# PHY151H1F – Experiment 4: Simple Harmonic Motion

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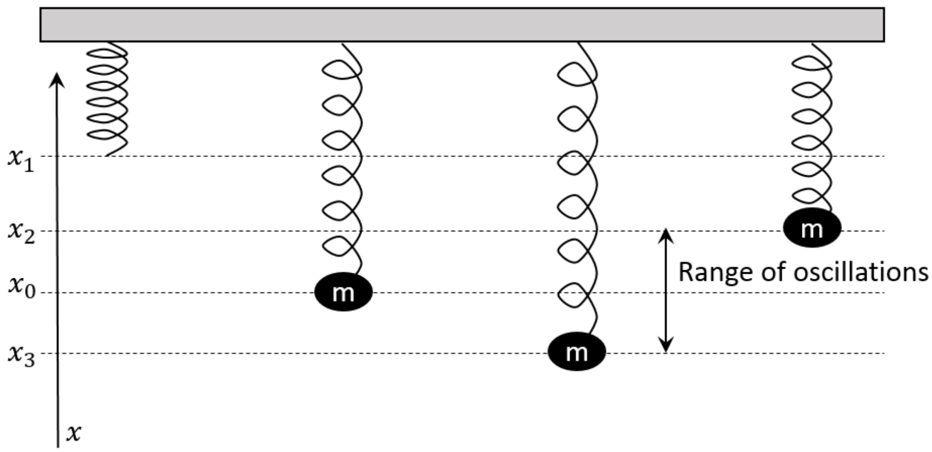
**Today's Textbook Reference for this lab:**
*"University Physics with Modern Physics"* 1st Edition by W. Bauer and G.D. Westfall ©2011
**Chapter 6** "Potential Energy and Energy Conservation"
**Section 6.6** "Work and Energy for the Spring Force"
Pg. 185: Potential Energy of an Object Hanging from a Spring.

## Preparatory Questions

Before we begin, let us familiarize ourselves with the system of a mass on a vertical spring. Begin by defining the following positions: $x_0$ is where the net force is zero; $x_1$ is where the spring force is zero; $x_2$ is the highest point that an oscillating mass reaches; and $x_3$ is the lowest point that the mass reaches. [Compare this with Fig.6.13 in your text.]




Assume that $x_1$ us higher than $x_2$ (this need not be true – why not?). List the following points:

A. Point(s) where the kinetic energy ($K$) is at a maximum; and at a minimum.
B. Point(s) where the gravitational potential energy ($U_g$) is at a maximum; and at a minimum.
C. Point(s) where the spring potential energy ($U_s$) is at a maximum; and at a minimum.

You can show with a free-body diagram and Hooke's Law that Newton's second law, with the $+x$ direction being up, is:

$$F_{\text{net}} = ma = -k(x - x_1) - mg \qquad [1]$$

where $a = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$.

D. Show that a solution to this equation is of the form:

$$x(t) = A \sin(\omega_0 t + \theta_0) + x_0 \qquad [2]$$

where $A$, $\omega_0$, $\theta_0$ and $x_0$ are constants. We call $A$ the **amplitude** of the oscillation: $A = |x_2 - x_0| = |x_3 - x_0|$.

E. Find $\omega_0$, the angular **frequency** of the oscillation, in terms of $k$ and $m$. Note the oscillation frequency in cycles per second is $f_0 = \omega_0/2\pi$.

F. Find the equilibrium height $x_0$ in terms of $x_1$, $m$, $g$ and $k$.

**The Experiment**

The main purpose of the experiment is to verify equation [2].

Suspend the spring from a stand, and position the Motion Detector under the oscillating mass. Remember that the Motion Detector will not detect motion at less than 15 cm. Short pieces of string between the spring and the stand and the spring and the mass can prevent swinging and twisting of the oscillating mass. Run DataStudio using a mass in the mid-range available and small values for the amplitude. Obtain data for 20 or so cycles of oscillation of the spring.

Study the resulting Position, Velocity, Acceleration graphs and make qualitative comments about them. What are the relative phases of these graphs? Why do they have the form they have?

Use *Zoom Select* to expand a few cycles of the Position graph and use the *Smart Tool* to obtain estimates of $x_0$, $A$ and the period. Calculate $\theta_0$ using equation [2] and the value of $x$ at $t$ = 0. Pay attention to the sign of $\theta_0$. A positive value of $\theta_0$ is equivalent to moving the origin of the axes to the right at $t = 0$. Make estimates of the errors in the quantities that you have measured.

Click on *Curve Fit* on the main toolbar (not *Fit* on the graph toolbar) to draw a curve of the form of Equation [2]. Click *New* → *Sine Fit* → *Manual*. Click on *Input* and choose the appropriate data set. Note that DataStudio writes equation [2] in an unconventional form but it is equivalent to equation [2]. Enter estimates of $x_0$, $A$, the period and C (which is derived from $\theta_0$) into the appropriate boxes and click *Accept*. Drag the appropriate *Fit Run* onto the graph of your data. You can change the colour of the *Fit* Run by double clicking on the *Fit Run* in the *Data Summary* window. Adjust the coefficients (and click *Accept*) until the fitted curve matches your data.

Does the curve which fits the data over the few cycles that you have selected fit the data many cycles away from your selected region? If not, why not?

Try the *Sine Fit* using *Automatic*. Do your measurements agree with DataStudio's calculations?

Now take measurements of Position versus time with several different masses, to obtain values of frequency, *f*. Determine the value of *f* by (a) using the *Smart Tool* and measuring the time for several oscillations or (b) counting the number of complete oscillations in a given time period or (c) alternatively, using a very quick method, the *Fast Fourier Transform* function (FFT) listed on the *Displays* window. We suggest that even if you use (a) or (b) that you try (c) to experience Fourier Transforms with the caveat that the FFT gives a small range of frequencies with better resolution as the number of points increases. We suggest that *(sample rate)* x *(number of seconds observed)* ≥1024. Drag the Position icon in the *Data* window onto the *FFT* icon in the *Displays* window. You will have to expand the scale on the x-axis to display the result clearly; once this is done you can read the frequency directly off the graph using the *Smart Tool.*

Display your data graphically either manually or using the DataStudio software as explained in *Hints* section of the Preliminary Exercise. Investigate the relation between $f$ and the mass of the oscillating object to extract the value of the spring constant, $k$. Check this value by recording the force when the spring is extended from zero by a measured amount.

Go back to your original data for $x(t)$ and $v(t)$. Use these data to plot

$$E(t) = 1/2\ m\ v^2 + 1/2\ k\ (x - x_0)^2$$

using your data, and the values of $m$ and $k$ that you now know. Is this a constant? There are two important issues. One is the small scale jitters one expects from experimental data, and the other is the long term trend associated with the inevitable non-conservative forces. Make sure you comment on both issues.