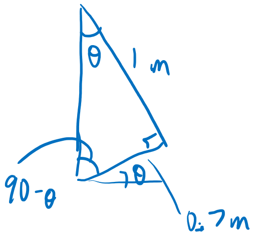
Assume centre of mass is at geometric centre

SIMPLIFY & DIAGRAM

Max. tipped:



REPRESENT MATHEMATICALLY



$$\tan\theta = \frac{0.7}{1\ m}$$

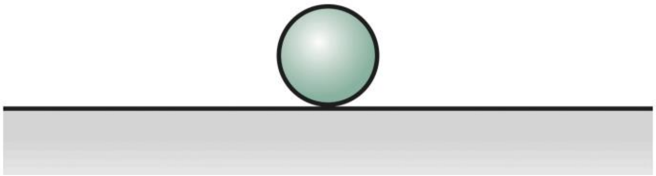
SOLVE & EVALUATE

$$\theta = \tan^{-1}\left(\frac{0.7}{1}\right) = \boxed{35°}$$

Less than 90°,
So that's good.

STABILITY JEOPARDY!

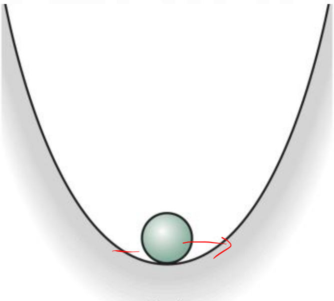
RIP Alex Trebek
1940-2020



It could look like this.

A. What is “Stable Equilibrium”?
[✓ B.] What is “Neutrally Stable Equilibrium”?
C. What is “Unstable Equilibrium”?
D. What is “Metastable Equilibrium”?

STABILITY JEOPARDY!



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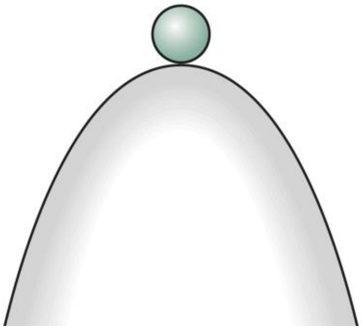
STABILITY JEOPARDY!



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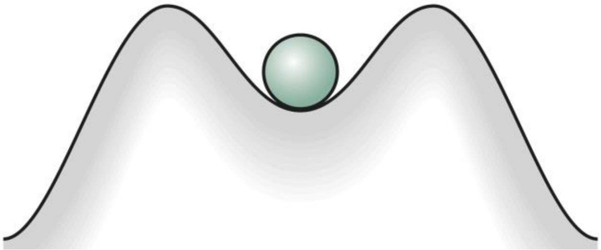
STABILITY JEOPARDY!



It could look like this.

A. What is “Stable Equilibrium”?
B. What is “Neutrally Stable Equilibrium”?
C. What is “Unstable Equilibrium”?
[✓ D. What is “Metastable Equilibrium”?]

Midterm Assessment 4

- There will be two problems you must solve using the 4-step method.
- The solutions must be in your handwriting and written upon an Answer Template Sheet or something very similar.
- You will see both problems at once, starting tomorrow at 8:10pm Toronto time. One will be from Chapter 7, and the other from Chapter 8.
- You will get an email from crowdmark, and also there will be a link on the Quercus under Module 4.
- You have 30 minutes to write out your solutions to both.
- There is an additional 5 minutes which you should allow for uploading the file.
- All uploads must be complete by 8:45pm, 30 minutes after the start time.
- Your images should ideally be PDFs or JPEG images.
- Worst-case scenario if crowdmark fails is you can attach your images in an email to phy131fall@physics.utoronto.ca . (But please only do this if crowdmark fails.)

Before Class 26 on Wednesday

- Don’t forget to do the quiz on Tuesday evening!
- Also, please read:
- 9.1 Rotational Kinematics
- 9.2 Rotational Inertia

Have you ever wondered: How do figure skaters spin so quickly?