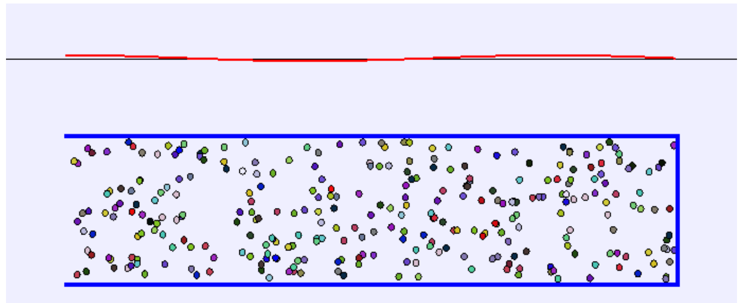
PHY131H1F - Class 34



Today:

11.7 Sound Waves, Beats

11.8 Standing Waves on Strings

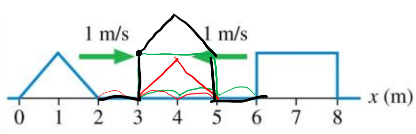
11.9 Standing Waves in Air Columns

This is a standing wave of sound in an open-closed tube.

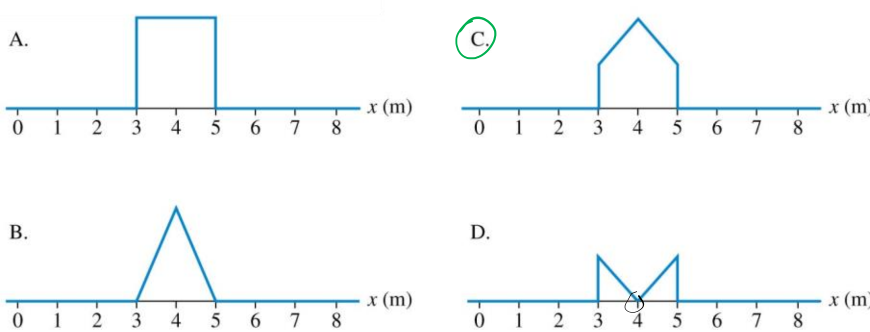
1

i-Clicker Discussion Question

Two wave pulses on a string approach each other at speeds of 1 m/s. How does the string look at $t = 3$ s?



Approaching pulses at $t = 0$ s



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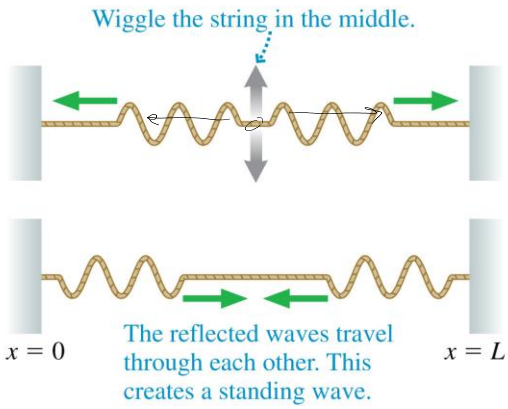
Standing Waves on a String



Reflections at the ends of the string cause waves of *equal amplitude and wavelength* to travel in opposite directions along the string, which results in a standing wave.

3

The Mathematics of Standing Waves

According to the principle of superposition, the net displacement of a medium when waves with displacements y_R (right traveling wave) and y_L (left traveling wave) are present is

$$y = y_R + y_L = \overbrace{A\cos\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]}^{\text{sine wave traveling to the right}} + \overbrace{A\cos\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]}^{\text{sine wave traveling to the left.}}$$

We can simplify this by using a trigonometric identity, and arrive at (proof on next slide):

$$y = \underbrace{2A\cos\left(\frac{2\pi}{\lambda}x\right)}_{\text{Position-dependent Amplitude.}}\underbrace{\cos\left(\frac{2\pi}{T}t\right)}_{\text{Simple Harmonic Motion (SHM)}}$$

For a standing wave, the pattern is not propagating!

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y_R = wave traveling toward the right.

y_L = wave traveling toward the left

Superposition:

$$y = y_R + y_L = A\cos\left(2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right) + A\cos\left(2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right)$$

set $\theta_1 = \frac{2\pi t}{T}$, $\theta_2 = \frac{2\pi x}{\lambda}$

$$y = A\cos(\theta_1 - \theta_2) + A\cos(\theta_1 + \theta_2)$$

Use trig. identities:

$$y = A\left[\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2\right] + A\left[\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2\right]$$

$$y = 2A\cos\theta_1 \cos\theta_2$$

$$y = 2A\cos\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

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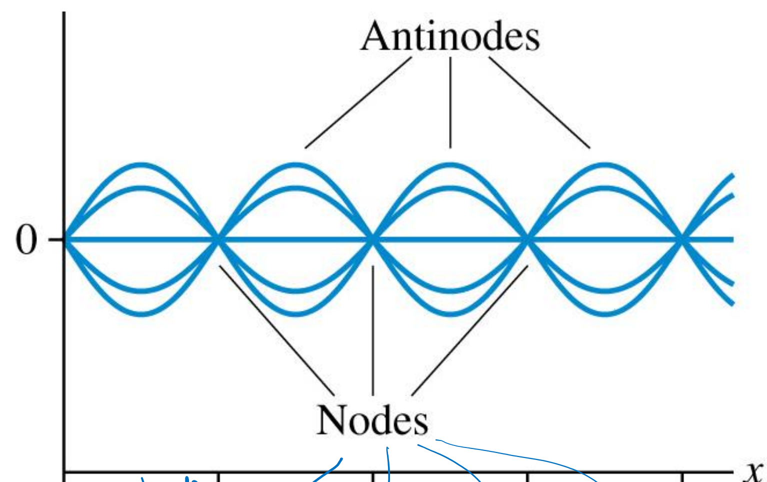
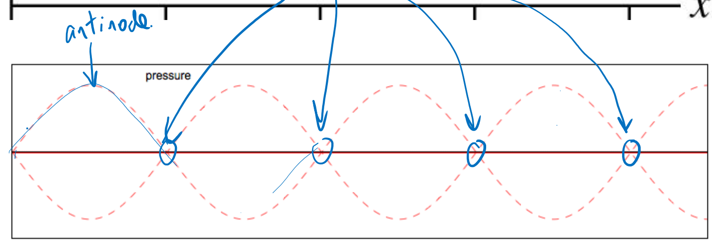
Many textbooks draw this:



What is really happening is this:



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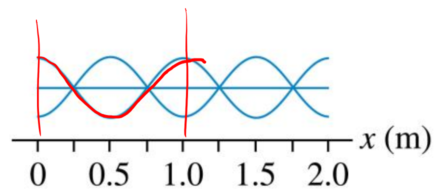
Poll Question

What is the wavelength
of this standing wave?

A. 0.25 m.
B. 0.5 m.
C. 1.0 m.
D. 2.0 m.



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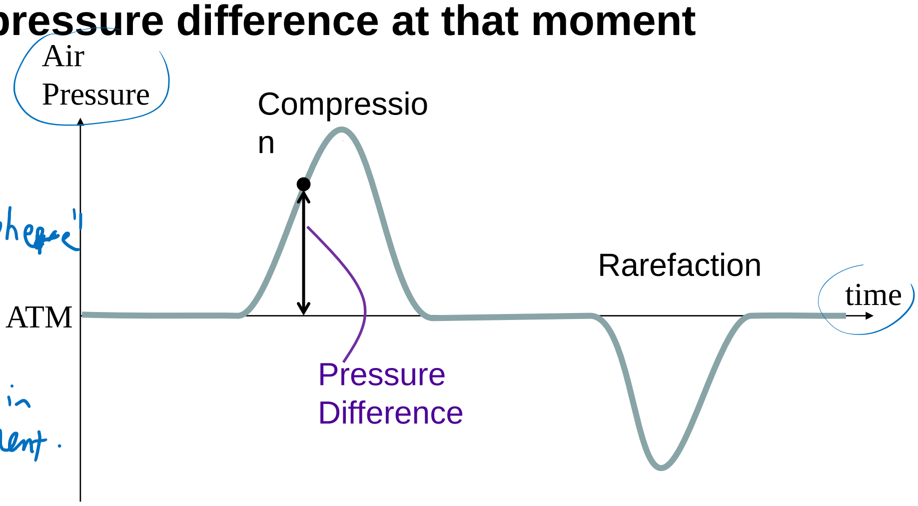
At each moment of time:
Add up all of the compressions
Subtract all of the rarefactions
= The pressure difference at that moment

Principle of Superposition
for SOUND WAVES.

1 "Atmosphere"
→
= ambient pressure in environment.



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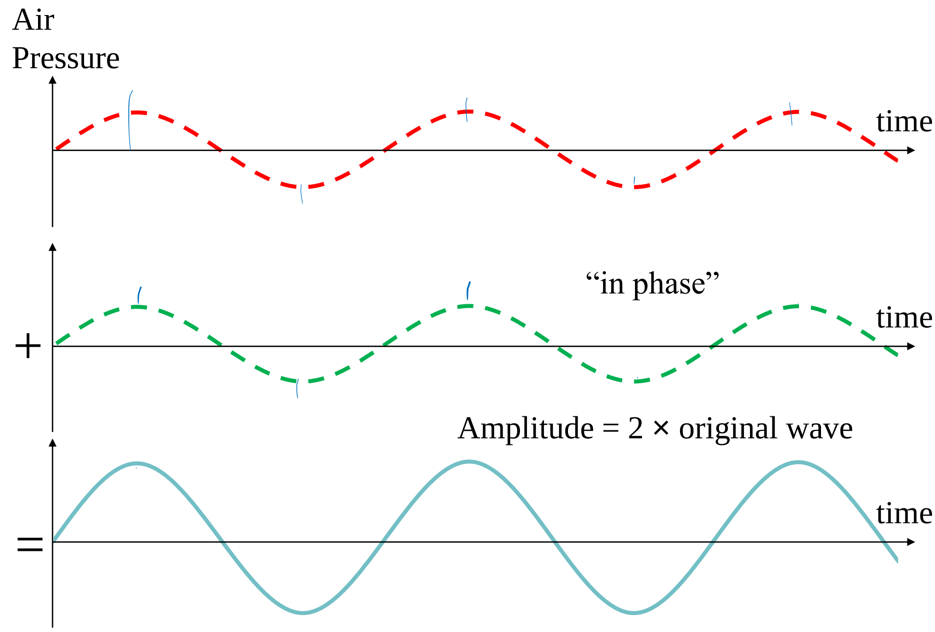
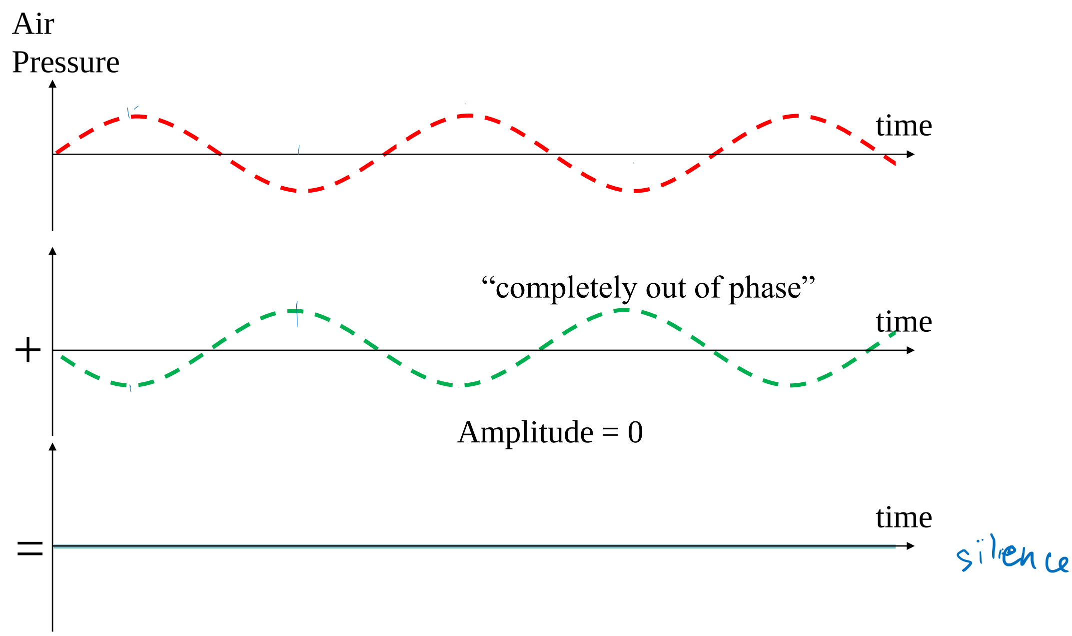

Air
Pressure
time
+
"in phase"
time
Amplitude = 2 × original wave
=
time

9


Air
Pressure
time
+
"completely out of phase"
time
Amplitude = 0
=
time
silence

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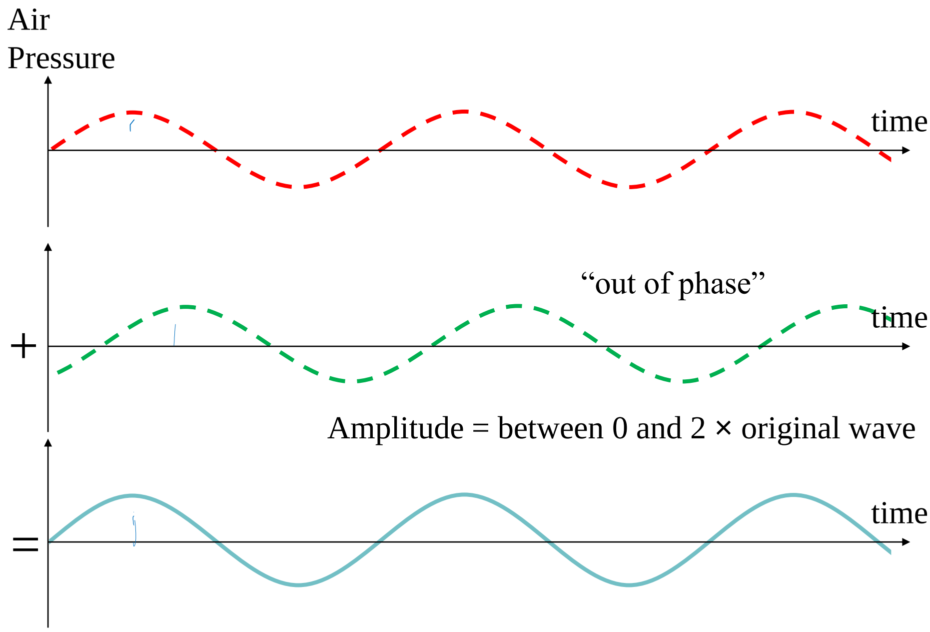

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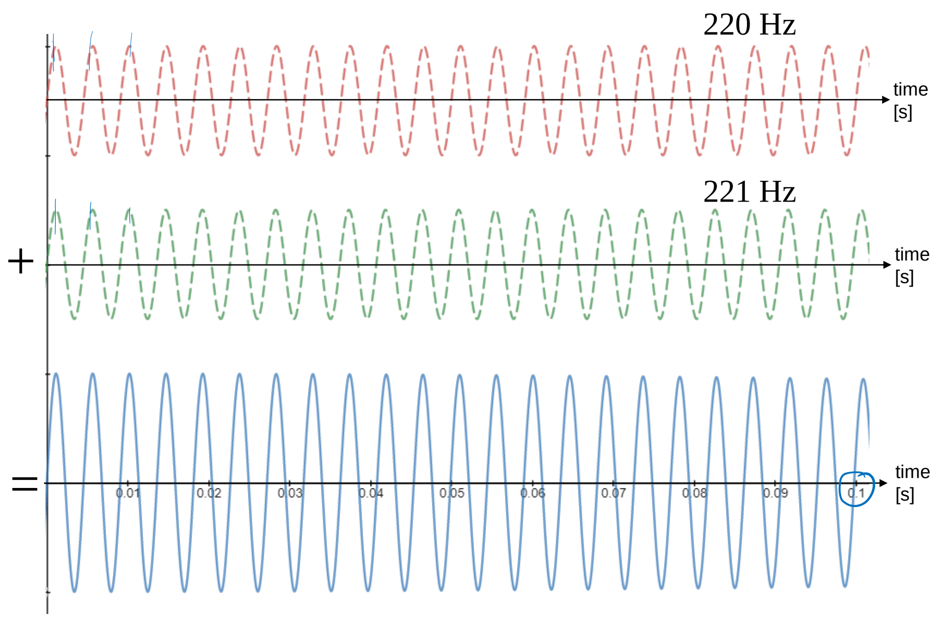
- The first 0.1 seconds.
- Both tones start “in phase”
- There are 22 full oscillations of the first tone.
- There are 22.1 full oscillations of the second tone.

Beats



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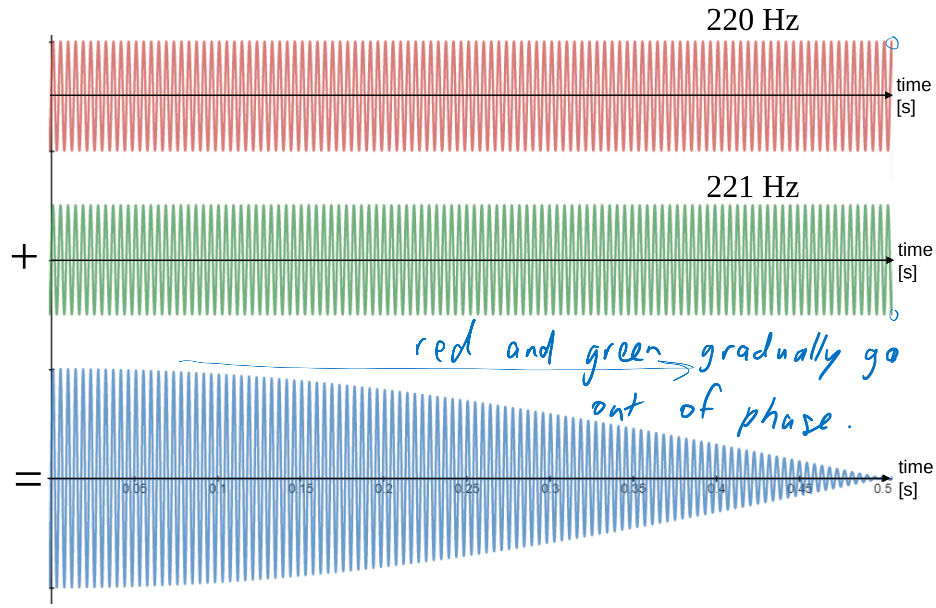
Beats

- The first 0.5 seconds.
- Both tones start "in phase"
- There are 110 full oscillations of the first tone.
- There are 110.5 full oscillations of the second tone.
- They end up "out of phase"



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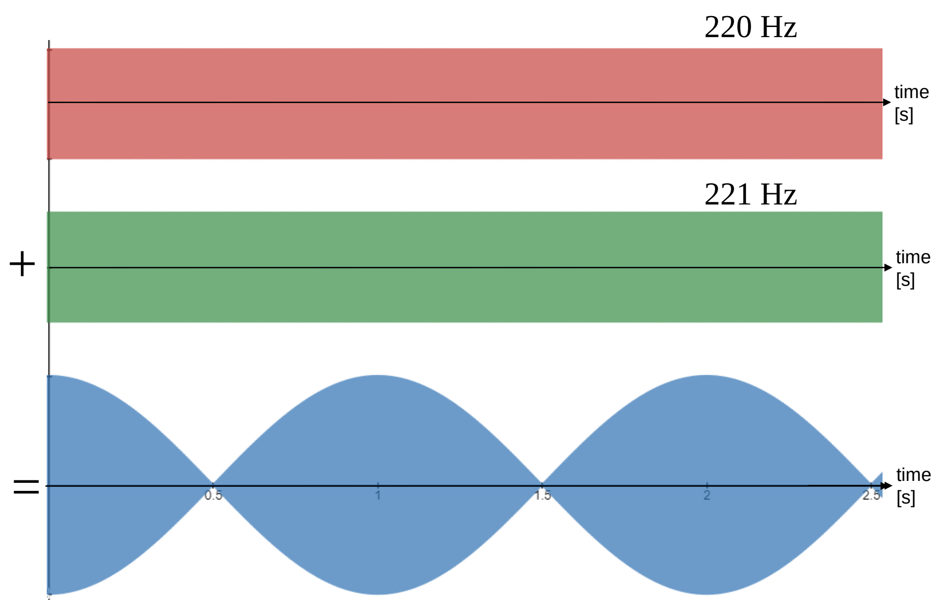
Beats

- The first 2.5 seconds.
- There are so many oscillations that the wiggly line just looks like a solid colour!
- The blue bumps are called "beats"



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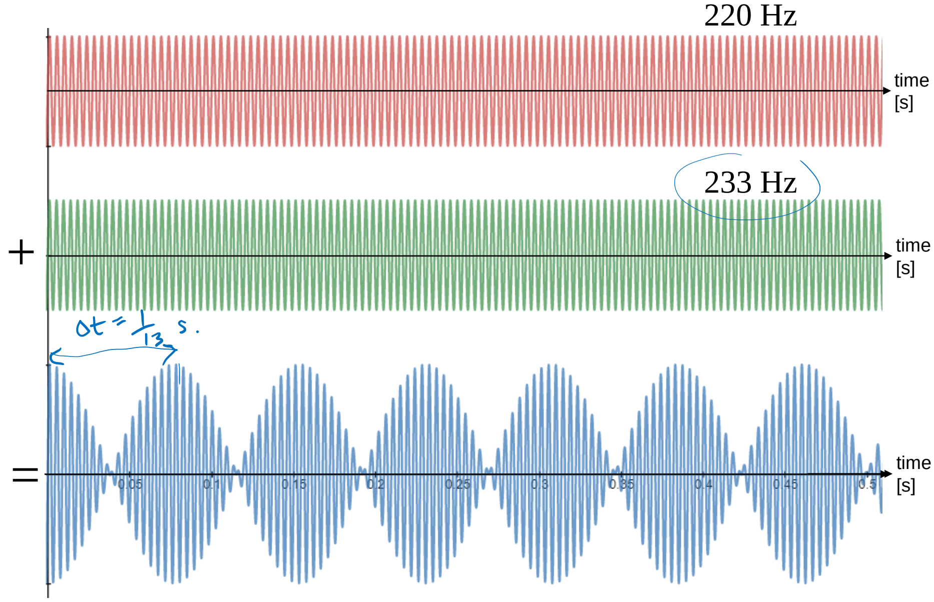
Beats

$f_{beat} = 233 - 220 = 13\ Hz$

- The first 0.5 seconds of two different tones.
- Both tones start "in phase"
- Every 1/13th of a second, the red curve has 16.92 oscillations, and the green curve has 17.92 oscillations.
- So they end up in phase again.



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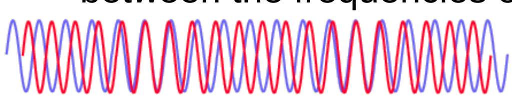
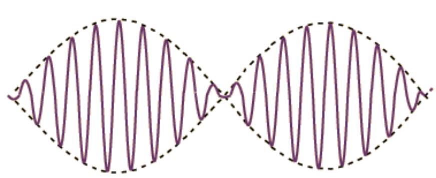
Beats

- Periodic variations in the loudness of sound due to interference
- Occur when two waves of similar, but not equal frequencies are superposed.
- Provide a comparison of frequencies
- Frequency of beats is equal to the **difference** between the frequencies of the two waves.





[Demonstration]

[image from http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html]

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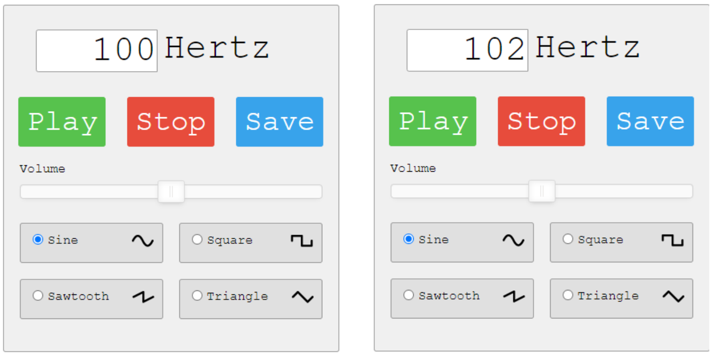
Beats

- https://onlinetonegenerator.com/



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Beats

- Applications
  - Piano tuning by listening to the disappearance of beats from a known frequency and a piano key
  - Tuning instruments in an orchestra by listening for beats between instruments and piano tone

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Beat and beat frequencies

A **beat** is a wave that results from the superposition of two waves of about the same frequency. The beat (the net wave) has a frequency equal to the average of the two frequencies and has variable amplitude. The frequency with which the amplitude of the net wave changes is called the **beat frequency** f_{beat}; it equals the difference in the frequencies of the two waves:

$$f_{beat} = |f_1 - f_2|$$

(Equation 11.9)
Pg. 335

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Poll Question

- If you combine the sounds of two pure tones, one with a frequency of 440 Hertz, and the other with a frequency of 220 Hertz, what do you get?

A. Beats with a frequency of 2 Hertz
B. Beats with a frequency of 220 Hertz
C. Beats with a frequency of 440 Hertz
D. A continuous sound which humans perceive to be two tones played at once

For beats to occur, f_1 and f_2 must be "pretty close"

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Poll Question

- You are tuning a piano and you want to make the frequency of an A key to be 440 Hertz, but you suspect it is out of tune.
- You have a reference sound source that you know for sure makes a pure tone of 440 Hertz.
- When you sound the reference at the same time as the piano A key, you hear 3 beats per second.
- What is the frequency of your out-of-tune piano A key?

A. 440 Hz
B. 443 Hz
C. 437 Hz
D. It's impossible to know with the information given

$f_{beat} = 3\ Hz = |f_1 - f_2|$

$f_2 = 440\ Hz.$

$f_1 = 437 Hz$ or $443\ Hz$

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Poll Question

- You are tuning a piano and you want to make the frequency of the A key to be 440 Hertz, but you suspect it is out of tune. You have a reference sound source that you know for sure makes a pure tone of 440 Hertz. When you sound the reference at the same time as your out-of-tune piano A key, you hear 3 beats per second. ← either ~~437~~ Hz or 443 Hz
- You then tighten the string on the piano, which you know raises the frequency of the A key a bit. When you sound the reference at the same time now, you hear 7 beats per second! What is the new frequency of the out-of-tune piano A key? $f \uparrow$, it gets worse.

A. 440 Hz
B. 447 Hz
C. 433 Hz
D. It's impossible to know with the information given

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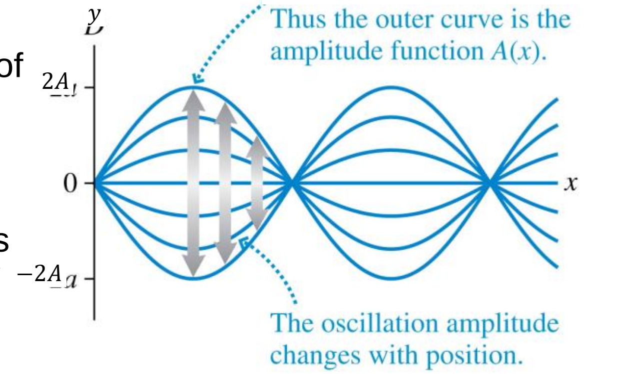
The Mathematics of Standing Waves

- Shown is the graph of $y(x,t)$ at several instants of time.
- The nodes occur at $x_m = m\lambda/2$, where m is an integer.



$$y(x,t) = \boxed{A(x)}\sin\left(\frac{2\pi}{T}t\right) \qquad \boxed{A(x)} = 2A\sin\left(\frac{2\pi}{\lambda}x\right)$$

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Standing Waves on a String

For a string of fixed length L, the boundary conditions can be satisfied only if the wavelength has one of the values:

$$\lambda_m = \frac{2L}{m} \qquad m = 1, 2, 3, 4, \ldots$$

v = wave speed on string

$v = \sqrt{F/\mu}$

Because $\lambda f = v$ for a sinusoidal wave, the oscillation frequency corresponding to wavelength λ_m is:

$$f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = m\frac{v}{2L} \qquad m = 1, 2, 3, 4, \ldots$$

← m = "mode number"

The lowest allowed frequency is called the **fundamental frequency**: $f_1 = v/2L$.

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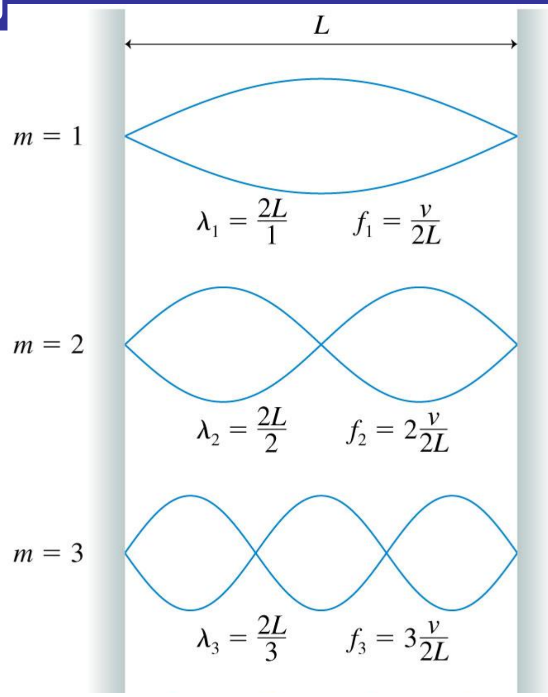
Standing Waves on a String

- Shown are various standing waves on a string of fixed length L.
- These possible standing waves are called the **modes** of the string, or sometimes the *normal modes.*
- Each mode, numbered by the integer m, has a unique wavelength and frequency.



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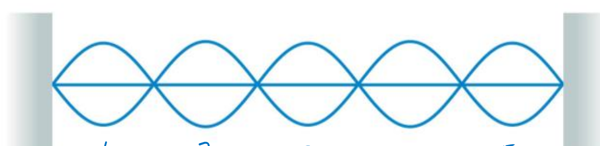
Poll Question

What is the mode number of this standing wave?

A. m = 1
B. m = 2
C. m = 3
D. m = 4
E. m = 5



1 2 3 4 5

↑
count the number of antinodes to get m for standing wave on a string.

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Standing Waves on a String

There are three things to note about the normal modes of a string:

1. m is the number of *antinodes* on the standing wave.
2. The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
3. The frequencies of the normal modes form a series: $f_1, 2f_1, 3f_1, \ldots$ These are also called **harmonics**. $2f_1$ is the "second harmonic", $3f_1$ is the "third harmonic", etc.

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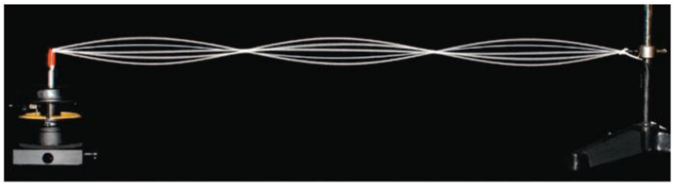
Standing Waves on a String

- m is the number of *antinodes* on the standing wave.
- The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
- The frequencies of the normal modes form a series: $f_1, 2f_1, 3f_1, \ldots$
- The fundamental frequency f_1 can be found as the *difference* between the frequencies of any two adjacent modes: $f_1 = \Delta f = f_{m+1} - f_m$.
- Below is a time-exposure photograph of the $m = 3$ standing wave on a string.



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Piano Key Num.	Note name	Freq. [Hz]	Piano Key Num.	Note name	Freq. [Hz]	Piano Key Num.	Note name	Freq. [Hz]	Piano Key Num.	Note name	Freq. [Hz]
1	A0	27.5	23	G2	98.0	45	F4	349.2	67	D#6	1244.5
2	A#0	29.1	24	G#2	103.8	46	F#4	370.0	68	E6	1318.5
3	B0	30.9	25	A2	110.0	47	G4	392.0	69	F6	1396.9
4	C1	32.7	26	A#2	116.5	48	G#4	415.3	70	F#6	1480.0
5	C#1	34.6	27	B2	123.5	49	A4	440.0	71	G6	1568.0
6	D1	36.7	28	C3	130.8	50	A#4	466.2	72	G#6	1661.2
7	D#1	38.9	29	C#3	138.6	51	B4	493.9	73	A6	1760.0
8	E1	41.2	30	D3	146.8	52	C5	523.3	74	A#6	1864.7
9	F1	43.7	31	D#3	155.6	53	C#5	554.4	75	B6	1975.5
10	F#1	46.2	32	E3	164.8	54	D5	587.3	76	C7	2093.0
11	G1	49.0	33	F3	174.6	55	D#5	622.3	77	C#7	2217.5
12	G#1	51.9	34	F#3	185.0	56	E5	659.3	78	D7	2349.3
13	A1	55.0	35	G3	196.0	57	F5	698.5	79	D#7	2489.0
14	A#1	58.3	36	G#3	207.7	58	F#5	740.0	80	E7	2637.0
15	B1	61.7	37	A3	220.0	59	G5	784.0	81	F7	2793.8
16	C2	65.4	38	A#3	233.1	60	G#5	830.6	82	F#7	2960.0
17	C#2	69.3	39	B3	246.9	61	A5	880.0	83	G7	3136.0
18	D2	73.4	40	C4	261.6	62	A#5	932.3	84	G#7	3322.4
19	D#2	77.8	41	C#4	277.2	63	B5	987.8	85	A7	3520.0
20	E2	82.4	42	D4	293.7	64	C6	1046.5	86	A#7	3729.3
21	F2	87.3	43	D#4	311.1	65	C#6	1108.7	87	B7	3951.1
22	F#2	92.5	44	E4	329.6	66	D6	1174.7	88	C8	4186.0

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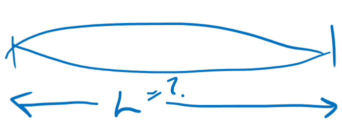
A steel guitar string has a mass density of 0.01 kg/m, and is held at a tension of 100 Newtons. What should be the effective length of string between two fixed ends to produce the note E2?

SKETCH & TRANSLATE.



$\mu = 0.01 \ \frac{kg}{m}$

$F = 100 \ N$

$v = \sqrt{F/\mu}$

Note E2 : 82.4 Hz from online table.

SIMPLIFY & DIAGRAM

Assume string is in fundamental mode m=1. $f_1 = \frac{v}{2L}$

REPRESENT MATHEMATICALLY

Eq. 11.11: $f_1 = \frac{v}{2L}$

$L = \frac{v}{2f_1} = \sqrt{F/\mu}\ \frac{1}{2f_1}$

SOLVE & EVALUATE

$L = \sqrt{\frac{100}{0.01}}\ \frac{1}{2(82.4)}$

$L = 0.6068$

$\boxed{L = 0.6 \ m}$ ← seems about right for a guitar.

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Poll Question

The frequency of the third harmonic of a string is

A. One-third the frequency of the fundamental.
B. Equal to the frequency of the fundamental.
C. Three times the frequency of the fundamental.
D. Nine times the frequency of the fundamental.

$f_3 = 3 f_1$

$f_m = \frac{mv}{2L}$ $\quad$ $f_m = m f_1$

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Sound Waves

- Your ears are able to detect sinusoidal sound waves with frequencies between about $20\ \mathrm{Hz}$ and $20\ \mathrm{kHz}$.

- Low frequencies are perceived as "low pitch" bass notes, while high frequencies are heard as "high pitch" treble notes.
- Sound waves with frequencies above $20\ \mathrm{kHz}$ are called *ultrasonic* frequencies.
- Oscillators vibrating at frequencies of many MHz generate the ultrasonic waves used in ultrasound medical imaging.

[Demonstration]

Image from http://www.weblocal.ca/uc-baby-3d-ultrasound-brampton-on.html

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Before Class 35 on Wednesday

- Please finish reading Chapter 11:
- Section 11.10 on Doppler Effect

- We will also be starting a course review and we'll look at what exactly to expect on the Final Assessment on Dec.17

- Note that Wednesday is the Last Day of Classes!

- Thursday Dec. 10 is a "Make-up Day" for the missed Monday class due to Thanksgiving. I'll be here in this zoom call on Thursday with more Course Review, and some Liquid Nitrogen Demonstrations including Levitating Superconductors

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