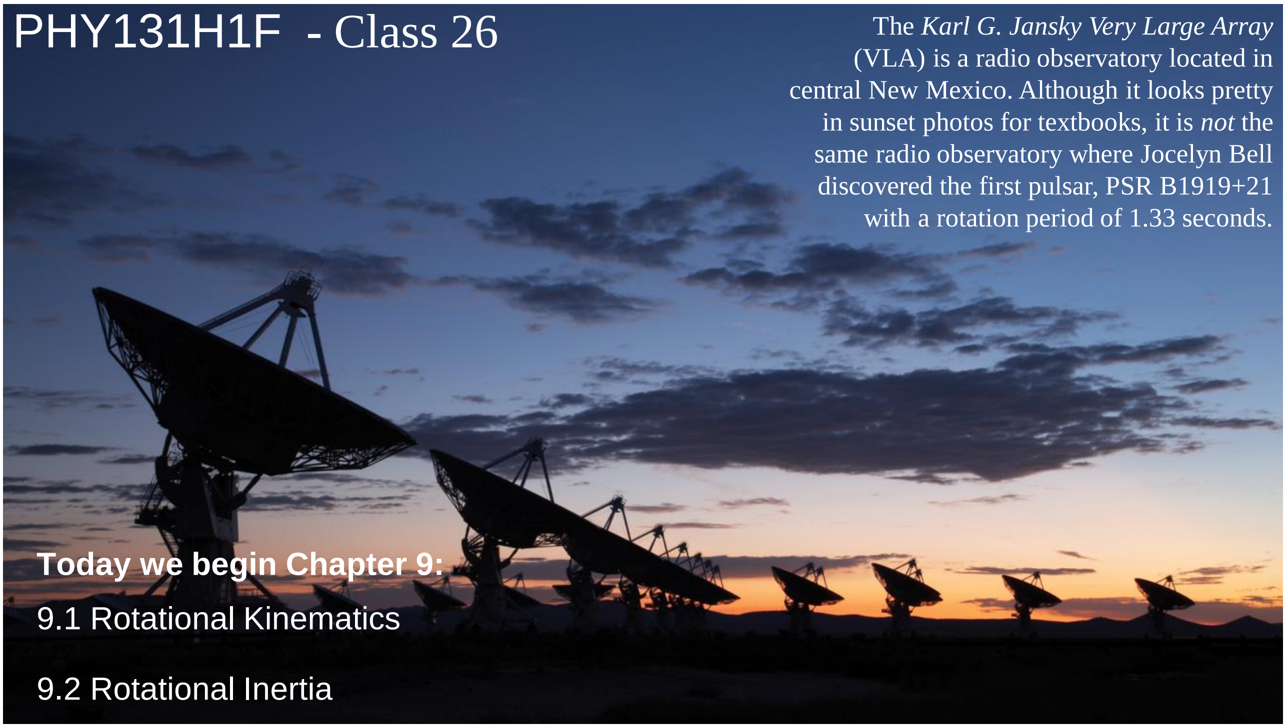
PHY131H1F - Class 26



The *Karl G. Jansky Very Large Array* (VLA) is a radio observatory located in central New Mexico. Although it looks pretty in sunset photos for textbooks, it is *not* the same radio observatory where Jocelyn Bell discovered the first pulsar, PSR B1919+21 with a rotation period of 1.33 seconds.

Today we begin Chapter 9:

9.1 Rotational Kinematics

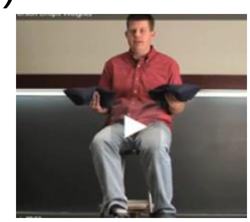
9.2 Rotational Inertia

1

Open MyLab & Mastering

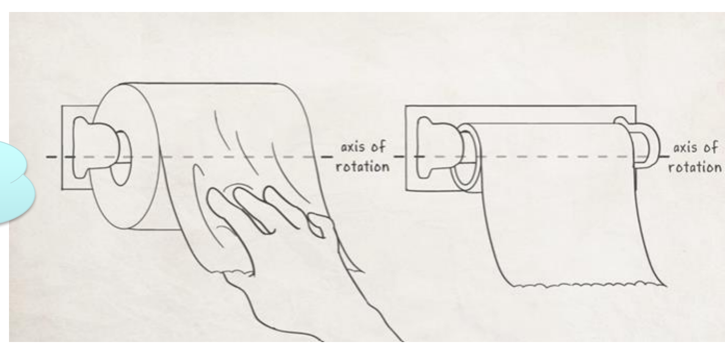
Videos and Practice for Chapter 9 (not for homework credit)

- Featuring, "Buzzcut Guy Gets Really Dizzy!"



- And, going to the toilet will never be the same again!

Which toilet paper roll has more ***rotational inertia****??*



2

Homework Credits

PHY131H1F Marking Scheme:

• Practicals:	20%
• Homework Credits:	x%, where $0 \le x \le 20$
• Midterm Assessments (best 4 out of 5):	$(50 - (x/2))$%
• Final Assessment:	$(30 - (x/2))$%

- All homework and surveys not directly related to the Practicals is optional.
- As you accumulate homework credits, the weighting of your midterms and final assessments will be reduced from 80% down to a minimum of 60%. The breakdown of all the maximum available points this semester is:
  - 11 MasteringPhysics homeworks: 138 points
  - 8 In-Class Friday TeamUp Group Quizzes: 120 points
  - 3 Practices Quizzes for Midterms: 42 points
  - Getting to Know you Survey and LASSO Surveys: 5 points
  - **Total Possible Points: 305**
- If you earn n homework credits, then:

$$x = \frac{20\,n}{305}$$

$$\frac{x}{x} = 1 = 100\%$$

3

Poll: How did Midterm 2 and 4 compare?

- As long as we're in COVID, for midterms and the final I want every student in this course to simultaneously write out, by hand, the solutions to two problems using the 4-step method in 30 minutes, and then upload images immediately for marking. I know this is not easy, especially when the internet is not perfectly reliable, either on U of T's end or your end.
- Midterm 2 on Oct.13 was submitted via Quercus.
- Midterm 4, last night, was submitted via Crowdmark.
- How did they compare for you?

A. Crowdmark and Quercus were equally good; I had no major issues with the technology. 155 (43%)

B. Crowdmark was a better experience for me than Quercus. 129 (36%)

C. Crowdmark was a worse experience for me; I actually preferred using Quercus. 68 (19%)

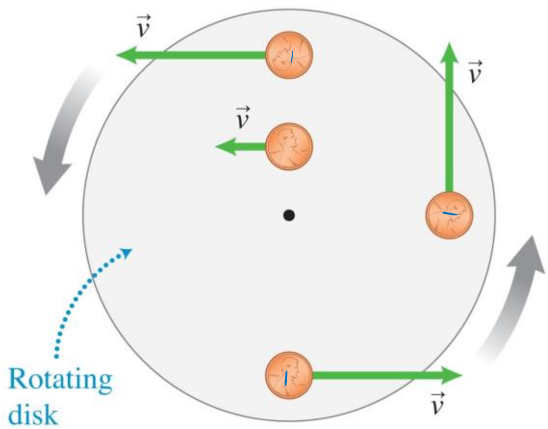
D. Both Crowdmark and Quercus were equally bad; I had major technical issues with both. 13 (4%)

E. I can't compare as I missed one or the other or both. 4 (1%)

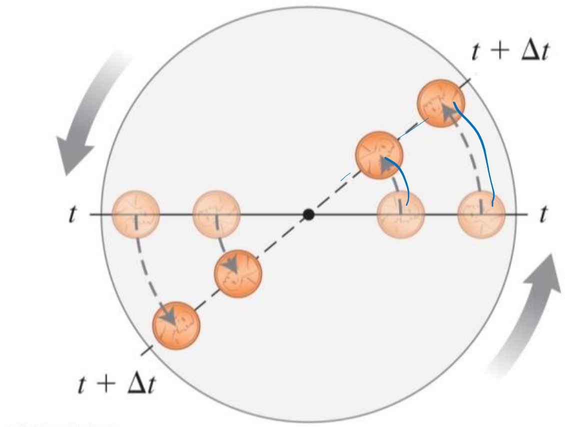
4

Suppose a horizontal disk is rotating on a lab bench and your are looking down on it. The rotation axis passes through the centre, and is perpendicular to the disk (out of page)

The direction of the velocity $\vec{v}$ for each coin changes continually.



Coins at the edge travel farther during Δt than those near the center. The speed v will be greater for coins near the edge than for coins near the center.



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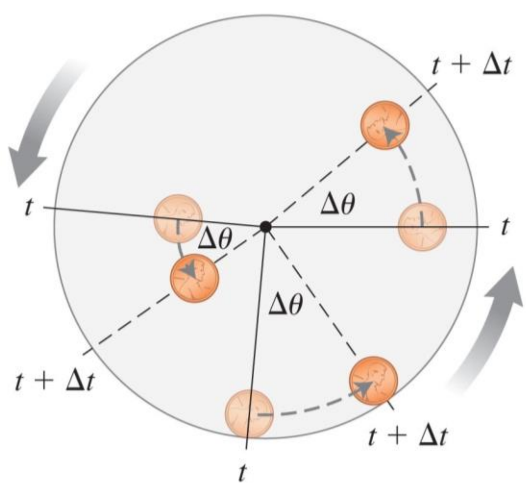
Rotational kinematics

- There are similarities between the motions of different points on a rotating rigid body.
  - During a particular time interval, all coins at the different points on the rotating disk turn through the same angle.
  - **Perhaps we should describe the rotational position of a rigid body using an angle.**

All coins turn through the same angle in Δt, regardless of their position on the disk.



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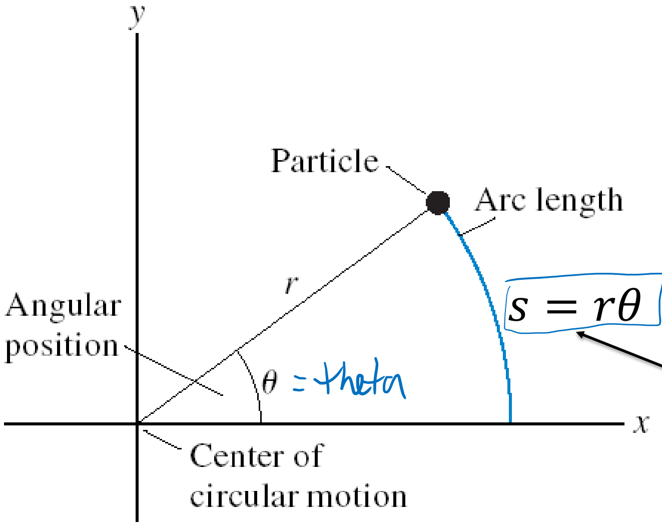
Rotational (Angular) Position



NOTE: This equation only works if θ is measured in **radians!**

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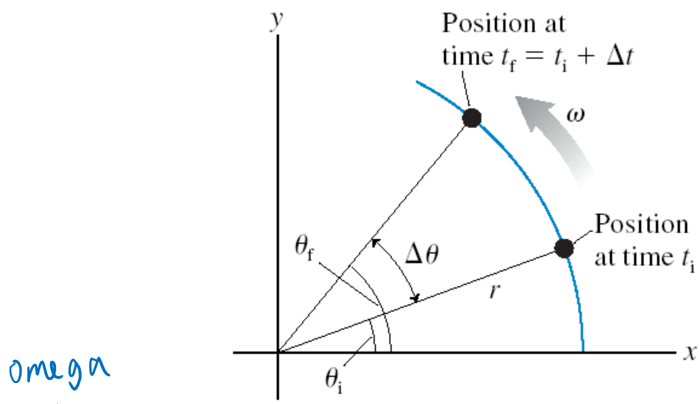
Angular Velocity



$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt} \qquad \text{(angular velocity)}$$

$$\theta_f = \theta_i + \omega \Delta t \qquad \text{(uniform circular motion)}$$

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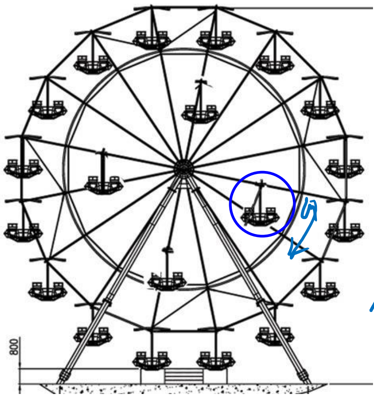
Poll



$v = \omega r$, r = distance from axis

A carnival has a Ferris wheel where some seats are located halfway between the center and the outside rim. Compared with the seats on the outside rim, the inner cars have

All cars sweep out equal angles in equal time. ω = same for all.

A. Smaller angular speed and greater tangential speed
B. Greater angular speed and smaller tangential speed
C. The same angular speed and smaller tangential speed
D. Smaller angular speed and the same tangential speed
E. The same angular speed and the same tangential speed

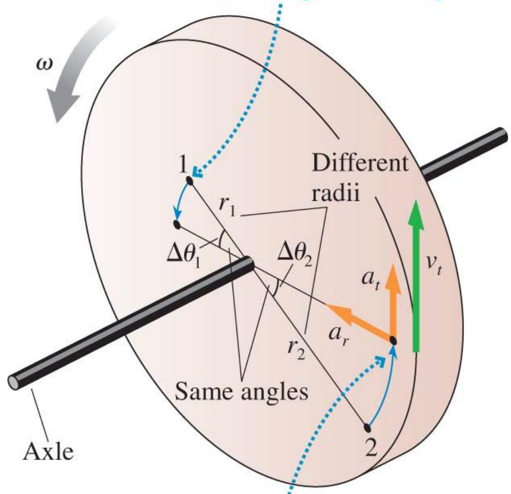
9

Rigid Body Rotation

- Angular velocity, ω, is the rate of change of angular position, θ.
- The units of ω are rad/s.
- If the rotation is speeding up or slowing down, then its angular acceleration, α, is the rate of change of angular velocity, ω.
- The units of α are rad/s^2.
- All points on a rotating rigid body have the same ω and the same α.

alpha \alpha

Every point on the wheel turns through the same angle and thus undergoes circular motion with the same angular velocity ω.



All points on the wheel have a tangential velocity and a radial (centripetal) acceleration. They also have a tangential acceleration if the wheel has angular acceleration.

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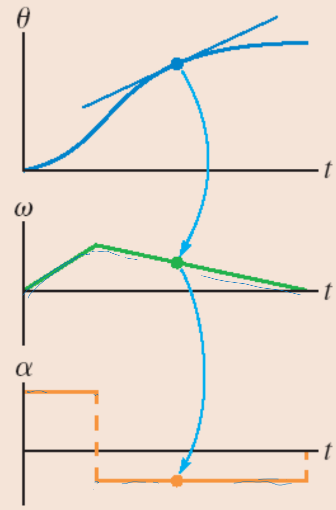
Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.



- Arc length: $s = \theta r$
- Tangential velocity: $v_t = \omega r$
- Tangential acceleration: $a_t = \alpha r$

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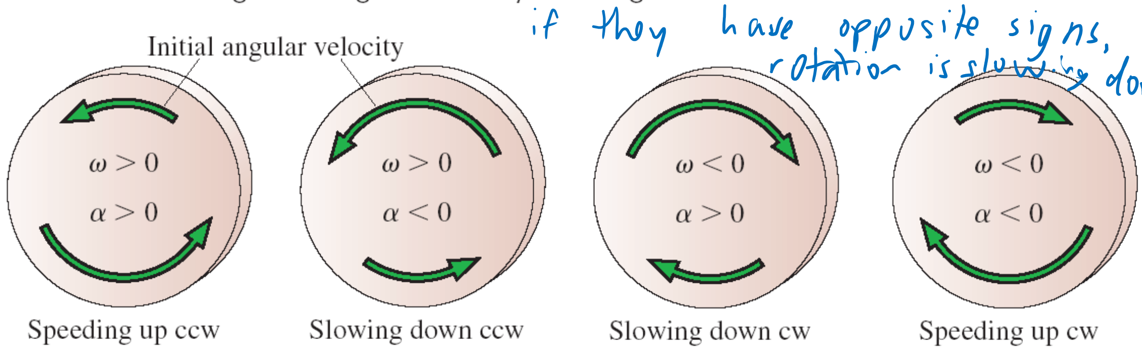
Rotational Kinematics

if ω and α have same sign, then rotation is speeding up.

The signs of angular velocity and angular acceleration.

if they have opposite signs, rotation is slowing down.



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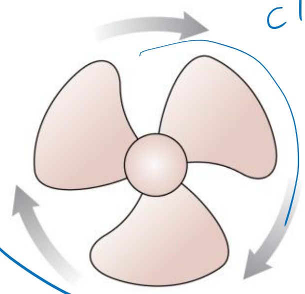
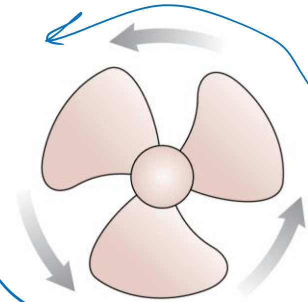
Poll 1/2. [Please use the historical convention in which we define **positive** angular displacement to be **counter-clockwise**.]



The fan blade is speeding up.
What are the signs of ω and α?

A. ω is positive and α is positive.
B. ω is positive and α is negative.
C. ω is negative and α is positive.
D. ω is negative and α is negative.

Poll 2/2. [Please use the historical convention in which we define **positive** angular displacement to be **counter-clockwise**.]



The fan blade is slowing down.
What are the signs of ω and α?

A. ω is positive and α is positive.
B. ω is positive and α is negative.
C. ω is negative and α is positive.
D. ω is negative and α is negative.

Rotational Kinematics

$v_x = \omega r$

Linear	Rotational Analogy
• *x* specifies position. The S.I. Unit is metres.	• θ is **angular position**. The S.I. Unit is radians, where 2π radians $= 360°$.
• Velocity, v_x, is the slope of the *x* vs *t* graph. [m/s]	• **Angular velocity**, ω, is the slope of the θ vs *t* graph. [rad/s]
• Acceleration, a_x, is the slope of the v_x vs *t* graph. [m/s²]	• **Angular Acceleration**, α, is the slope of the ω vs *t* graph. [rad/s²]

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Radians are the Magical Unit!

- Radians appear and disappear as they please in your equations!!!
- They are the only unit that is allowed to do this!
- Example: $v_t = \omega r$

Units check:

$$\underset{\omega}{\left[\frac{rad}{s}\right]} \times \underset{r}{[m]} = \left[\frac{\cancel{rad} \cdot m}{s}\right] = \left[\frac{m}{s}\right]$$

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Rotational Kinematics

Table 9.1, Page 256

Translational motion	Rotational motion	
$v_x = v_{0x} + a_x t$	$\omega = \omega_0 + \alpha t$	(9.3)
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	(9.6)
$2a_x(x - x_0) = v_x^2 - v_{0x}^2$	$2\alpha(\theta - \theta_0) = \omega^2 - \omega_0^2$	(9.7)

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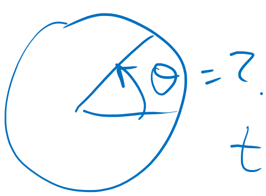
A bicycle wheel has an initial angular velocity of 1.50 rad/s, and a constant angular acceleration of 0.200 rad/s^2. Through what angle has the wheel turned between t = 0 and t = 2.50 s?

SKETCH & TRANSLATE.



$\omega_0 = 1.5\ \frac{rad}{s}$

$\alpha = +0.2\ \frac{rad}{s^2}$

$t = 2.5\ s$

SIMPLIFY & DIAGRAM

constant angular acceleration.

use eqs. from table 9.1

REPRESENT MATHEMATICALLY

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

Define $\theta_0 = 0 \leftarrow$ at $t = 0$

SOLVE & EVALUATE

$\theta = 0 + 1.5(2.5) + \frac{1}{2}(0.2)2.5^2$

$\boxed{\theta = 4.37\ rad}$

$2\pi \sim 6$ rad is full circle, so this is less than that.

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The "Rolling Without Skidding" Constraints

When a round object rolls without skidding, the distance the axis, or centre of mass, travels is equal to the change in angular position times the radius of the object.

distance = $s = \theta R$

The speed of the centre of mass is

$$v = \omega R$$

The acceleration of the centre of mass is

$$a = \alpha R$$

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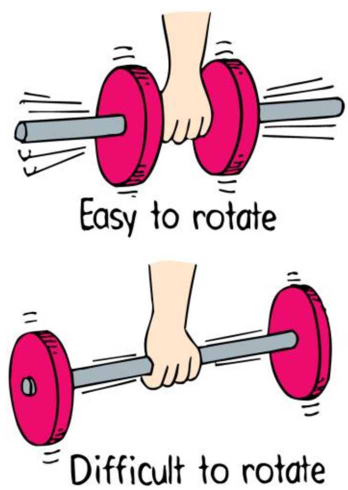
Rotational Inertia

Depends upon:

- mass of object.
- distribution of mass around axis of rotation.
  - The greater the distance between an object's mass concentration and the axis, the greater the rotational inertia.



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Rotational Inertia

Consider a body made of N particles, each of mass m_i, where $i = 1$ to N. Each particle is located a distance r_i from the axis of rotation. For this body made of a countable number of particles, the rotational inertia is:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_{i=1}^{N} m_i r_i^2$$

The units of rotational inertia are kg m^2. An object's rotational inertia depends on the axis of rotation.

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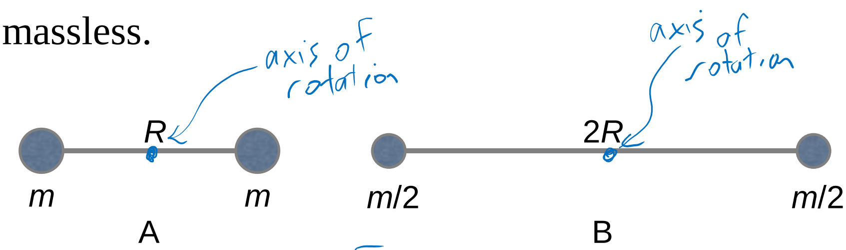
Poll

Which dumbbell has the larger rotational inertia about the midpoint of the rod? The connecting rod is massless.



$I_A = 2\, m\left(\frac{R}{2}\right)^2 = \frac{1}{2} m R^2$ $\quad I = \sum m r^2$

A. Dumbbell A.

B. Dumbbell B $\quad I_B = 2\, \frac{m}{2} (R)^2 = m R^2 \quad I_B = 2 I_A$

C. Their rotational inertias are the same.

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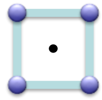
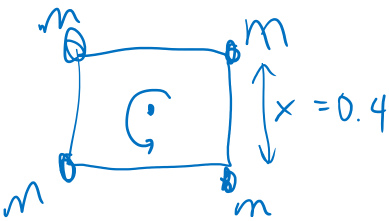
Four small metal spheres, each with mass 0.2 kg, are arranged in a square 0.40 m on a side and connected by extremely light rods.

Find the rotational inertia about an axis through the centre of the square, perpendicular to its plane.



SKETCH & TRANSLATE.

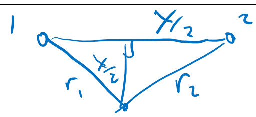


$m = 0.2$ kg

$x = 0.4$

$I = ?$

SIMPLIFY & DIAGRAM



$r_1 = r_2 = r_3 = r_4 = r$

REPRESENT MATHEMATICALLY

$I = m_1 r_1^2 + m_2 r_2^2 + \dots$

$= 4mr^2$

SOLVE & EVALUATE

Pythagoras:

$r^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^2$

$r = \sqrt{2\left(\frac{0.4}{2}\right)^2}$

$r = \sqrt{0.08}$

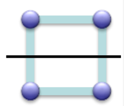
$I = 4(0.2)(0.08)$

$I = 0.064 \text{ kg m}^2$

23

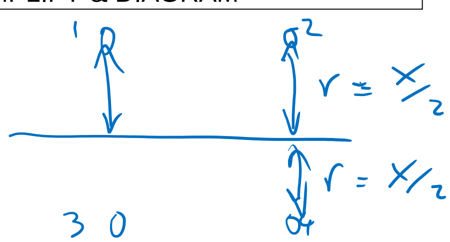
Four small metal spheres, each with mass 0.2 kg, are arranged in a square 0.40 m on a side and connected by extremely light rods.

Find the rotational inertia about an axis through the centre of the square, parallel to its plane.



SKETCH & TRANSLATE.

SIMPLIFY & DIAGRAM



REPRESENT MATHEMATICALLY

$I = 4mr^2$

SOLVE & EVALUATE

$I = 4(0.2)(0.2)^2$

$I = 0.032 \text{ kg m}^2$

$\sim \frac{1}{2}$ of I for rotation axis perp. to plane.

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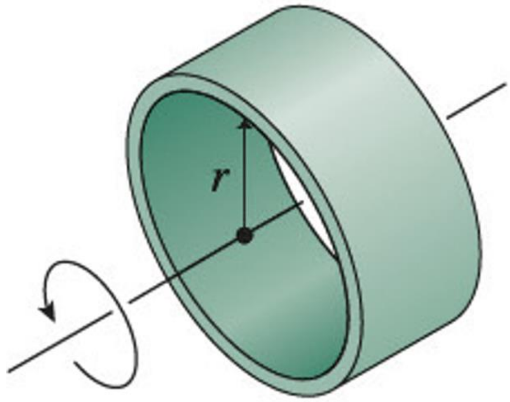
Rotational Inertias of Simple Objects



Thin ring or hollow cylinder about its axis

$I = MR^2$

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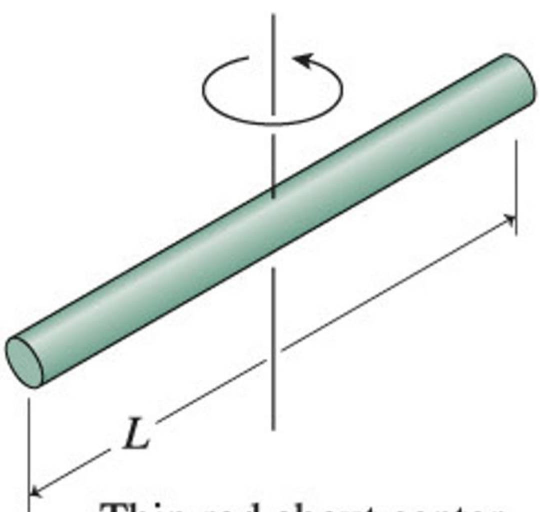
Rotational Inertias of Simple Objects



Thin rod about center

$I = \frac{1}{12}ML^2$

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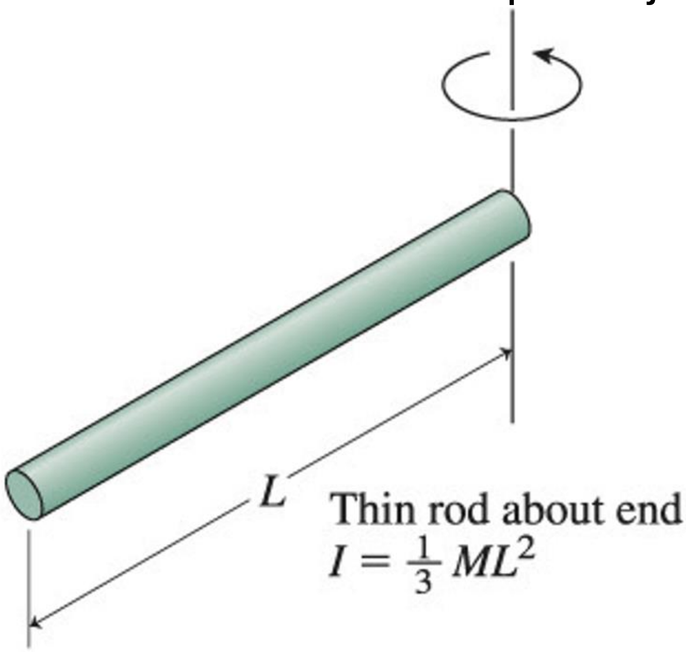
Rotational Inertias of Simple Objects



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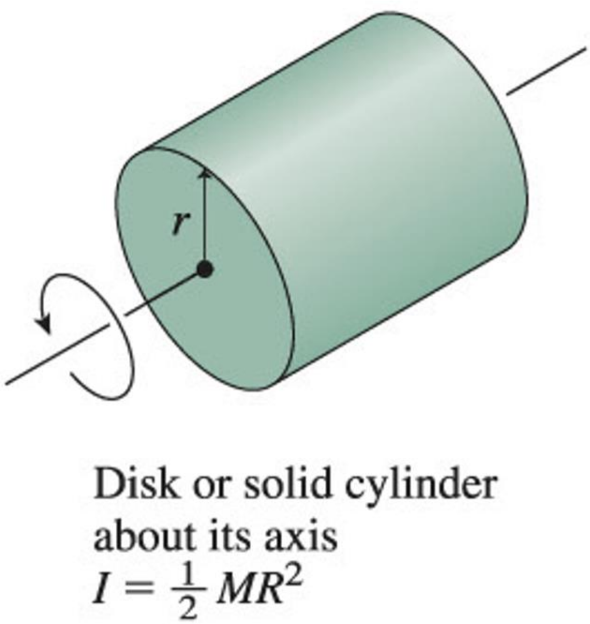
Rotational Inertias of Simple Objects



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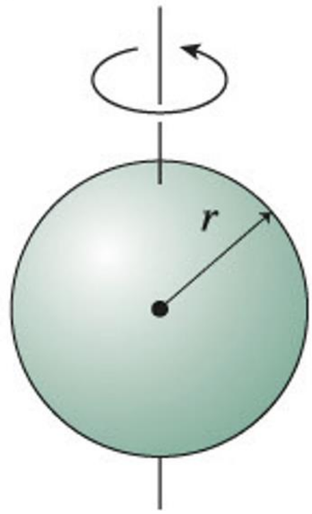
Rotational Inertias of Simple Objects

Solid sphere about diameter

$I = \frac{2}{5} MR^2$



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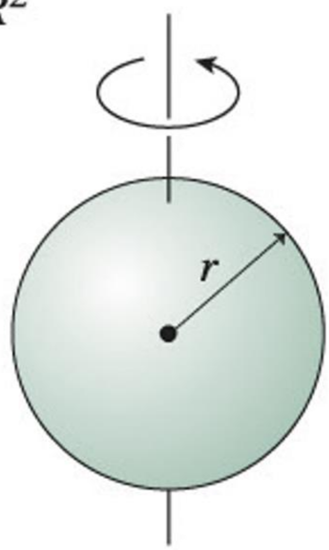
Rotational Inertias of Simple Objects

Hollow spherical shell about diameter

$I = \frac{2}{3} MR^2$



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Poll

Rotational inertia, I, is

A. the rotational analog of kinetic energy.
B. the rotational analog of mass.
C. the rotational analog of momentum.
D. the tendency for anything that is rotating to continue rotating.

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Next up: Rotational **Dynamics**…

Linear	Rotational Analogy
• x	• θ
• v_x	• ω
• a_x	• α
• Force: F_x	• Torque: τ
• Mass: m	• Rotational Inertia: I

Newton's Second Law:

$$a_x = \frac{(F_{net})_x}{m} \qquad \alpha = \frac{\tau_{net}}{I}$$

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Before Class 27 on Friday

- Please continue reading Chapter 9:
- 9.3 Newton's Second Law for Rotational Motion
- 9.4 Rotational Momentum

- Plan to meet up with your Practical Pod during Friday's class – you should be able to turn on your microphone in order to participate in the TeamUp Quiz Module 5 Ch.9.
- If you cannot do the TeamUp quiz during class, it can be done either with your pod or on your own at any time over the weekend.