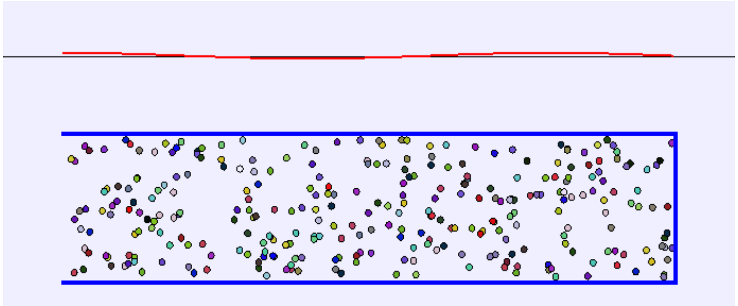
PHY131H1F - Class 34



This is a standing wave of sound in an open-closed tube.

Today:

11.7 Sound Waves, Beats

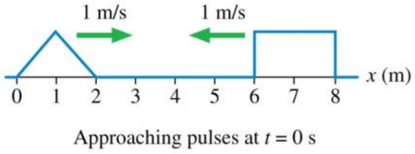
11.8 Standing Waves on Strings

11.9 Standing Waves in Air Columns

---

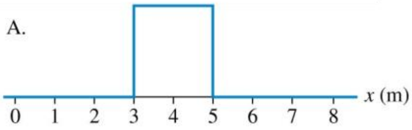
i-Clicker Discussion Question

Two wave pulses on a string approach each other at speeds of $1\ \mathrm{m/s}$. How does the string look at $t = 3\ \mathrm{s}$?

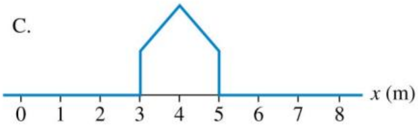


Approaching pulses at $t = 0$ s

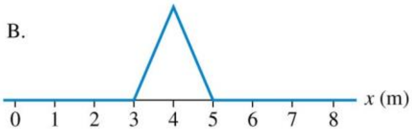
A.



C.

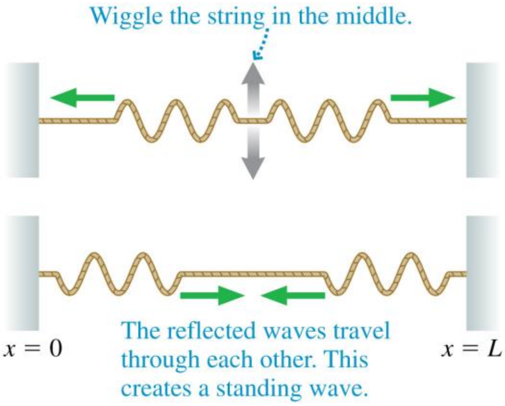


B.



D.

Standing Waves on a String



Reflections at the ends of the string cause waves of *equal amplitude and wavelength* to travel in opposite directions along the string, which results in a standing wave.

The Mathematics of Standing Waves

According to the principle of superposition, the net displacement of a medium when waves with displacements y_{R} (right traveling wave) and y_{L} (left traveling wave) are present is

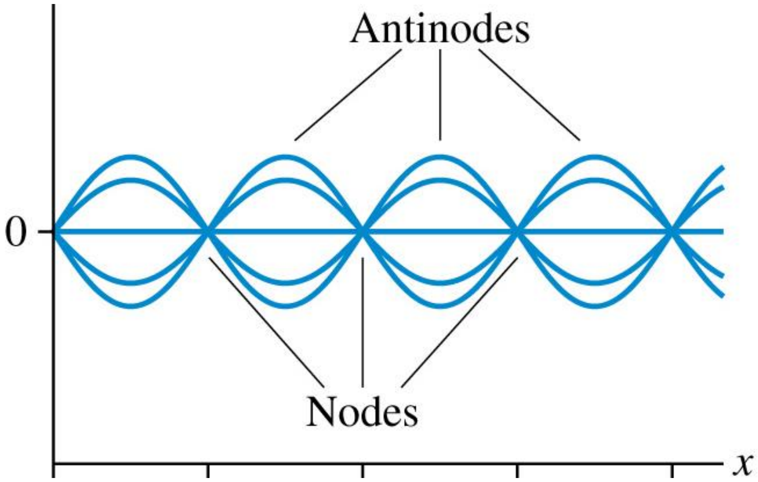
$$y = y_{\mathrm{R}} + y_{\mathrm{L}} = A\cos\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] + A\cos\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

We can simplify this by using a trigonometric identity, and arrive at:

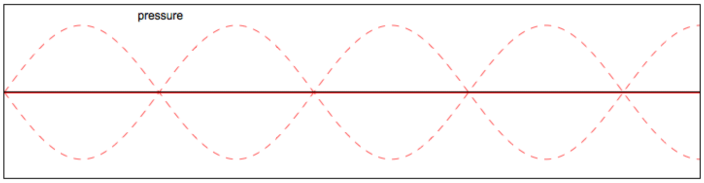
$$y = 2A\sin\left(\frac{2\pi}{\lambda}x\right)\sin\left(\frac{2\pi}{T}t\right)$$

For a standing wave, the pattern is not propagating!

Many textbooks draw this:

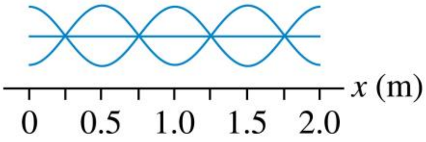


What is really happening is this:



Poll Question

What is the wavelength of this standing wave?



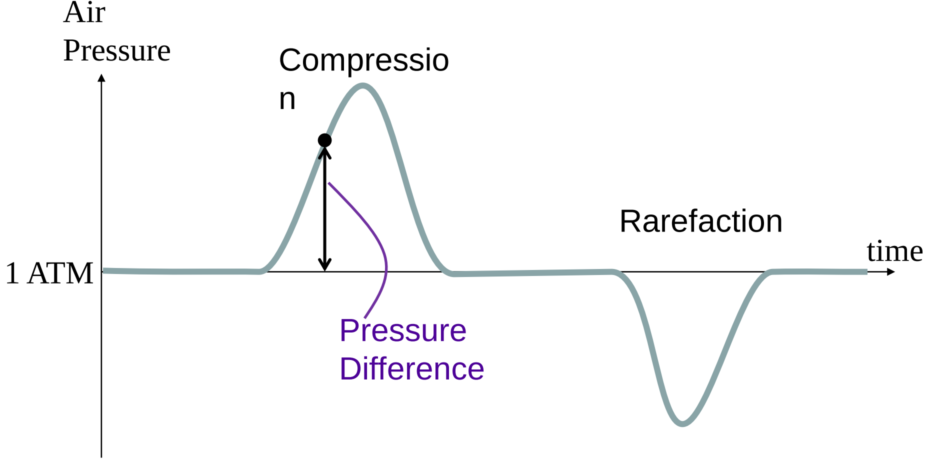
- A. 0.25 m.
- B. 0.5 m.
- C. 1.0 m.
- D. 2.0 m.

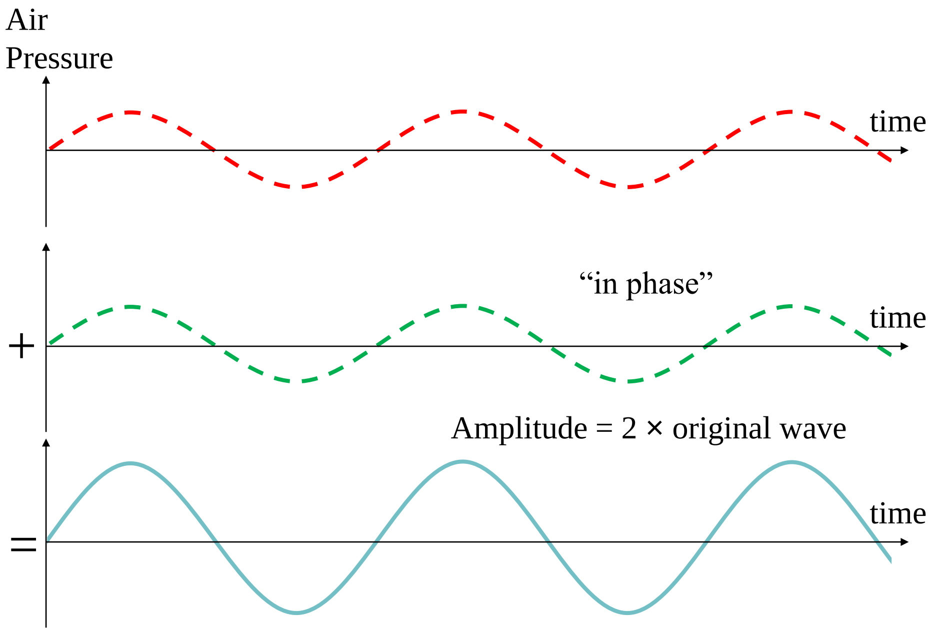
At each moment of time:

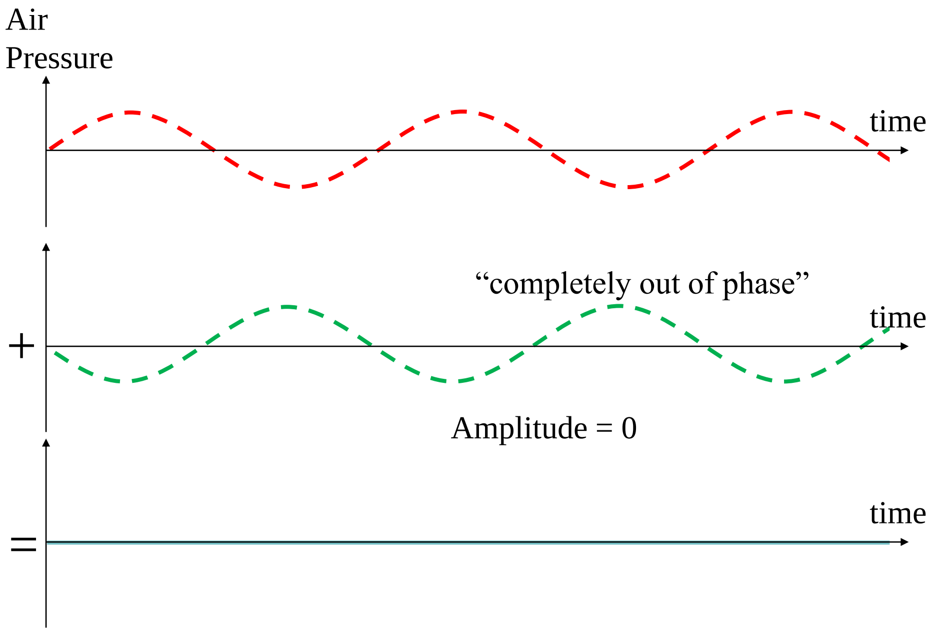
Add up all of the compressions

Subtract all of the rarefactions

= The pressure difference at that moment

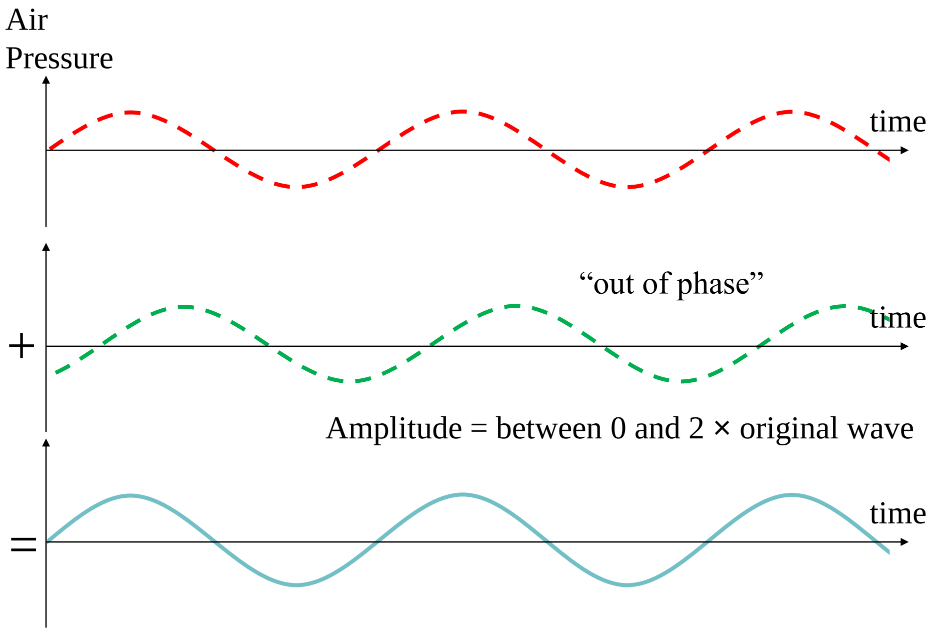






“completely out of phase”

Amplitude = 0

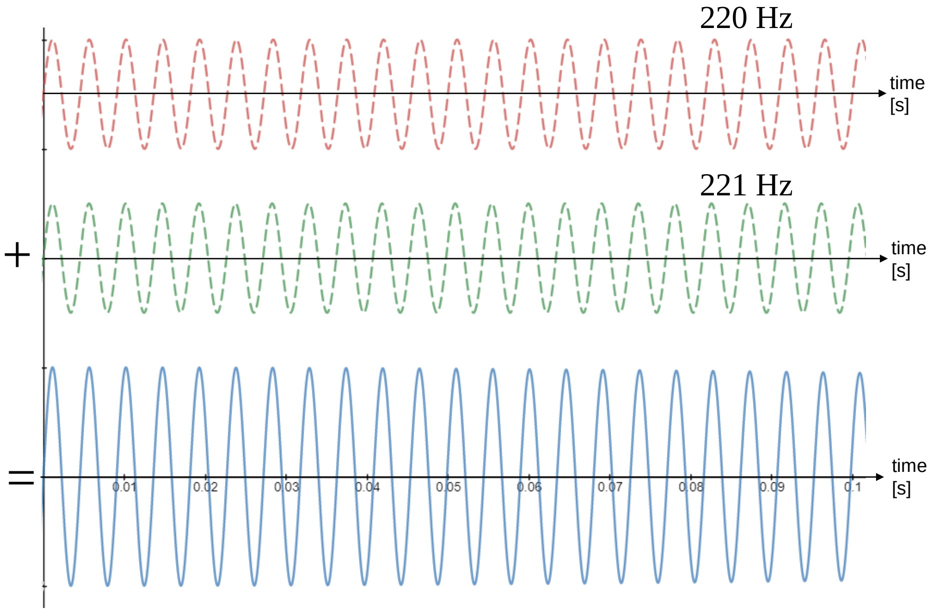


“out of phase”

Amplitude = between 0 and 2 × original wave

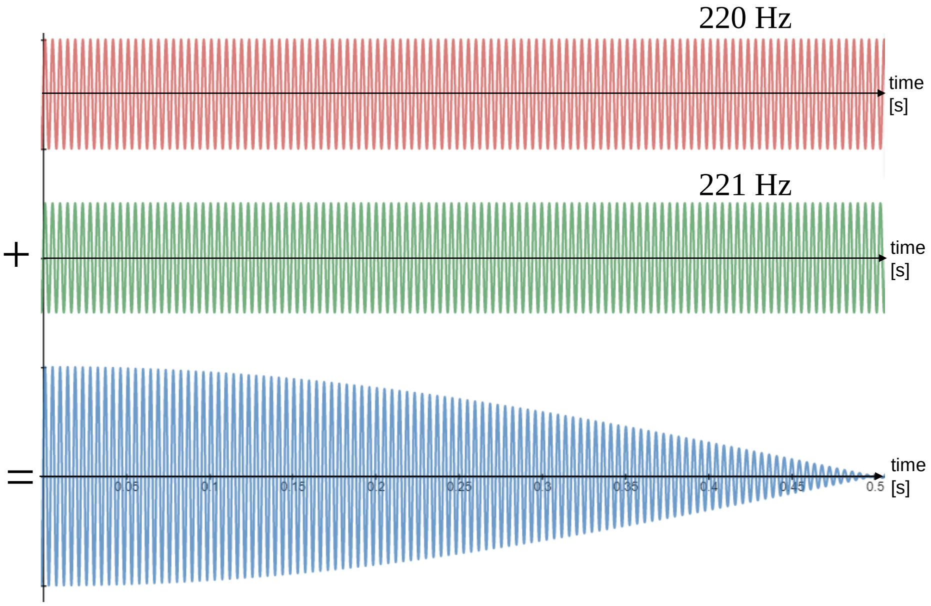
Beats

- The first 0.1 seconds.
- Both tones start “in phase”
- There are 22 full oscillations of the first tone.
- There are 22.1 full oscillations of the second tone.



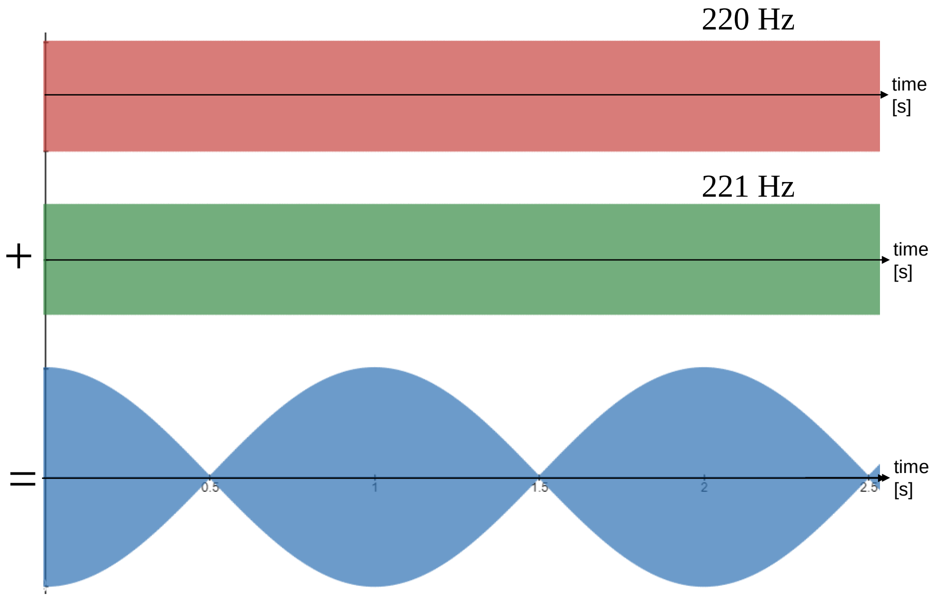
Beats

- The first 0.5 seconds.
- Both tones start “in phase”
- There are 110 full oscillations of the first tone.
- There are 110.5 full oscillations of the second tone.
- They end up “out of phase”



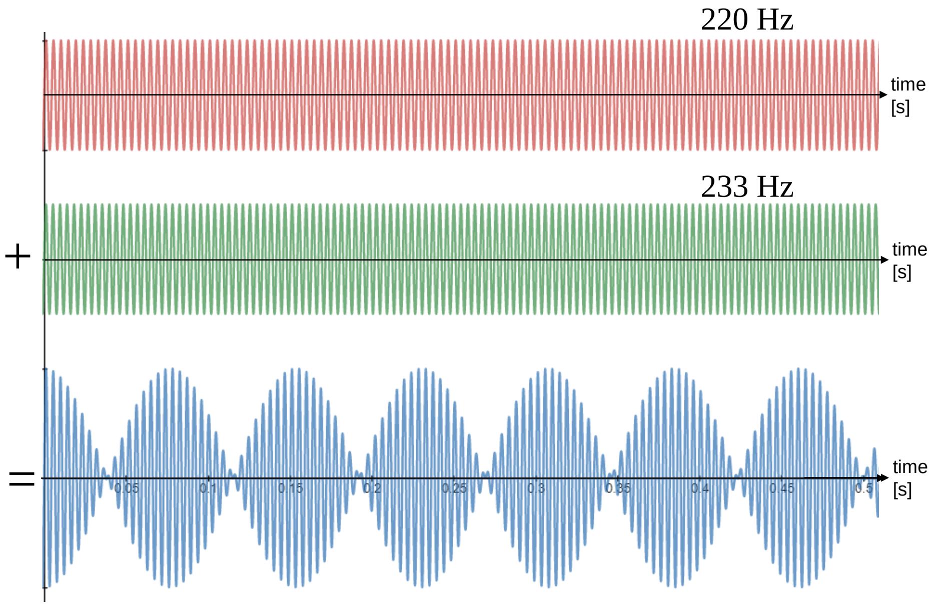
Beats

- The first 2.5 seconds.
- There are so many oscillations that the wiggly line just looks like a solid colour!
- The blue bumps are called "beats"



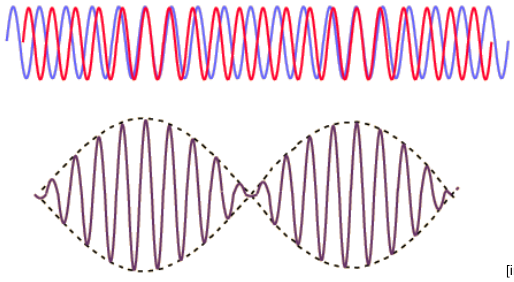
Beats

- The first 0.5 seconds of two different tones.
- Both tones start "in phase"
- Every 1/13th of a second, the red curve has 16.92 oscillations, and the green curve has 17.92 oscillations.
- So they end up in phase again.



Beats

- Periodic variations in the loudness of sound due to interference
- Occur when two waves of similar, but not equal frequencies are superposed.
- Provide a comparison of frequencies
- Frequency of beats is equal to the **difference** between the frequencies of the two waves.

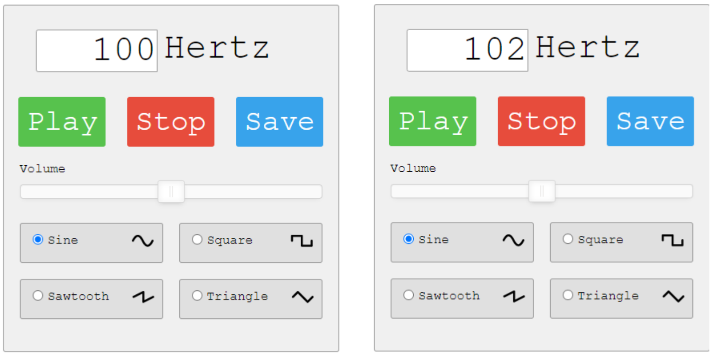


[Demonstration]

[image from http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html]

Beats

- https://onlinetonegenerator.com/



100 Hertz

● Sine ○ Square ○ Sawtooth ○ Triangle

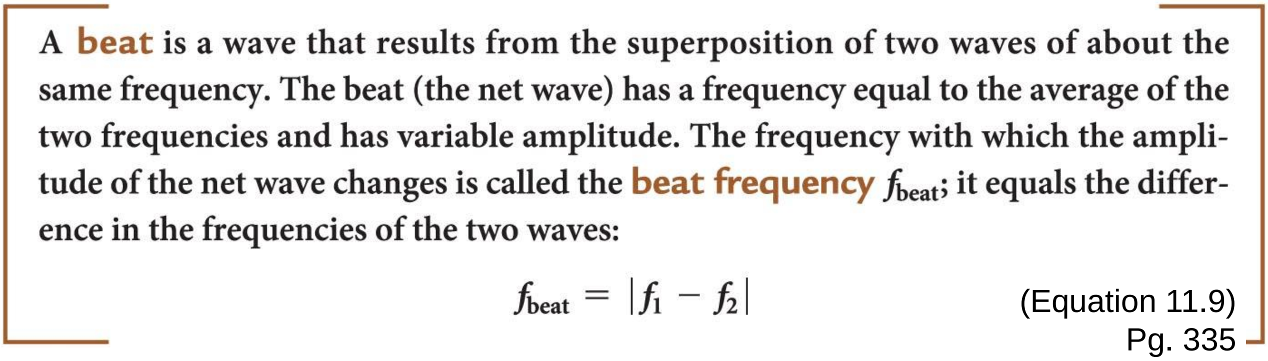
102 Hertz

● Sine ○ Square ○ Sawtooth ○ Triangle

Beats

- Applications
  - Piano tuning by listening to the disappearance of beats from a known frequency and a piano key
  - Tuning instruments in an orchestra by listening for beats between instruments and piano tone

Beat and beat frequencies



A **beat** is a wave that results from the superposition of two waves of about the same frequency. The beat (the net wave) has a frequency equal to the average of the two frequencies and has variable amplitude. The frequency with which the amplitude of the net wave changes is called the **beat frequency** f_{beat}; it equals the difference in the frequencies of the two waves:

$$f_{\text{beat}} = |f_1 - f_2|$$

(Equation 11.9)
Pg. 335

Poll Question

- If you combine the sounds of two pure tones, one with a frequency of 440 Hertz, and the other with a frequency of 220 Hertz, what do you get?

A. Beats with a frequency of 2 Hertz

B. Beats with a frequency of 220 Hertz

C. Beats with a frequency of 440 Hertz

D. A continuous sound which humans perceive to be two tones played at once

Poll Question

- You are tuning a piano and you want to make the frequency of an A key to be 440 Hertz, but you suspect it is out of tune.
- You have a reference sound source that you know for sure makes a pure tone of 440 Hertz.
- When you sound the reference at the same time as the piano A key, you hear 3 beats per second.
- What is the frequency of your out-of-tune piano A key?

A. 440 Hz

B. 443 Hz

C. 437 Hz

D. It’s impossible to know with the information given

Poll Question

- You are tuning a piano and you want to make the frequency of the A key to be 440 Hertz, but you suspect it is out of tune. You have a reference sound source that you know for sure makes a pure tone of 440 Hertz. When you sound the reference at the same time as your out-of-tune piano A key, you hear 3 beats per second.
- You then tighten the string on the piano, which you know raises the frequency of the A key a bit. When you sound the reference at the same time now, you hear 7 beats per second! What is the new frequency of the out-of-tune piano A key?

A. 440 Hz

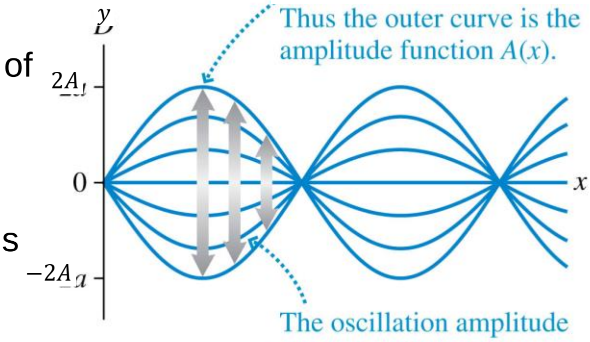
B. 447 Hz

C. 433 Hz

D. It’s impossible to know with the information given

The Mathematics of Standing Waves

- Shown is the graph of $y(x,t)$ at several instants of time.
- The nodes occur at $x_m = m\lambda/2$, where m is an integer.



At this moment, sin($2\pi t/T$) =1. Thus the outer curve is the amplitude function $A(x)$.

The oscillation amplitude changes with position.

$$y(x,t) = A(x)\sin\left(\frac{2\pi}{T}t\right) \qquad A(x) = 2A\sin\left(\frac{2\pi}{\lambda}x\right)$$

Standing Waves on a String

For a string of fixed length L, the boundary conditions can be satisfied only if the wavelength has one of the values:

$$\lambda_m = \frac{2L}{m} \qquad m = 1, 2, 3, 4, \ldots$$

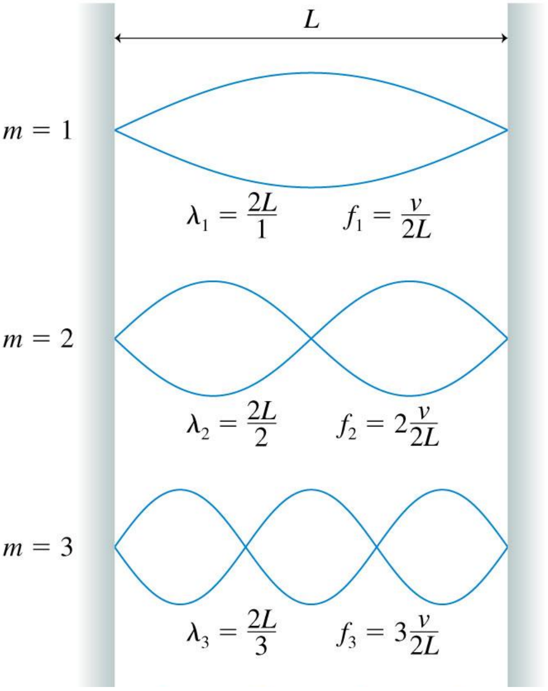
Because $\lambda f = v$ for a sinusoidal wave, the oscillation frequency corresponding to wavelength λ_m is:

$$f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = m\frac{v}{2L} \qquad m = 1, 2, 3, 4, \ldots$$

The lowest allowed frequency is called the **fundamental frequency**: $f_1 = v/2L$.

Standing Waves on a String

- Shown are various standing waves on a string of fixed length L.
- These possible standing waves are called the **modes** of the string, or sometimes the *normal modes.*
- Each mode, numbered by the integer m, has a unique wavelength and frequency.



Poll Question

What is the mode number of this standing wave?

A. m = 1
B. m = 2
C. m = 3
D. m = 4
E. m = 5

Standing Waves on a String

There are three things to note about the normal modes of a string:

1. *m* is the number of *antinodes* on the standing wave.
2. The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
3. The frequencies of the normal modes form a series: f_1, $2f_1$, $3f_1$, …These are also called **harmonics**. $2f_1$ is the "second harmonic", $3f_1$ is the "third harmonic", etc.

Standing Waves on a String

- m is the number of *antinodes* on the standing wave.
- The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
- The frequencies of the normal modes form a series: $f_1, 2f_1, 3f_1, \ldots$
- The fundamental frequency f_1 can be found as the *difference* between the frequencies of any two adjacent modes: $f_1 = \Delta f = f_{m+1} - f_m$.
- Below is a time-exposure photograph of the $m = 3$ standing wave on a string.

Piano Key Num.	Note name	Freq. [Hz]
1	A0	27.5
2	A#0	29.1
3	B0	30.9
4	C1	32.7
5	C#1	34.6
6	D1	36.7
7	D#1	38.9
8	E1	41.2
9	F1	43.7
10	F#1	46.2
11	G1	49.0
12	G#1	51.9
13	A1	55.0
14	A#1	58.3
15	B1	61.7
16	C2	65.4
17	C#2	69.3
18	D2	73.4
19	D#2	77.8
20	E2	82.4
21	F2	87.3
22	F#2	92.5
23	G2	98.0
24	G#2	103.8
25	A2	110.0
26	A#2	116.5
27	B2	123.5
28	C3	130.8
29	C#3	138.6
30	D3	146.8
31	D#3	155.6
32	E3	164.8
33	F3	174.6
34	F#3	185.0
35	G3	196.0
36	G#3	207.7
37	A3	220.0
38	A#3	233.1
39	B3	246.9
40	C4	261.6
41	C#4	277.2
42	D4	293.7
43	D#4	311.1
44	E4	329.6
45	F4	349.2
46	F#4	370.0
47	G4	392.0
48	G#4	415.3
49	A4	440.0
50	A#4	466.2
51	B4	493.9
52	C5	523.3
53	C#5	554.4
54	D5	587.3
55	D#5	622.3
56	E5	659.3
57	F5	698.5
58	F#5	740.0
59	G5	784.0
60	G#5	830.6
61	A5	880.0
62	A#5	932.3
63	B5	987.8
64	C6	1046.5
65	C#6	1108.7
66	D6	1174.7
67	D#6	1244.5
68	E6	1318.5
69	F6	1396.9
70	F#6	1480.0
71	G6	1568.0
72	G#6	1661.2
73	A6	1760.0
74	A#6	1864.7
75	B6	1975.5
76	C7	2093.0
77	C#7	2217.5
78	D7	2349.3
79	D#7	2489.0
80	E7	2637.0
81	F7	2793.8
82	F#7	2960.0
83	G7	3136.0
84	G#7	3322.4
85	A7	3520.0
86	A#7	3729.3
87	B7	3951.1
88	C8	4186.0

A steel guitar string has a mass density of 0.01 kg/m, and is held at a tension of 100 Newtons. What should be the effective length of string between two fixed ends to produce the note E2?

SKETCH & TRANSLATE.

SIMPLIFY & DIAGRAM

REPRESENT MATHEMATICALLY

SOLVE & EVALUATE

Poll Question

The frequency of the third harmonic of a string is

A. One-third the frequency of the fundamental.
B. Equal to the frequency of the fundamental.
C. Three times the frequency of the fundamental.
D. Nine times the frequency of the fundamental.

Sound Waves

- Your ears are able to detect sinusoidal sound waves with frequencies between about 20 Hz and 20 kHz.

- Low frequencies are perceived as “low pitch” bass notes, while high frequencies are heard as “high pitch” treble notes.
- Sound waves with frequencies above 20 kHz are called *ultrasonic* frequencies.
- Oscillators vibrating at frequencies of many MHz generate the ultrasonic waves used in ultrasound medical imaging.

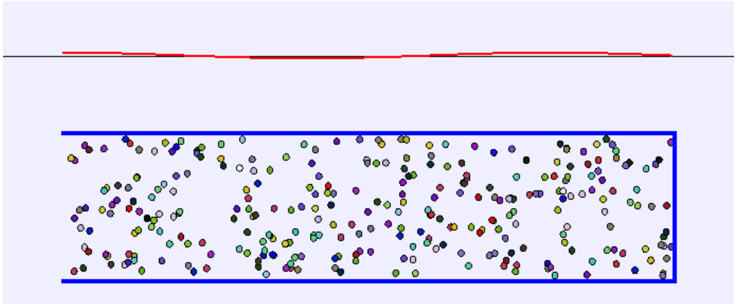
[Demonstration]

Image from http://www.weblocal.ca/uc-baby-3d-ultrasound-brampton-on.html

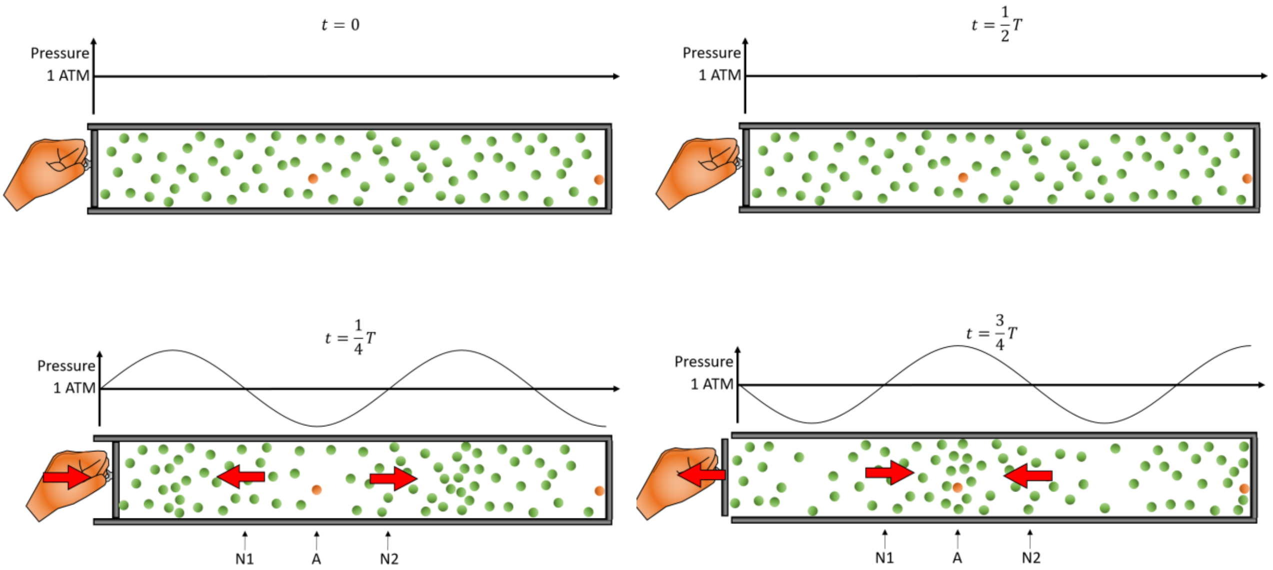
31

Standing Sound Waves

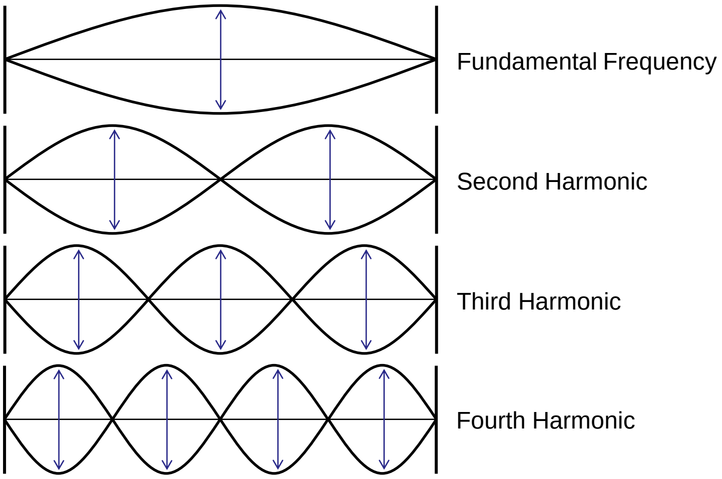
- A long, narrow column of air, such as the air in a tube or pipe, can support a longitudinal standing sound wave.
- An open end of a column of air must be a pressure node (always at ambient pressure), thus the boundary conditions—nodes at the ends—are the same as for a standing wave on a string.
- A closed end forces a pressure antinode.



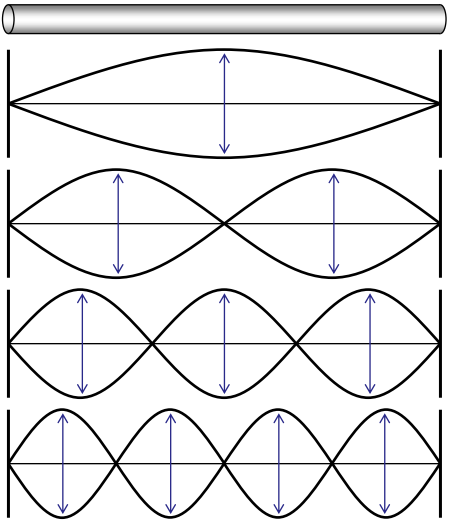
32



Harmonics On a String



Harmonics in an Open-Open Wind Instrument



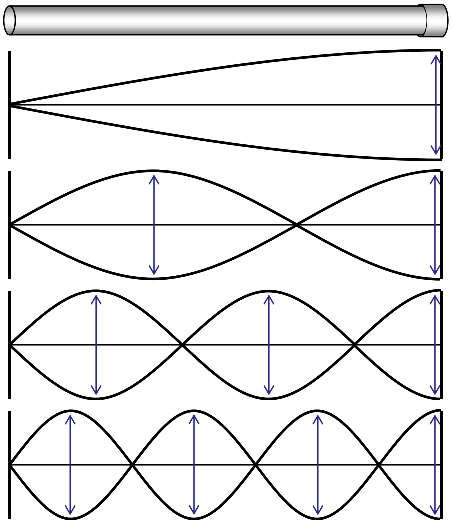
$$\lambda_{1o} = \frac{2L}{1} \qquad f_{1o}$$

$$\lambda_{2o} = \frac{2L}{2} \qquad f_{2o} = 2f_{1o}$$

$$\lambda_{3o} = \frac{2L}{3} \qquad f_{3o} = 3f_{1o}$$

$$\lambda_{4o} = \frac{2L}{4} \qquad f_{4o} = 4f_{1o}$$

Harmonics in an Open-Closed Wind Instrument



$$\lambda_{1c} = \frac{4L}{1} \qquad f_{1c} = \frac{f_{1o}}{2}$$

$$\lambda_{3c} = \frac{4L}{3} \qquad f_{3c} = 3f_{1c}$$

$$\lambda_{5c} = \frac{4L}{5} \qquad f_{5c} = 5f_{1c}$$

$$\lambda_{7c} = \frac{4L}{7} \qquad f_{7c} = 7f_{1c}$$

Musical Instruments

- With a wind instrument, blowing into the mouthpiece creates a standing sound wave inside a tube of air.
- The player changes the notes by using her fingers to cover holes or open valves, changing the length of the tube and thus its fundamental frequency:

$$f_1 = \frac{v}{2L}$$ for an open-open tube instrument, such as a flute

$$f_1 = \frac{v}{4L}$$ for an open-closed tube instrument, such as a clarinet

- In both of these equations, v is the speed of sound in the air *inside* the tube.
- Overblowing wind instruments can sometimes produce higher harmonics such as $f_2 = 2f_1$ and $f_3 = 3f_1$.

$$\begin{cases} \lambda_m = \dfrac{2L}{m} \\ f_m = m\dfrac{v}{2L} = mf_1 \end{cases} \qquad \begin{matrix} m = 1, 2, 3, 4, \ldots \\ \text{(open-open or closed-closed tube)} \end{matrix}$$

$$\begin{cases} \lambda_m = \dfrac{4L}{m} \\ f_m = m\dfrac{v}{4L} = mf_1 \end{cases} \qquad \begin{matrix} m = 1, 3, 5, 7, \ldots \\ \text{(open-closed tube)} \end{matrix}$$

A clarinet acts like an open-closed tube. A particular note being played on a clarinet has an upper harmonic with a frequency of 1310 Hz, and the next higher strong harmonic has a frequency of 1834 Hz. What is the fundamental frequency?

SKETCH & TRANSLATE.

SIMPLIFY & DIAGRAM

REPRESENT MATHEMATICALLY

SOLVE & EVALUATE

39

Before Class 35 on Wednesday

- Please finish reading Chapter 11:
- Section 11.10 on Doppler Effect

- We will also be starting a course review and we'll look at what exactly to expect on the Final Assessment on Dec.17

- Note that Wednesday is the Last Day of Classes!

- Thursday Dec. 10 is a "Make-up Day" for the missed Monday class due to Thanksgiving. I'll be here in this zoom call on Thursday with more Course Review, and some Liquid Nitrogen Demonstrations including Levitating Superconductors

40