PHY131H1F - Class 27

Today:

9.3 Newton's Second Law for Rotational Motion

9.4 Rotational Momentum

1

Poll

Crazy Friday: Let's Choose a Zoom-Filter my face today

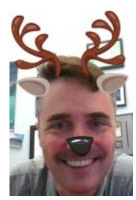
What Studio Filter would you prefer on my face today?

A. Happy Sprout

B. Pizza on Head

C. Shades

D. Reindeer antlers

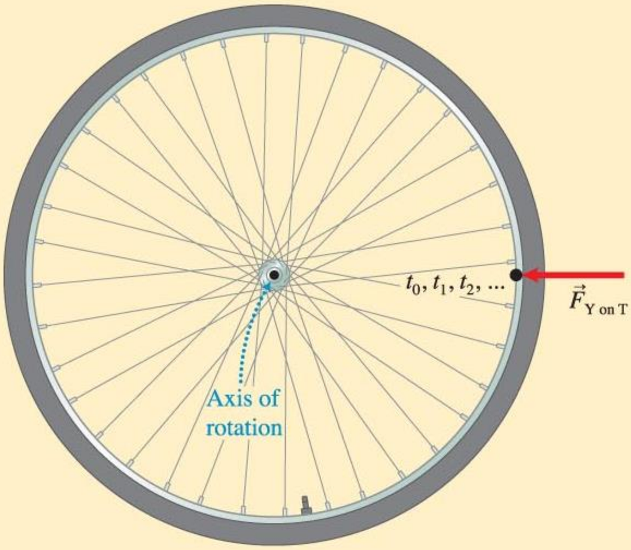


E. Bunny rabbit

2

Observational Experiment #1

Experiment 1: Your bike sits upside down. You push on the front tire toward the axle. The tire does not turn.



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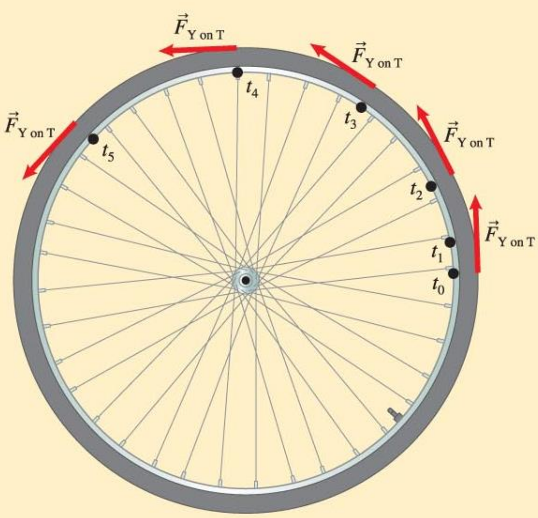
Observational Experiment #2

Experiment 2: You push lightly and continuously on the outside of the tire in a counterclockwise (ccw) direction tangent to the tire. As you continue to push, the tire rotates ccw faster and faster.



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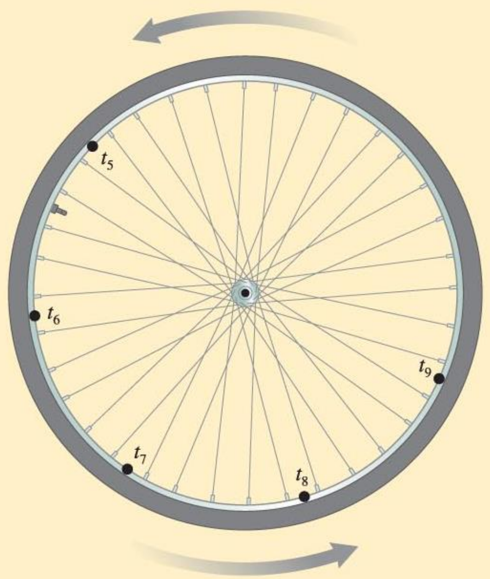
Observational Experiment #3

Experiment 3: You release the spinning tire and watch it. The tire continues rotating ccw at a constant rate.



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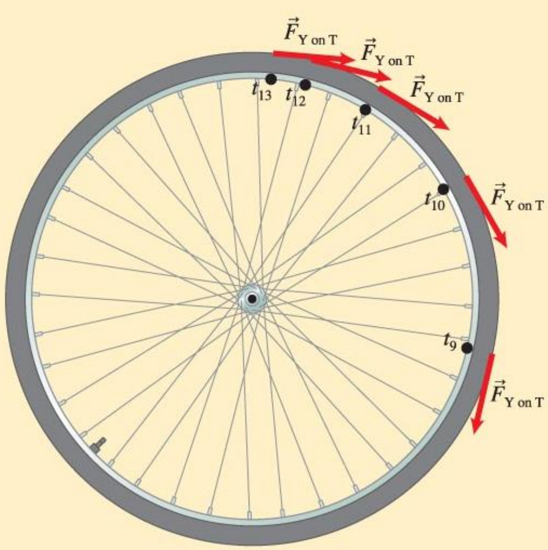
Observational Experiment #4

Experiment 4: With the tire still rotating ccw fast, you gently and continuously push clockwise (cw) against the tire. The rotational speed decreases.



6

Rotational form of Newton's Second Law

- One or more objects exert forces on a rigid body with rotational inertia I that can rotate about some axis.
- The sum of the torques $\Sigma\tau$ (net torque) due to these forces about that axis causes the object to have a rotational acceleration α:

$$\alpha = \frac{1}{I}\Sigma\tau$$

7

Analogy chart

Linear	Rotational Analogy
x	θ
v_x	ω
a_x	α
Force: F_x	Torque: τ
Mass: m	Rotational Inertia: I

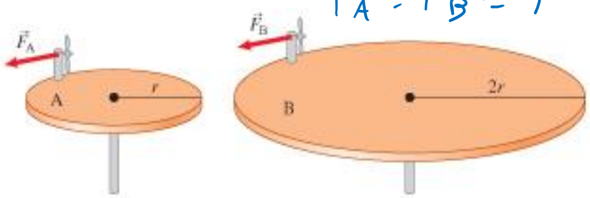
Newton's Second Law:

$$a_x = \frac{(F_{net})_x}{m} \qquad \alpha = \frac{\tau_{net}}{I}$$

8

Two disks are cut from the same uniform board. The radius of disk B is twice the radius of the disk A. The disks can rotate around axes with negligible friction. Two very light battery-powered fans are attached to the disks, as shown. When switched on, the fans exert equal forces on the disks. What is the angular acceleration of disk A, in terms of that of disk B?

$F_A = F_B = F$



SKETCH & TRANSLATE.

In both cases, we will use Newton's second Law for rotation: $\alpha = \frac{\Sigma\tau}{I}$, $\Sigma\tau$ = torque due to fan.

SIMPLIFY & DIAGRAM

A: $I_A = \frac{1}{2} m_A r^2$ (solid disk eq.).

Assume disks are made of same material of area-density:

"sigma" $\sigma = \frac{\text{mass}}{\text{area}} = \frac{m_A}{\pi r^2} \Rightarrow m_A = \sigma\pi r^2$

$I_A = \frac{1}{2}\sigma\pi r^2 r^2 = \frac{\pi\sigma}{2} r^4$

$\tau_A = Fr$

B: $I_B = \frac{1}{2} m_B (2r)^2 = 2 m_B r^2$

$\sigma = \frac{m_B}{\pi(2r)^2} = \frac{m_B}{4\pi r^2}$

$m_B = 4\pi\sigma r^2$

$I_B = 2(4\pi\sigma r^2)r^2 = 8\pi\sigma r^4$

$\tau_B = F(2r) = 2Fr$

9

SIMPLIFY & DIAGRAM

A: $I_A = \frac{1}{2} m_A r^2$ (solid disk)

Assume disks are made of same material of area-density

$\sigma = \frac{m}{\text{area}} = \frac{m}{\pi r^2} \Rightarrow m_A = \sigma\pi r^2$

$I_A = \frac{1}{2}\sigma\pi r^2 r^2 = \frac{\pi\sigma}{2} r^4$

$\tau_A = Fr$

B: $I_B = \frac{1}{2} m_B (2r)^2 = 2 m_B r^2$

$\sigma = \frac{m_B}{\pi(2r)^2} = \frac{m_B}{4\pi r^2}$

$m_B = 4\pi\sigma r^2$

$I_B = 2(4\pi\sigma r^2)r^2 = 8\pi\sigma r^4$

$\tau_B = F(2r) = 2Fr$

REPRESENT MATHEMATICALLY

$\alpha_A = \frac{\tau_A}{I_A} = \frac{Fr}{\frac{\pi\sigma}{2} r^4} = 2\frac{F}{\pi\sigma r^3}$

$\alpha_B = \frac{\tau_B}{I_B} = \frac{2Fr}{8\pi\sigma r^4} = \frac{1}{4}\frac{F}{\pi\sigma r^3}$

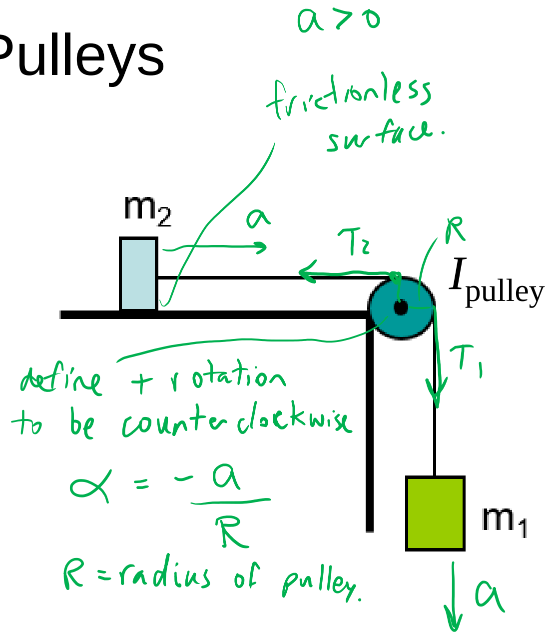
SOLVE & EVALUATE

$\boxed{\alpha_A = 8\alpha_B}$ ← makes sense, since $\tau_B = 2\tau_A$ and $I_B = 16 I_A$

10

Massive Pulleys

- A real pulley has some rotational inertia, I_{pulley}.
- This means that it will require a net torque in order to cause it to have rotational acceleration.
- For an accelerating system, the tension must **change** as it goes over the pulley.



11

Crazy Friday TV Show Bracket
Today: Quarterfinals

11:43

After the Team-Up Quiz we will be having four quick (15 seconds each!) polls to determine which four shows make it to the semifinals next week. Pick your favourite for each pair.

- Queen's Gambit vs
- Better Call Saul

- The Good Place vs
- BoJack Horseman

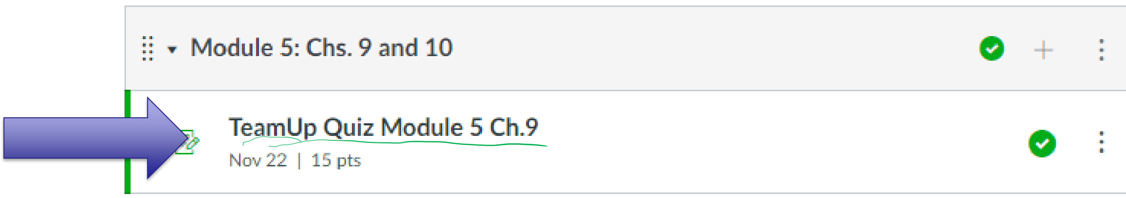
- What We Do in the Shadows vs
- Tiger King

- Schitt's Creek vs
- Parks and Recreation

12

TeamUp Time!!

- Today you will be doing three multiple choice questions, all from Chapter 9, as a team of 2-4 students in your Practicals Pod.
- Your pod-team shares the mark!
- Right now you should open Microsoft Teams and someone (most recent Facilitator) should place a **video call** to all 3 or 4 members of your Pod-Chat.



13

Now: TeamUp! You have 10 minutes

- The first step is to decide who will be the TeamUp **Driver**
- All students must log-in to Quercus [You will now have three windows open: my zoom lecture, Microsoft Teams, and Quercus]
- **Non-drivers**: Wait!
- **Driver**: Go to the TeamUp Quiz Ch.9 in Module 5, click Go to Tool, then Create a Group. Let everyone in the Breakout Room know the session ID. Then WAIT – don't drive off alone!
- **Non-drivers**: Once you get the session ID, go to the TeamUp Quiz in this module, click Go to Tool, then Join Session and type the ID you were given.
- Once everyone in your room arrives in TeamUp, start going through the questions. Please **achieve consensus** before the driver submits.
- YOU MAY BEGIN! I'm going to go on mute for 10 minutes. Note: if your pod-mates are available on Microsoft Teams right now, go to the PHY131 Help Centre and I'll set up breakout rooms there. Zoom Meeting ID: 938 0964 2256, Passcode: 723874

14

Crazy Friday TV Show Bracket
Today: Quarterfinals

15 second poll. Which do you prefer?

A. Queen's Gambit
B. Better Call Saul

15

Crazy Friday TV Show Bracket
Today: Quarterfinals

15 second poll. Which do you prefer?

A. What We Do in the Shadows
B. Tiger King

16

Crazy Friday TV Show Bracket
Today: Quarterfinals

15 second poll. Which do you prefer?

A. The Good Place
B. BoJack Horseman

17

Crazy Friday TV Show Bracket
Today: Quarterfinals

15 second poll. Which do you prefer?

A. Schitt's Creek
B. Parks and Recreation

18

Question 1 Discussion

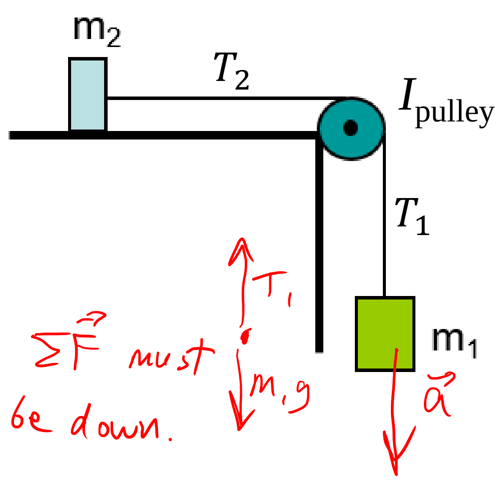
- Mass m_2 is on a frictionless table. It is attached to a cord that wraps around a massive pulley of radius R, and is attached to a hanging mass m_1.
- After the system is released, how does the magnitude of the tension in the rope attached to m_1 compare to the magnitude of the force of gravity, $m_1 g$, on m_1?

A. $T_1 > m_1 g$

B. $T_1 < m_1 g$

C. $T_1 = m_1 g$

D. T_1 and $m_1 g$ cannot be compared



Question 2 Discussion

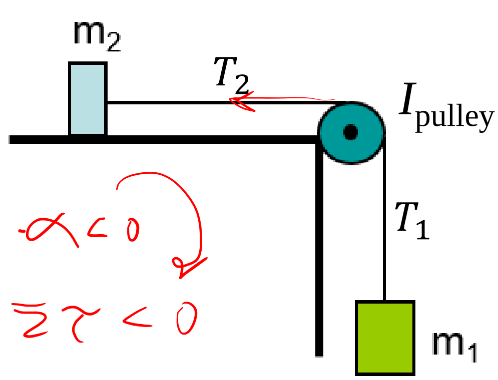
- Mass m_2 is on a frictionless table. It is attached to a cord that wraps around a massive pulley of radius R, and is attached to a hanging mass m_1.
- After the system is released, how do the tensions in the two ropes compare?

A. $T_1 > T_2$

B. $T_1 < T_2$

C. $T_1 = T_2$

D. T_1 and T_2 cannot be compared



Question 3 Discussion

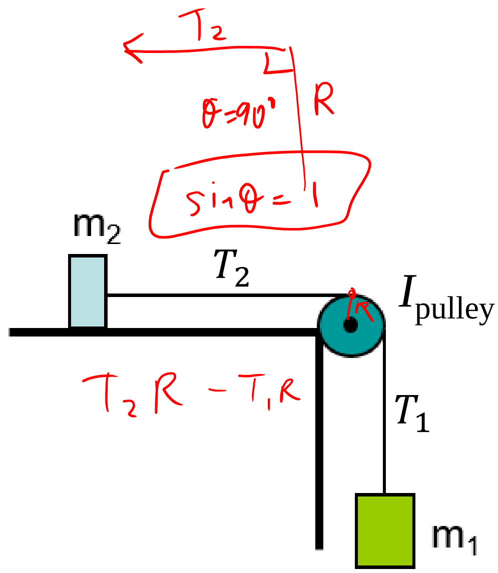
- Mass m_2 is on a frictionless table. It is attached to a cord that wraps around a massive pulley of radius R, and is attached to a hanging mass m_1.
- After the system is released, what is the torque on the pulley [define counter-clockwise = positive]?

A. $T_1 R - T_2 R$

B. $T_2 R - T_1 R$

C. $T_1 R + T_2 R$

D. $T_1 R$



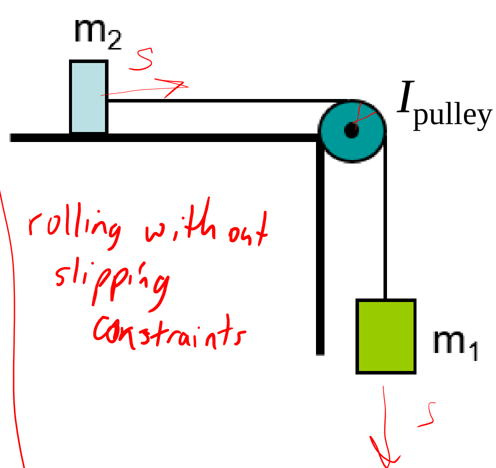
21

The Massive Pulley Constraints

When an object is attached to a pulley, the distance the object travels is equal to the change in angular position of the pulley times the radius of the pulley.

- Distance m_2 and m_1 move is: $s = \theta R$, where θ is the angle the pulley turns through.
- Speeds of m_2 and m_1 are related to the angular speed of the pulley by: $v = \omega R$
- Accelerations of m_2 and m_1 are related to the angular acceleration of the pulley by:

$$a = \alpha R$$

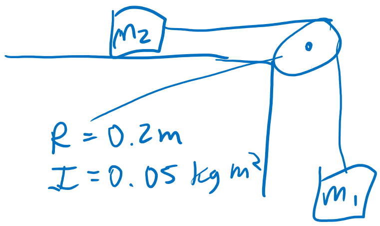


22

Mass $m_2 = 2.5$ kg is on a frictionless table. It is attached to a cord that wraps around a massive pulley of radius $R = 0.20$ m, and rotational inertia $I = 0.050$ kg m^2. The same cord is attached to a hanging mass $m_1 = 0.75$ kg. After the system is released, what is the acceleration of m_1 as it falls?

SKETCH & TRANSLATE.

$m_2 = 2.5$ kg.



$R = 0.2$ m

$I = 0.05$ kg m^2

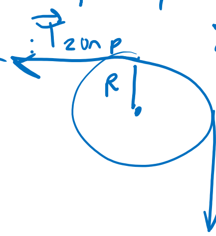
$m_1 = 0.75$ kg. $\downarrow a$

find a

SIMPLIFY & DIAGRAM

Assume no friction on either disk. or on pulley.

Pulley system:

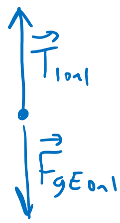


$\Sigma\tau = T_2R - T_1R$

$\alpha = \frac{\Sigma\tau}{I}$

$\alpha = \frac{R}{I}(T_2 - T_1)$ (1)

m_1 system:



Define $+y \downarrow$

$\Sigma F_y = m_1 g - T_1$

$a = \frac{\Sigma F_y}{m_1}$

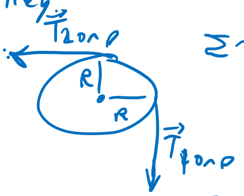
$a = \frac{m_1 g - T_1}{m_1}$ (2)

23

SIMPLIFY & DIAGRAM

Assume no friction on either block or on pulley.

Pulley system:

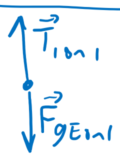


$\Sigma\tau = T_2R - T_1R$

$\alpha = \frac{\Sigma\tau}{I}$

$\alpha = \frac{R}{I}(T_2 - T_1)$ (1)

m_1 system:



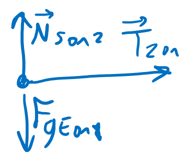
$\Sigma F_y = m_1 g - T_1$ $\downarrow +y$

$a = \Sigma F_y / m_1$

$a = \frac{m_1 g - T_1}{m_1}$ (2)

m_2 system:

Define $+x \rightarrow$



$\Sigma F_x = T_2$

$a = \frac{T_2}{m_2}$ (3)

Fourth eq. is acceleration constraint. Note that when m_1 falls, a is +. but α is clockwise, negative, so

$a = -\alpha R$ (4)

REPRESENT MATHEMATICALLY

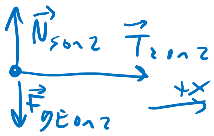
4 eqs, 4 unknowns: T_1, T_2, α, a.

Solve for a.

SOLVE & EVALUATE

24

m_2 system:



$\Sigma F_x = T_2$

$$a = \frac{T_2}{m_2} \qquad (3)$$

4th eq. is acceleration constraint.
Note that when m_1 falls, a is $+$,
but α is clockwise, negative, so

$$a = -\alpha R \qquad (4)$$

REPRESENT MATHEMATICALLY

4 eqs, 4 unknowns: T_1, T_2, α, a
solve for a.

$(2) \Rightarrow m_1 a = m_1 g - T_1$

$T_1 = m_1 g - m_1 a$

$(3) \Rightarrow T_2 = m_2 a$

plug into (1):

$$\Rightarrow \alpha = \frac{R}{I}(m_2 a - m_1 g + m_1 a)$$

plug into (4):

$$a = -\frac{R^2}{I}\left((m_1+m_2)a - m_1 g\right)$$

$$a\left(\frac{I}{R^2} + m_1 + m_2\right) = m_1 g$$

SOLVE & EVALUATE

$$a = \frac{m_1 g}{I/R^2 + m_1 + m_2} = \frac{0.75(9.8)}{\left(\frac{0.05}{0.2^2}\right) + 0.75 + 2.5}$$

$$\boxed{a = 1.6\ m/s^2}$$ ← less than g.

25

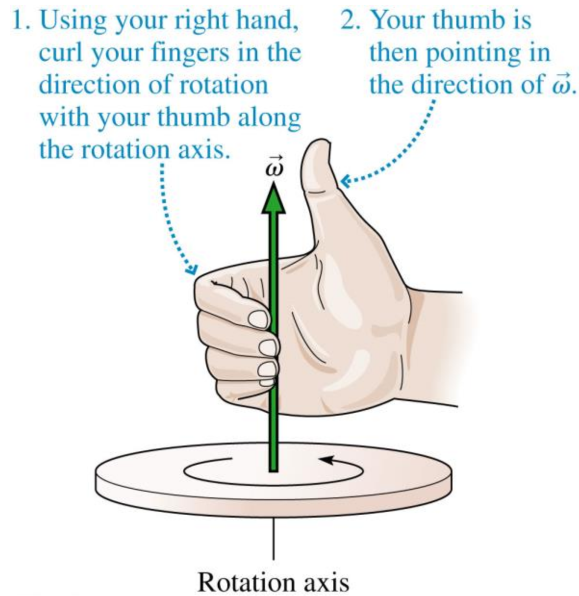
The Vector Description of Rotational Motion

- One-dimensional motion uses a scalar velocity v and force F.
- A more general understanding of motion requires **vectors** $\vec{v}$ and $\vec{F}$.
- Similarly, a more general description of rotational motion requires us to replace the scalars ω and τ with the vector quantities $\vec{\omega}$ and $\vec{\tau}$.
- Doing so will lead us to the concept of *angular momentum*.

26

"The Right Hand Rule"

- The magnitude of the angular velocity $\vec{\omega}$ vector is ω.
- The angular velocity vector points along the axis of rotation in the direction given by the right-hand rule as illustrated.



27

Rotational Momentum

- Rotational momentum

  = rotational inertia × rotational velocity

$$\vec{L} = I\vec{\omega}$$

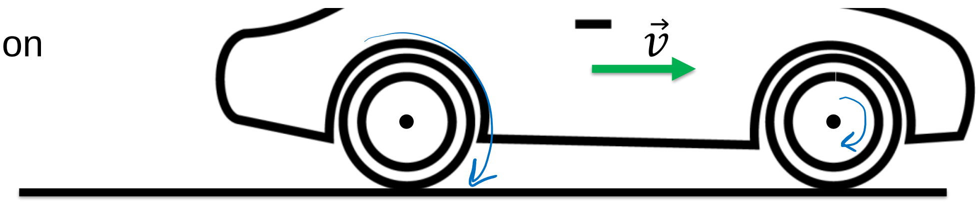
  – This is analogous to

  Linear momentum = mass × velocity

$$\vec{p} = m\vec{v}$$

28

Poll Question



You are driving your car towards the right.

What is the direction of the angular momentum, $\vec{L}$, of the wheels?

also direction of $\vec{\omega}$

A. Left

B. Right

C. Up

D. Down

E. Into the page

F. Out of the page

fingers.

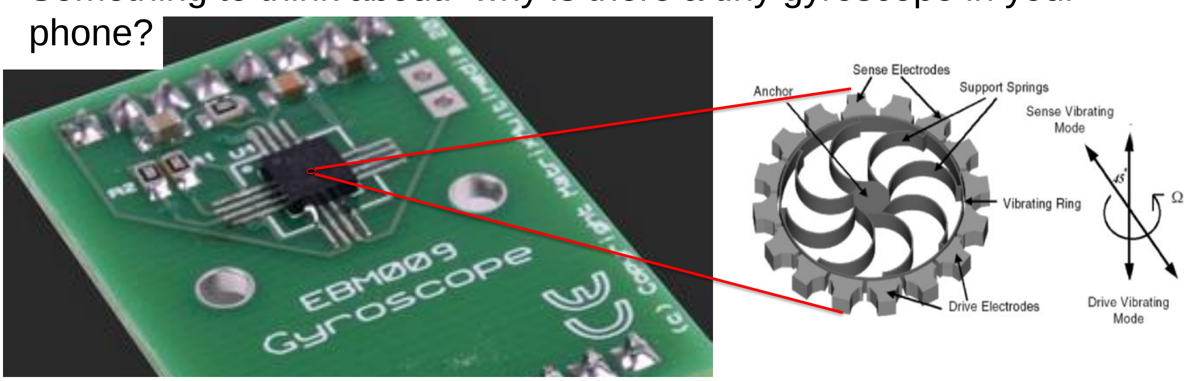
thumb points away

29

Before Class 28 on Monday

- Please finish reading all of Chapter 9:
- 9.5 Rotational Kinetic Energy
- 9.6 Tides and Earth's Day

- Something to think about: why is there a tiny gyroscope in your phone?



30

INFORMAL OFFICE HOUR SLIDES ATTACHED BELOW

31

‹ Homework 9

Flywheel Kinematics

$\omega = \frac{\Delta\theta}{\Delta t}$

$\alpha = \frac{\Delta\omega}{\Delta t}$

$\omega_i = 0$

$\alpha = \frac{\omega_f - \omega_i \to 0}{t_f - t_i \to 0}$

$\alpha = \frac{\omega_f}{t_1}$

$t_1 = \frac{\omega_f}{\alpha}$

$\theta_f = \theta_i \to 0 + \omega t + \frac{1}{2}\alpha t^2$

$\theta_1 = \omega_1 t_1 + \frac{1}{2}\alpha t_1^2$

$\alpha = \frac{\omega_1}{t_1}$

$\Delta\omega = \alpha \Delta t$

$\omega_1 = \alpha t_1$

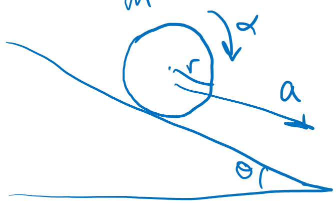
$t_2 = \frac{-\omega_1}{-5\alpha} = \frac{+\omega_1 t_1}{5\omega_1} = t_1/5$

$\omega_f = 0 \quad \Delta\omega_2 = -\omega_1$

$\alpha_2 = -5\alpha$

32

Hoop on a Ramp



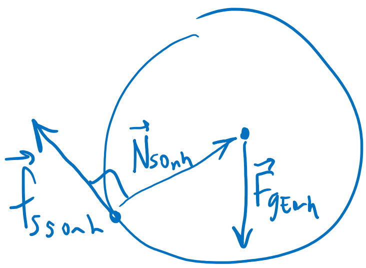
hoop: $I = mr^2$

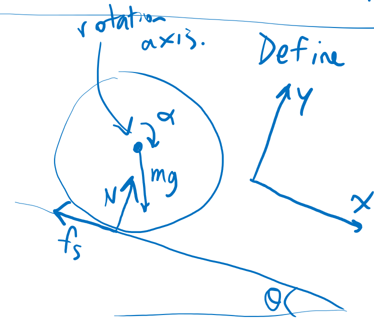
Define + rotation to be clockwise..

$$\alpha = \frac{a}{r}$$ (1). ← rolling without slipping.

↑ static friction prevents slipping

System = hoop.
force diagram.





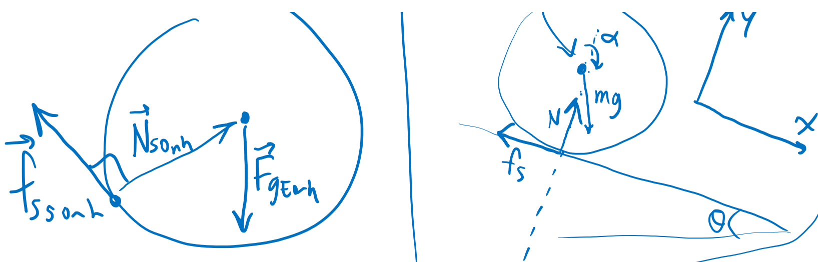
$a_y = 0$
$a_x = a$

$$\Sigma F_y = 0$$
$$\Sigma F_x = ma$$
$$\Sigma \tau = I\alpha$$

33

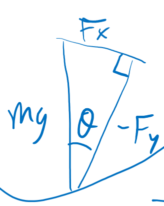
< Homework 9

Hoop on a Ramp



$a_x = a$

$$\Sigma F_y = 0$$
$$\Sigma F_x = ma$$
$$\Sigma \tau = I\alpha$$



	x	y	τ
mg	$mg\sin\theta$	$-mg\cos\theta$	0
N	0	N	0
f_s	$-f_s$	0	$+f_s r$
sum:	$mg\sin\theta - f_s$	$N - mg\cos\theta$	$f_s r$

$$N - mg\cos\theta = 0$$
$$mg\sin\theta - f_s = ma$$
$$f_s r = mr^2\alpha$$
$$\alpha = \frac{a}{r}$$

← 4 eqs, 4 unknowns: N, f_s, a, α
algebra, algebra, algebra. solve for a

34