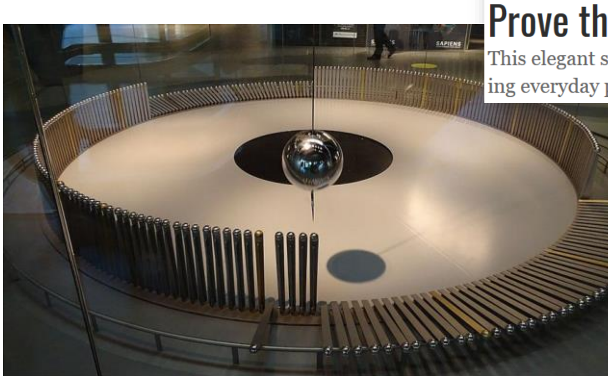
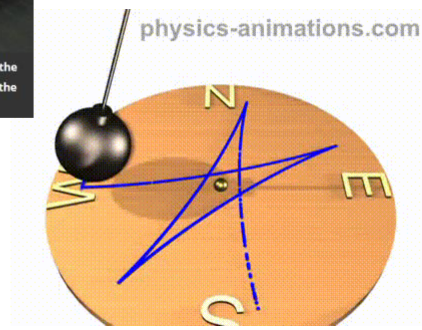
PHY131H1F - Class 30


A Foucault-inspired pendulum apparatus at the CosmoCaixa museum in Barcelona, Spain. As the path of the pendulum shifts due to Earth's rotation, the bob will gradually knock over all of the vertical rods around the circle's circumference. (Wikimedia Commons)

How Does Foucault's Pendulum Prove the Earth Rotates?

This elegant scientific demonstration has been delighting everyday people for nearly 200 years

https://www.smithsonianmag.com/smithsonian-institution/how-does-foucaults-pendulum-prove-earth-rotates-180968024/



Today:

10.3 Dynamics of Simple Harmonic Motion

10.4 Energy in Simple Harmonic Motion

10.5 The Simple Pendulum

1

Poll

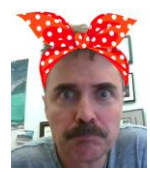
Crazy Friday: Let's Choose a Zoom-Filter my face today

What Zoom Video Filter would you prefer on my face today?

A. Mouse 

B. Pirate hat and eye-patch 

C. Pig snout and ears 

D. Red headband 

E. Red beret 

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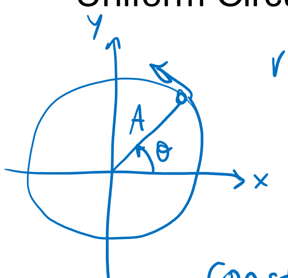
Official Course Evaluations for H1F Courses Have Begun

- An essential component of our commitment to teaching excellence is the regular evaluation of courses by students.
- Today at 3:00am, I believe, you were sent an email by course.evaluations@utoronto.ca inviting you to evaluate PHY131H1F on the Quercus.
- It only takes 10 minutes to answer the questions and enter your typed thoughts about the course.
- Your answers and thoughts are **anonymous**, but are very important to me and Professor Sealfon.
- I promise you that when the results become available to us in January, Carolyn and I will **read** every comment and scrutinize the responses to see if it can help us improve the course or our teaching in the future.

Uniform Circular Motion

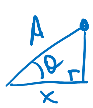


radius = A

object moves with constant angular velocity ω [rad/s]

$\theta_f = \theta_i + \omega t$, $\theta_i = 0$

$\omega = \frac{2\pi}{T}$ $\qquad \theta = \frac{2\pi}{T} t$



$$\boxed{x = A \cos\left(\frac{2\pi}{T} t\right)}$$

Simple Harmonic Motion

$$a_x = -\left(\frac{4\pi^2}{T^2}\right) x$$

Trial solution:

$$\boxed{x = A \cos\left(\frac{2\pi}{T} t\right)}$$

where A = constant.

here

$$\boxed{\frac{2\pi}{T} = \sqrt{\frac{k}{m}}}$$

Mathematically, S.H.M. is identical to one component of uniform circular motion!

What are v_{max} and a_{max}?

- If the position function is given by:

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

Note $\cos\theta$ varies between +1 an -1. $\Rightarrow x_{max} = A$

- Then the velocity and acceleration functions are:

$$v_x = -\left(\frac{2\pi}{T}\right)A\sin\left(\frac{2\pi}{T}t\right)$$

$$a_x = -\left(\frac{2\pi}{T}\right)^2 A\cos\left(\frac{2\pi}{T}t\right)$$

Same for $\sin\theta$. $\Rightarrow \boxed{v_{max} = \frac{2\pi}{T}A}$

$\Rightarrow a_{max} = \left(\frac{2\pi}{T}\right)^2 A$

- A is the amplitude of the vibration; T is the period of the vibration.

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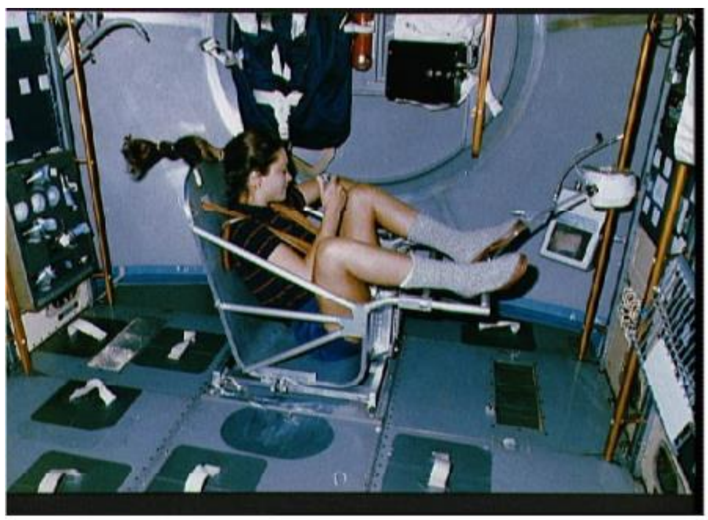
Poll question

Eq. 16.7: $T = 2\pi\sqrt{\frac{m}{k}}$

The Body Mass Measurement Device chair (mass = 32 kg) has a vibrational period of 1.2 s when empty. When an astronaut sits on the chair, what will be the vibrational period?

A. More than 1.2 s
B. Less than 1.2 s
C. 1.2 s



Astronaut Tamara Jernigan (Shuttle Columbia during STS-40, 5-14 June 1991) is weighed into space. This is the first type of "chair pose space." As the chair moves forward and backward, a calculation of the weight counter how astronaut retards the movement of the chair.

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The Body Mass Measurement Device chair (mass = 32 kg) has a vibrational period of 1.2 s when empty. When an astronaut sits on the chair, the period changes to 2.1 s. Determine the mass of the astronaut.

$m_c = 32$ kg
$T_c = 1.2$ s
$m_a = ?$
$T_{ca} = 2.1$ s

SKETCH & TRANSLATE.

① system = chair + spring

wall m_c T_c

② system: chair + astronaut + spring

wall $m_a + m_c$ T_{ca}

SIMPLIFY & DIAGRAM

Use Eq. 10.7 from pg. 294

$T = 2\pi\sqrt{m/k}$.

System ① → Solve for k.

System ② → Solve for m_a

REPRESENT MATHEMATICALLY

①: $T_c = 2\pi\sqrt{\frac{m_c}{k}}$

$\frac{T_c^2}{4\pi^2} = \frac{m_c}{k}$

$k = \frac{4\pi^2 m_c}{T_c^2}$

$\frac{k}{4\pi^2} = \frac{m_c}{T_c^2}$

②: $T_{ca} = 2\pi\sqrt{\frac{m_c+m_a}{k}}$

$\frac{T_{ca}^2}{4\pi^2} = \frac{m_c+m_a}{k}$

$m_c + m_a = \frac{k}{4\pi^2}T_{ca}^2$

$m_a = \frac{k}{4\pi^2}T_{ca}^2 - m_c$

SOLVE & EVALUATE

$m_a = m_c\frac{T_{ca}^2}{T_c^2} - m_c = m_c\left(\left(\frac{T_{ca}}{T_c}\right)^2 - 1\right)$

$m_a = 32\left(\left(\frac{2.1}{1.2}\right)^2 - 1\right) = \boxed{66 \text{ kg}}$

~145 lbs, reasonable for a human.

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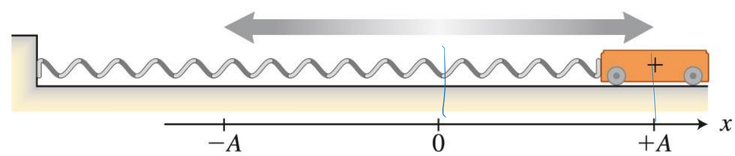
Section 10.4 Energy of Vibrational Systems

As a cart-spring system vibrates, the energy of the system continuously changes from all elastic to all kinetic.



start. →

Clock reading t	Displacement	Elastic potential energy U_s	Kinetic energy K	Total energy U_{tot}
$\frac{1}{2}T$	$-A$	$\frac{1}{2}kA^2$	0	$U_{tot} = \frac{1}{2}kA^2$
$\frac{1}{4}T$	0	0	$\frac{1}{2}mv_{max}^2$	$U_{tot} = \frac{1}{2}mv_{max}^2$
$\frac{3}{4}T$	0	0	$\frac{1}{2}mv_{max}^2$	
0	A	$\frac{1}{2}kA^2$	0	$U_{tot} = \frac{1}{2}kA^2$
T	A	$\frac{1}{2}kA^2$	0	

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$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(v_{max})^2 \quad \text{(conservation of energy)}$$

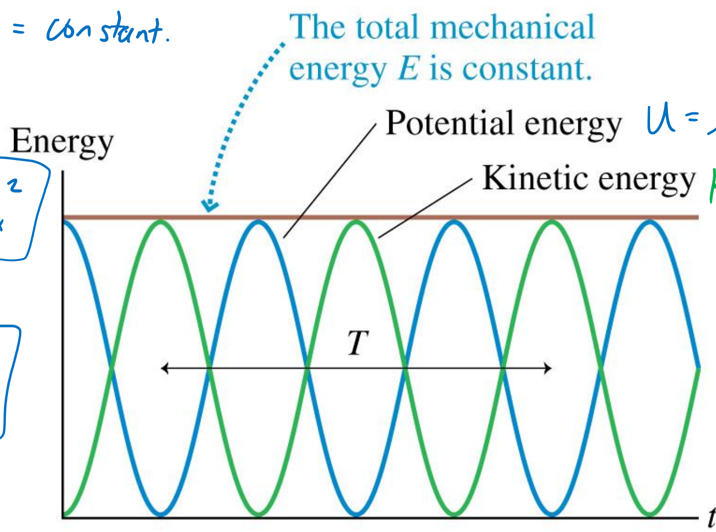
$E = K + U_s = \text{constant.}$

Either:

$$\boxed{E = \frac{1}{2}m v_{max}^2}$$

or

$$\boxed{E = \frac{1}{2}kA^2}$$



$U = \frac{1}{2}kx^2$

$K = \frac{1}{2}mv^2$

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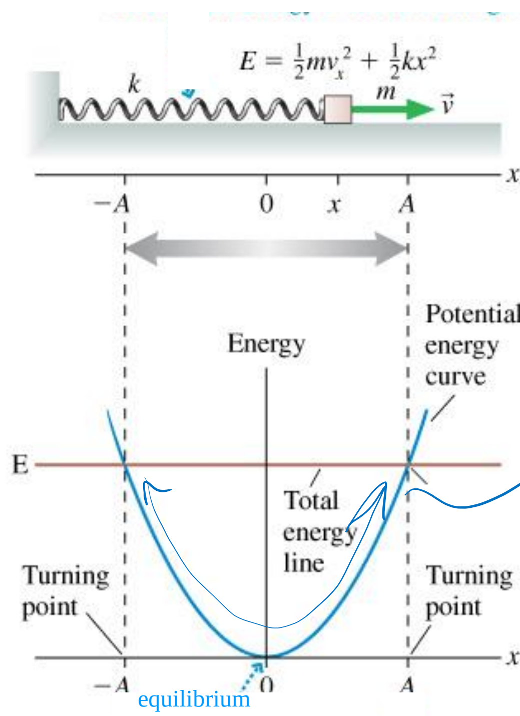

$U_s = \frac{1}{2}kx^2$

parabola.

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Relationship between the amplitude of the vibration and the cart's maximum speed

- The equation $U = \frac{1}{2}kA^2 = \frac{1}{2}mv^2_{max} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ can be rearranged to give:

$$v_{max} = \sqrt{\frac{k}{m}}A$$

- This makes sense conceptually:
  - When the mass of the cart is large, it should move slowly.
  - If the spring is stiff, the cart will move more rapidly.

TIP In the above discussion we neglected the interactions of the system with the surface of the track and with the air. These would both do negative work on the system and gradually decrease its energy, eventually bringing the vibrating system to rest.

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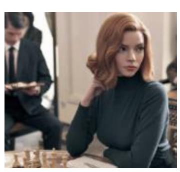
Crazy Friday TV Show Bracket
Today: Semifinals

After the Team-Up Quiz we will be having two quick (15 seconds each!) polls to determine which two shows make it to the finals next week. Pick your favourite for each pair.

- Queen's Gambit

vs

- Tiger King



- The Good Place
- vs
- Parks and Recreation

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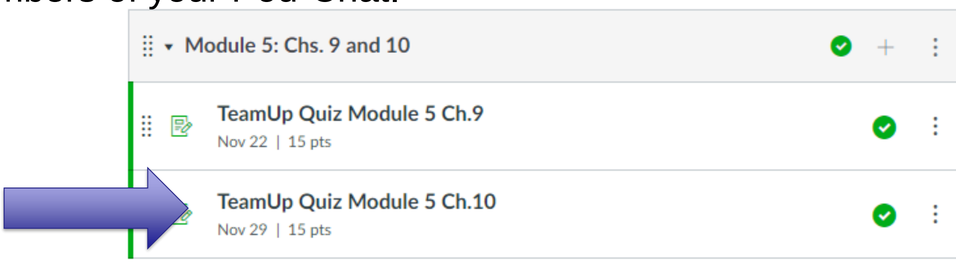
TeamUp Time!!

- Today you will be doing three multiple choice questions, all from Chapter 10, as a team of 2-4 students in your Practicals Pod.
- Your pod-team shares the mark!
- Right now you should open Microsoft Teams and someone (most recent Facilitator) should place a **video call** to all 3 or 4 members of your Pod-Chat.



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Now: TeamUp! You have 10 minutes

- The first step is to decide who will be the TeamUp **Driver**
- All students must log-in to Quercus [You will now have three windows open: my zoom lecture, Microsoft Teams, and Quercus] End 11:47
- **Non-drivers**: Wait!
- **Driver**: Go to the TeamUp Quiz Ch.10 in Module 5, click Go to Tool, then Create a Group. Let everyone in the Breakout Room know the session ID. Then WAIT – don't drive off alone!
- **Non-drivers**: Once you get the session ID, go to the TeamUp Quiz in this module, click Go to Tool, then Join Session and type the ID you were given.
- Once everyone in your room arrives in TeamUp, start going through the questions. Please **achieve consensus** before the driver submits.
- YOU MAY BEGIN! I'm going to go on mute for 10 minutes. Note: if your pod-mates are available on Microsoft Teams right now, go to the PHY131 Help Centre and I'll set up breakout rooms there. Zoom Meeting ID: 938 0964 2256, Passcode: 723874

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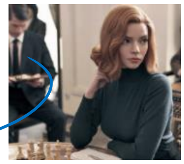
Crazy Friday TV Show Bracket
Today: Semifinals

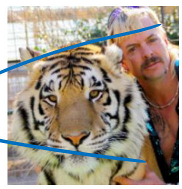
15 second poll. Which do you prefer?

A. Queen's Gambit



71%

B. ~~Tiger King~~



26%

15

Crazy Friday TV Show Bracket
Today: Semifinals

15 second poll. Which do you prefer?

A. The Good Place

65%

B. ~~Parks and Recreation~~

31%

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Question 1 Discussion

By what factor must we increase the amplitude of vibration of an object at the end of a spring in order to double its **maximum speed** during a vibration?

A. $\sqrt{2}$
B. 2
C. 4
D. 8

$v_{max} = \frac{2\pi}{T} A$, T = fixed.

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Question 2 Discussion

By what factor must we increase the amplitude of vibration of an object at the end of a spring in order to double the **total energy** of the system?

A. $\sqrt{2}$
B. 2
C. 4
D. 8

$E = \frac{1}{2} k A^2$

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Question 3 Discussion

A ball of mass m oscillates on a spring with spring constant $k = 2.0 \times 10^2$ N/m. The balls position is $\boxed{x = (0.35 \text{ m})\cos(15\,t)}$, where t is measured in seconds. What is the total mechanical energy of the ball+spring system?

meters, not mass!!

compare

A. 3.3 J

B. 6.7 J

C. 10 J

D. 12 J

wall

$E = \frac{1}{2} m v_{max}^2$

or

$E = \frac{1}{2} k A^2$

easiest to use this

$E = \frac{1}{2}(200)(0.35)^2$

$= 12.25$

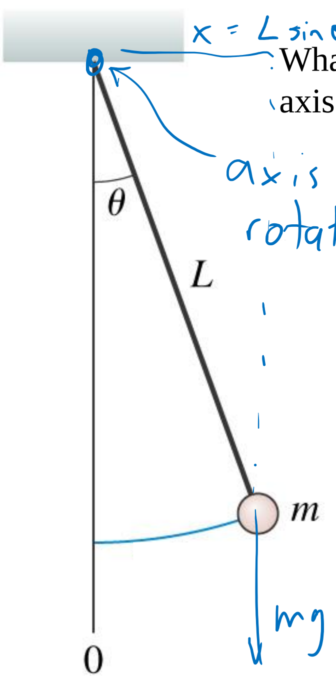
$m = ?$

$k = 200$ N/m

$$\boxed{x = A\cos\left(\frac{2\pi}{T}t\right)}$$

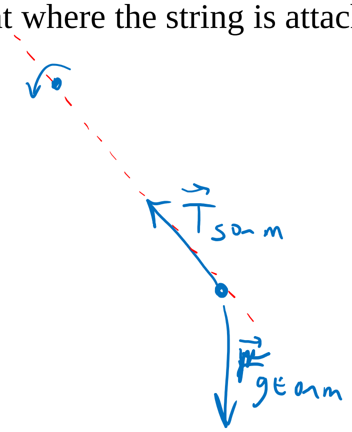
$A = 0.35$ meters

$\frac{2\pi}{T} = 15$ rad/s



What is the net torque on this pendulum? (Assume the rotation axis is the point where the string is attached to the ceiling.)

axis of rotation

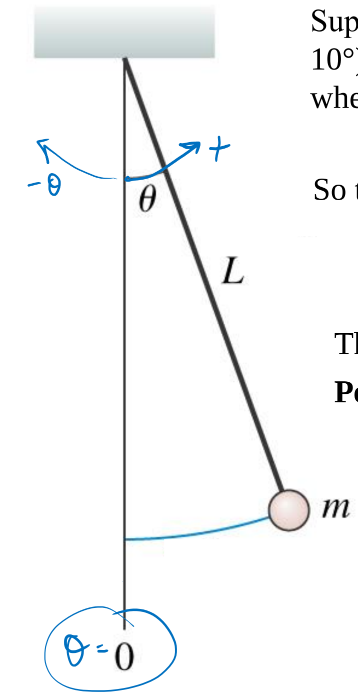


τ from tension $= 0$

$\Sigma\tau$ is from gravity

$$\boxed{\Sigma\tau = -mgL\sin\theta}$$

negative, since it is clockwise.



Suppose we restrict a pendulum's oscillations to small angles (< 10°). Then we may use the **small angle approximation** $\sin\theta \approx \theta$, where θ is measured in radians. The net torque on the mass is

$$\Sigma\tau = I\alpha \cong -mgL\theta$$

So the simple harmonic motion equation for θ as a function of time is:

$$\alpha = -\frac{mgL}{I}\theta$$

The solution to this is $\theta = A\cos\left(\frac{2\pi}{T}t\right)$, where A is a constant, and the **Period** of oscillations (in seconds) is:

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

But the rotational inertia of a point mass m a distance L from the rotation axis is $I = mL^2$, so

$$T = 2\pi\sqrt{\frac{\cancel{m}L^{\cancel{2}}}{\cancel{m}g\cancel{L}}} = 2\pi\sqrt{\frac{L}{g}}$$

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Mass on Spring versus Pendulum

	Mass on a Spring	Pendulum
Condition for S.H.M.	Small oscillations (Hooke's Law is obeyed)	Small angles
Period	$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$

[Demonstration]

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Before Class 31 on Monday

- Please finish reading all of Chapter 10:
- 10.6 Solving SHM Problems
- 10.7 Damped Vibrational Motion
- 10.8 Driven Vibrational Motion

- Remember on Tuesday at 8:10pm there will be a Midterm Assessment on Chapters 9 and 10.
- It will be 10 Multiple Choice questions, one at a time, in 30 minutes (same format as Midterms 1 and 3).

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Post Class Office Hour Notes.
Discussion of Homework 10, Problem 10
Problem 10.47

Seg. 1:

$$m_a v_i = (m_a + m_c) v_f$$

$$v_f = \frac{m_a}{m_a + m_c} v_i$$

Seg. 2:

$$v_f = v_{max} = 2\pi f A$$

$$v_i = \left(\frac{m_a + m_c}{m_a}\right) 2\pi f A$$

$$= \frac{0.53}{0.05} (2 \cdot 3.14159 \cdot 2.5 \cdot 0.21)$$

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Post Class Office Hour Notes Discussion of Homework 10, Problem 11   Problem 10.66 Part A

$$x = \sqrt{\frac{2(57\times10^{-3})9.8\cdot 1.0}{17\times10^{3}}} = 0.0081 m$$

8.1 mm

$\frac{1}{2}mv_f^2 = \frac{1}{2}kx^2$

2 segments: (1) free fall. $y=1 \rightarrow y=0$

(2) spring compression. $y=0 \rightarrow y=-x$

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\boxed{mgh = \frac{1}{2}mv_f^2}$$

$$v_f = \sqrt{2gh}$$

(1): Initial: $h = 1.0$ m

Final $h = 0$.

System = ball + earth

v_f

$mgh = \frac{1}{2}kx^2$

$\frac{2mgh}{k} = x^2$

$x = \sqrt{\frac{2mgh}{k}}$

(2) initial: $y=0$

$v_f = \sqrt{2gh}$

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

$$\frac{1}{2}mv_f^2 + 0 + 0 = 0 - mgx + \frac{1}{2}kx^2$$

Final: stops: $y=-x$

x

$h = 1m$, $mgh + \cancel{mgx} = \frac{1}{2}kx^2$

Neglect mgx .... $\rightarrow 0$.

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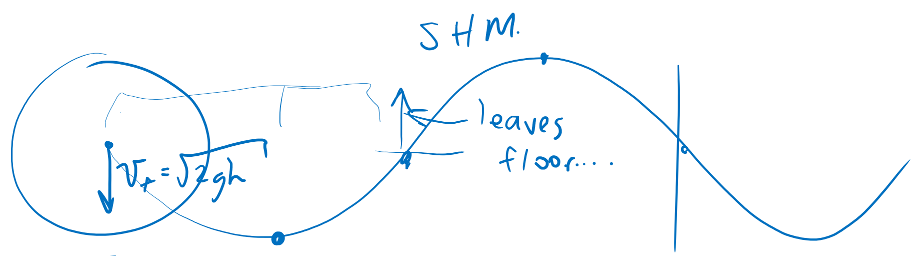
Post Class Office Hour Notes Discussion of Homework 10, Problem 11   Problem 10.66 Part B

Initial:



$\Delta t = T/4$

T

$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 0.0115$ s

$$\Delta t = \frac{2\pi}{4}\sqrt{\frac{m}{k}} = \frac{2\pi}{4}\sqrt{\frac{57\times10^{-3}}{17\times10^{3}}}$$

$\frac{T}{4}$ ← time for ball to stop $= 0.0029$ s

$\frac{T}{2}$ = time for ball to bounce: 0.0058 s

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