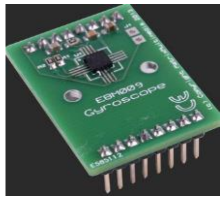
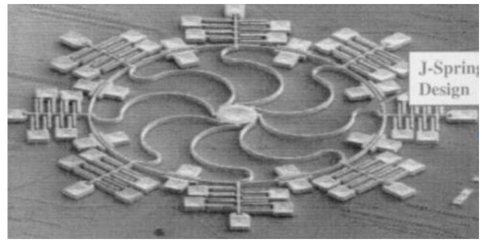
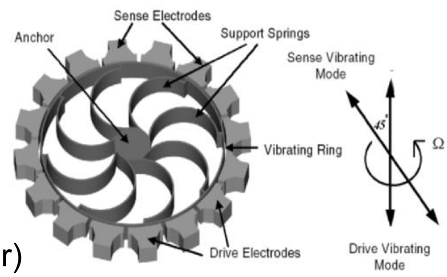
PHY131H1F - Hour 28







Today:

We finish up Chapter 9!

9.5 Rotational Kinetic Energy

(let's skip 9.6 on Tides and Earth's day this semester)

1

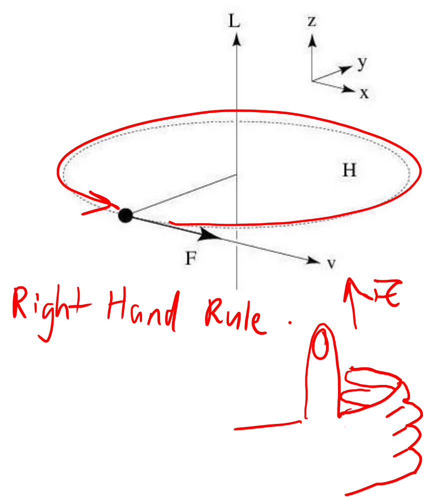
PollQuestion

A person spins a tennis ball on a string in a horizontal circle (so that the axis of rotation is vertical). At the point indicated below, the ball is given a sharp blow in the forward direction. This causes a change in rotational momentum dL in the

A. x-direction

B. y-direction

C. z-direction



2

Rotational momentum of an isolated system is constant

- If the net torque that external objects exert on a turning object is zero, or if the torques add to zero, then the rotational momentum L of the turning object remains constant:

$$L_f = L_i \quad \text{or} \quad I_f\,\omega_f = I_i\,\omega_i$$

(Eq. 9.13 from Etkina, pg.268)

> **TIP** Rotational momentum is sometimes called angular momentum.

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Demonstration 1

rotational speed [rad/s]

- Which has greater **rotational inertia**, I:

A. Chair + Harlow + two 5 kg masses, arms outstretched

B. Chair + Harlow + two 5 kg masses held near chest

- Assume external torque = zero, so $L = I\,\omega$ is constant.
- Harlow begins rotating at some particular value of ω is with his arms outstretched.
- He then brings the masses in to his chest.
- How does this affect the rotation speed ω?

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Demonstration 2

- Harlow is not rotating. He is holding a bicycle wheel which is rotating counterclockwise as viewed from above. What is the direction of the rotational momentum of the chair + Harlow + bicycle wheel system?

A. Up

B. Down

- Assume no external torque.
- If Harlow flips the wheel upside down, it is now rotating clockwise as viewed from above.
- The rotational momentum of the wheel is now down.
- What happens to Harlow+chair?

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Demonstration 1 Recap

- Harlow begins rotating at some particular value of ω is with his arms outstretched. I is large, ω is small, $L = I\,\omega$ is some value.
- He then brings the masses in to his chest. I decreases.
- But there is no external torque, so $L = I\,\omega$ value must stay the same.
- So ω is increases.

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Demonstration 2 Recap

- Harlow is not rotating. He is holding a bicycle wheel which is rotating counterclockwise as viewed from above.
- Rotational momentum of the system is up.
- Harlow flips the wheel upside down, so its rotational momentum is now down.
- No external torques, so the rotational momentum of the system must still be up.
- Harlow + chair rotate counterclockwise.

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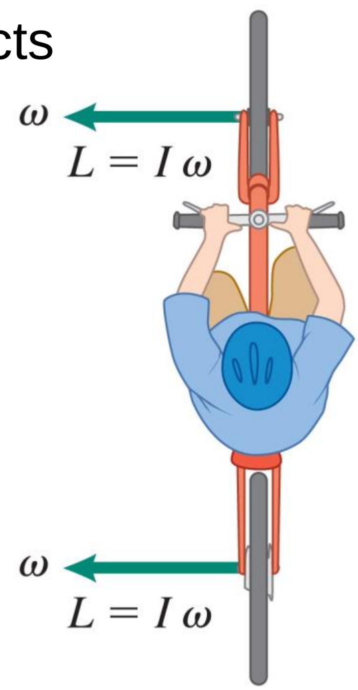
Stability of rotating objects

- If the rider's balance shifts a bit, the bike + rider system will tilt and the gravitational force exerted on it will produce a torque.
  - The rotational momentum of the system is large, so torque does not change its direction by much.
  - The faster the person is riding the bike, the greater the rotational momentum of the system and the more easily the person can keep the system balanced.



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A 20-cm-diameter, 2.0 kg solid disk is rotating at 200 rpm. A 20-cm-diameter, 1.0 kg circular loop is dropped straight down onto the rotating disk. Friction causes the loop to accelerated until it is "riding" on the disk. What is the final angular speed of the combined system?

$R_1 = 0.1$ m
$m_1 = 2$ kg
$R_2 = 0.1$ m
$m_2 = 1$ kg.

SKETCH & TRANSLATE.

2: — $\omega_{i2} = 0$

1: Disk $\omega_{i1} = 200$ rpm.

$\omega_f = ?$

System: both loop and disk.
→ friction is internal torque

SIMPLIFY & DIAGRAM

Assume totally inelastic collision.
No external torque.

$L = I\omega$

rotational momentum does not change.

$L_i + \Sigma \tau(t) = L_f$ (with $\Sigma \tau(t) = 0$)

$I_1 = \frac{1}{2} m_1 R_1^2$

$I_2 = m_2 R_2^2$

REPRESENT MATHEMATICALLY

$$\frac{1}{2} m_1 R_1^2 \omega_{i1} + 0 = \frac{1}{2} m_1 R_1^2 \omega_f + m_2 R_2^2 \omega_f$$

$R_1 = R_2 = R$, divide both sides by $R^2/2$

$$m_1 \omega_{i1} = m_1 \omega_f + 2 m_2 \omega_f$$

$$\omega_f = \frac{m_1}{m_1 + 2m_2} \omega_{i1}$$

SOLVE & EVALUATE

$$\omega_f = \frac{2}{2 + 2(1)} (200 \text{ rpm})$$

$$\boxed{\omega_f = 100 \text{ rpm}}$$

↑ Half of initial ω.
Don't need to convert to SI.

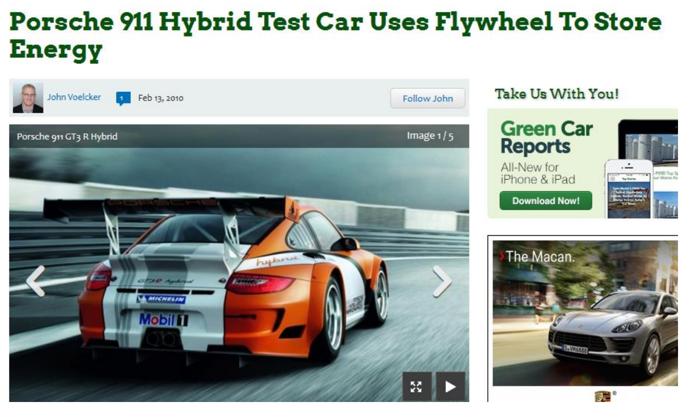
Rotational Kinetic Energy

A rotating rigid body has kinetic energy because all atoms in the object are in motion. The kinetic energy due to rotation is called **rotational kinetic energy**.

$$K_{rot} = \frac{1}{2} I \omega^2$$

Flywheels for storing and providing energy

- In a car with a flywheel, instead of rubbing a brake pad against the wheel and slowing it down, the braking system converts the car's translational kinetic energy into the rotational kinetic energy of the flywheel.
- As the car's translational speed decreases, the flywheel's rotational speed increases. This rotational kinetic energy could then be used later to help the car start moving again.



https://www.greencarreports.com/news/1042570_porsche-911-hybrid-test-car-uses-flywheel-to-store-energy

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Poll Question

A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended.

When she pulls in her arms, she reduces her rotational inertia and her angular speed increases.

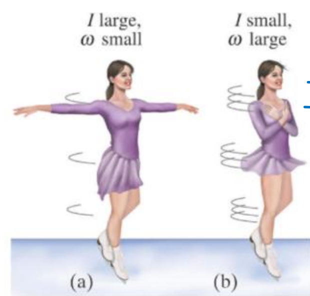
Compared to her initial rotational kinetic energy, her rotational kinetic energy after she has pulled in her arms must be:

A. the same because no work is done on her.

B. larger because she's rotating faster.

C. smaller because her rotational inertia is smaller.

Skater does work as she brings her arms in.



$I_i > I_f$

$\omega_f > \omega_i$

$I_i \omega_i = I_f \omega_f$

$K = \frac{1}{2} I \omega^2 = \frac{1}{2}(I\omega)\omega$

$L = I\omega =$ constant, as $\omega \uparrow$,

$K = \frac{1}{2} L \omega \uparrow$

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Complete Linear / Rotational Analogy Chart

Linear	Rotational Analogy
• $\vec{s}$, $\vec{v}$, $\vec{a}$	• θ, ω, α
• Force: $\vec{F}$	• Torque: τ
• Mass: m	• Rotational Inertia: I

• Newton's 2nd law: $\vec{a} = \frac{\Sigma \vec{F}_{net}}{m}$ $\qquad \alpha = \frac{\Sigma \tau_{net}}{I}$

• Kinetic energy: $K_{tran} = \frac{1}{2} m v^2$ $\qquad K_{rot} = \frac{1}{2} I \omega^2$

• Momentum: $\vec{p} = m\vec{v}$ $\qquad \vec{L} = I\vec{\omega}$

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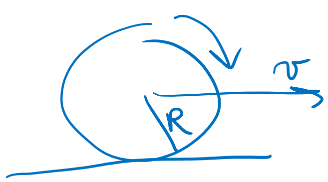
A 0.50 kg basketball rolls along the ground at 1.0 m/s. What is its *total* kinetic energy (translational plus rotational)? [Note that the rotational inertia of a hollow sphere is $I = 2/3\ MR^2$.]

SKETCH & TRANSLATE.



Assume rolling without slipping.

$K = ?$ $\quad v = \omega R$

SIMPLIFY & DIAGRAM

$K = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$I = \frac{2}{3} m R^2$ $\quad \omega = \frac{v}{R}$

REPRESENT MATHEMATICALLY

$K = \frac{1}{2} m v^2 + \frac{1}{2}\left[\frac{2}{3} m R^2\right]\left[\frac{v}{R}\right]^2$

$K = \frac{1}{2} m v^2 + \frac{2}{6} m R^2 \frac{v^2}{R^2}$

$K = \left(\frac{1}{2} + \frac{2}{6}\right) m v^2$

SOLVE & EVALUATE

$K = \left(\frac{3}{6} + \frac{2}{6}\right) m v^2$

$= \frac{5}{6} m v^2 = \frac{5}{6}(0.5)\,1^2$

$K = 0.42\ J$

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Summary of some Different Types of Energy:

- Kinetic Energy due to bulk motion of centre of mass: $K = ½\, mv^2$ (Sometimes called Translational Kinetic Energy K_{tran})
- Gravitational Potential Energy (System = object + Earth): $U_g = mgy$
- Spring Potential Energy (system = object + spring): $U_s = ½\, kx^2$
- Rotational Kinetic Energy: $K_{rot} = ½\, I\omega^2$
- Internal Thermal Energy: ΔU_{int} (If the system includes two surfaces rubbing against each other, then $\Delta U_{int} = |\, f_k d\, |$)
- A system can possess any or all of the above.
- One way of transferring energy in or out of a system is **work**:
- Work done by a constant force: $W = Fd\cos\theta$

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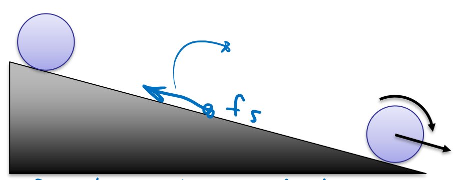
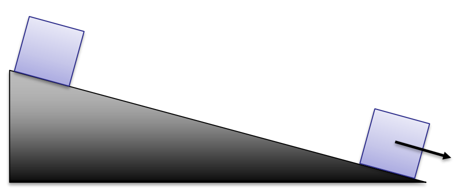
Is this ramp frictionless? If not, which way does friction act on disk as it rolls.



- A solid disk is released from rest and rolls without slipping down an incline. A box is released from rest and slides down a frictionless incline of the same angle. Which reaches the bottom first?
- A: disk wins
- B: box wins
- C: tie



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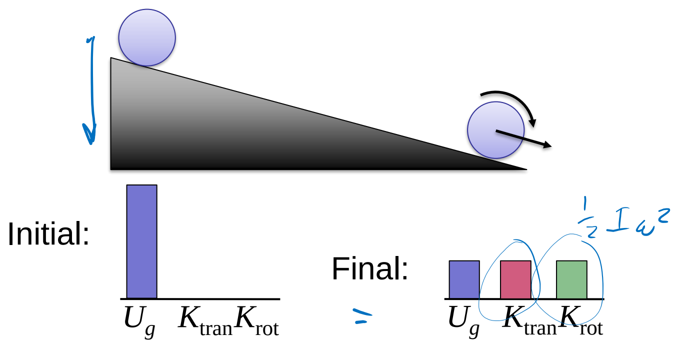
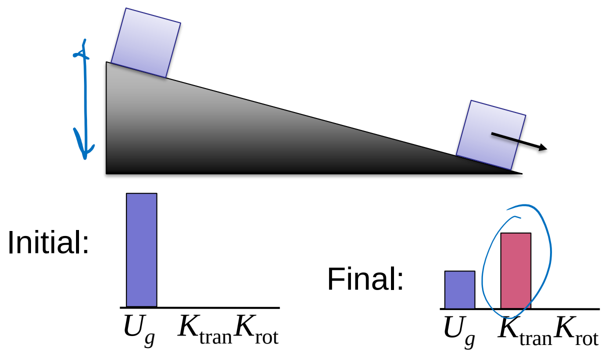
- Think about conservation of energy.

system = disk + Earth

- A rolling object has two forms of kinetic energy which must be **shared**





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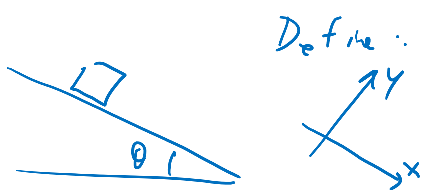
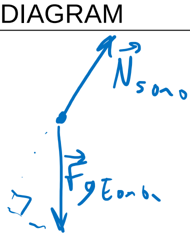
1. What is the acceleration of a sliding object down a ramp inclined at angle θ? [assume no friction]

SKETCH & TRANSLATE.



SIMPLIFY & DIAGRAM



REPRESENT MATHEMATICALLY

$$a_x = \frac{\Sigma F_x}{m} = \frac{mg\sin\theta}{m}$$

SOLVE & EVALUATE

$$a_x = g\sin\theta$$

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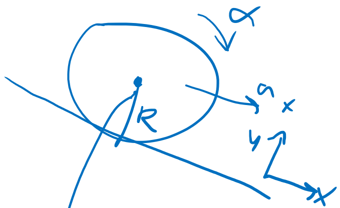
2. What is the acceleration of a **solid disk** ~~of~~ rolling down a ramp inclined at angle θ? [assume rolling without skidding]

$I = \frac{1}{2} m R^2$

SKETCH & TRANSLATE.

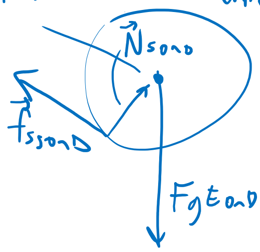


define +α clockwise,

$\alpha = \frac{a_x}{R}$ (1)

SIMPLIFY & DIAGRAM

axis of rotation = centre $\vec{N}$ toward axis → zero torque



zero gravitational torque

$\Sigma \tau = f_s R$

$\Sigma F_x = mg \sin\theta - f_s$

REPRESENT MATHEMATICALLY

$a_x = \frac{\Sigma F_x}{m} = \frac{mg \sin\theta - f_s}{m}$ (2)

$\alpha = \frac{\Sigma \tau}{I} = \frac{f_s R}{\frac{1}{2} m R^2}$

$\alpha = \frac{2 f_s}{mR}$ (3)

3 eqs, 3 unknowns: a_x, α, f_s. Solve for a_x.

(2) ⇒ $m a_x = mg \sin\theta - f_s$

⇒ $f_s = mg \sin\theta - m a_x$

plug this & (1) in (3).

$\frac{a_x}{R} = \frac{2}{mR}(mg \sin\theta - m a_x)$

$a_x = 2g \sin\theta - 2a_x$

REPRESENT MATHEMATICALLY

$a_x = \frac{\Sigma F_x}{m} = \frac{mg \sin\theta - f_s}{m}$ (2)

$\alpha = \frac{\Sigma \tau}{I} = \frac{f_s R}{\frac{1}{2} m R^2}$

$\alpha = \frac{2 f_s}{mR}$ (3)

3 eqs, 3 unknowns: α, a_x, f_s. Solve for a_x.

(2) ⇒ $m a_x = mg \sin\theta - f_s$

⇒ $f_s = mg \sin\theta - m a_x$

Plug (1) and (2) into (3):

$\frac{a_x}{R} = \frac{2}{mR}(mg \sin\theta - m a_x)$

$a_x = 2g \sin\theta - 2a_x$

SOLVE & EVALUATE

$3 a_x = 2g \sin\theta$

$a_x = \frac{2}{3} g \sin\theta$

Smaller than $g \sin\theta$

Before Class 29 on Wednesday

- Please start reading Chapter 10:
- 10.1 Period and Frequency
- 10.2 Simple Harmonic Motion

- We've now finished Chapters 8 and 9 on **Rotation and Torque** Stuff.
- Next class we will start in on Chapters 10 and 11, which are on **Vibrations and Waves**!
- Midterm Assessment 5 on Dec. 1 is on Chs. 9 and 10.
- Chapter 11 is the last chapter we will study in this course before the final exam on Dec.17.

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