Speaker A / B question answer. This is multiple choice question 18 of the APRIL/MAY 2008 PHY 138Y EXAMINATION – version 1, and question 3 of version 2.

QUESTION

Speakers A and B are sitting at the same large distance (many sound wavelengths) away from a stationary observer, emitting a sinusoidal wave of frequency $f = 1/T$. The waves they emit are identical except that they have a phase difference which causes the crest of the wave from Speaker B to encounter the observer a time t_0 after the crest of the wave from Speaker A. Speaker A is moved slightly further away from the observer, while Speaker B remains fixed, and the intensity of the combined sound from both speakers as heard by the observer increases at first. Which of the following is a possible value of t_0 for this situation?

Choices are:
0.3 T
0.6 T
0.7 T
0.9 T
1.57 T

ANSWER

As is done in Knight Chapter 21.6, let's model the waves as measured at the observer as:

$$D_A = a\sin(kx_A - \omega t + \phi_{0A})$$
$$D_B = a\sin(kx_B - \omega t + \phi_{0B})$$

Where x_A is the distance between the observer and Speaker A, and x_B is the distance between the observer and Speaker B. When $x_A = x_B$, the phase of $D_B(t = t_0)$ equals the phase of $D_A(t = 0)$. Therefore:

$$(kx - \omega t_0 + \phi_{0B}) = (kx - 0 + \phi_{0A})$$

solving for the phase difference:

$$\Delta\phi = \phi_{0B} - \phi_{0A} = \omega t_0$$

What happens to the phase difference $\Delta\phi$ when Speaker A is moved a small distance ε away from the observer, while Speaker B remains stationary? We will replace $x_A = x$ with $x_A = x + \varepsilon$. The new phase difference will be

$$\Delta\phi' = [kx + \phi_{0B}] - [k(x+\varepsilon) + \phi_{0A}] = \omega t_0 - k\varepsilon$$

The phase difference has *decreased* slightly from $\Delta\phi = 2\pi t_0 / T$ to something smaller.

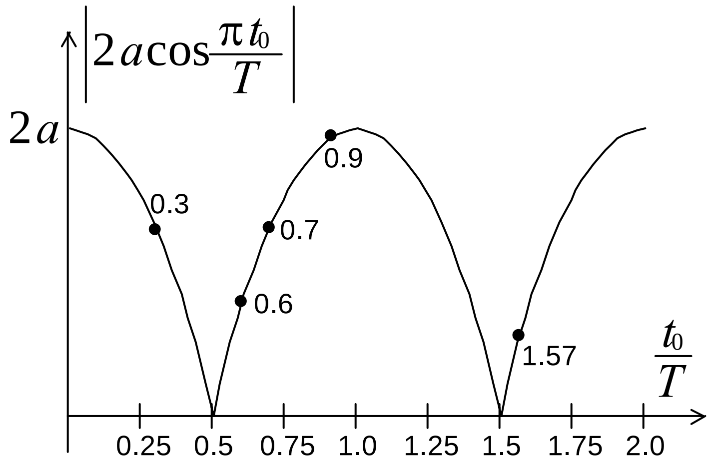
Note that the amplitude of the superposition of the two waves is:

$$A = \left| 2a\cos\left(\frac{\Delta\phi}{2}\right) \right|$$

or

$$A = \left| 2a\cos\left(\frac{\pi t_0}{T}\right) \right|$$

Here is a graph of combined wave amplitude versus t_0/T:



Phase difference increases with t_0. We are given a situation in which, when the phase difference decreases slightly, the combined sound intensity increases at first. Intensity increases with amplitude. So we must have a negative slope of amplitude versus phase difference. It is clear from the graph that of the five values given, only $t_0 = 0.3\ T$ gives a negative slope of amplitude versus phase difference.