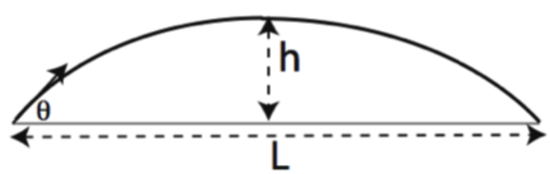
PHY151H1: MIDTERM 2016
WEDNESDAY, 26 OCT 2016, 6:15 PM-7:45 PM (90 minutes)

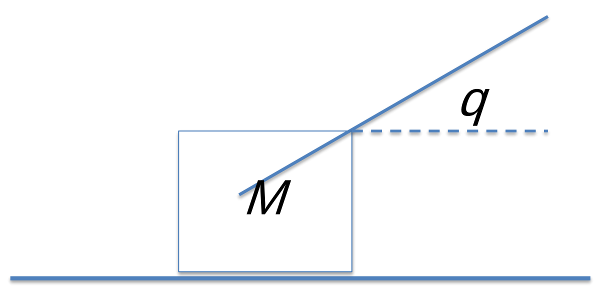
- Answer all 5 questions. Each question has equal weight
- The last page of this midterm is an aid sheet. You may use any of the formulas in the aid sheet without derivation, unless the derivation is specifically requested in the question. You can detach this sheet if you like.
- Other permitted aids: an electronic calculator, non-electronic language dictionary.
- Please include your name, student number, and your tutorial section on each exam book.
- It is recommended you write in ink without use of correction fluid or tape. If you write in pencil, or blank out things written in ink, you will NOT be able to dispute the grading after the test is returned.

1. (25 points) The acceleration of a particle restricted to motion in a straight line is given by $a(t) = 3t + (t+1)^{-3}$. Its position at t = 0s is x=0m. If its position at t =1s is $x = 0.25$m, What is the particle's velocity at t = 0s?

2. A projectile launched at an angle θ to the horizontal reaches a maximum height h and a total horizontal displacement L. Ignore air resistance. (See figure below)
   a) (15 points) Find the ratio h/L in terms of θ only.
   b) (10 points) Would the ratio h/L be larger, smaller or the same if the projectile was launched on the moon instead of on the Earth? Justify your answer.



3. A box of mass M sits on a rough horizontal surface. The coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k. The box is pulled by a massless rope as shown below.



   a) (10 points) What range of tensions can be applied to the rope without the box moving in the horizontal direction?
   b) (15 points) Once the box is moving, what tension should be applied to keep the box moving at constant speed?

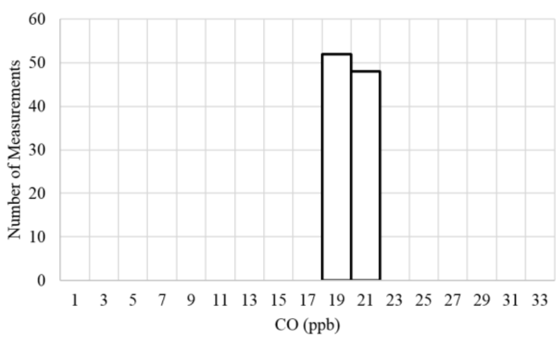
4. (a) (10 points) Use the Lorentz transforms to demonstrate that for any two events at space-time points (x_1, t_1) and (x_2, t_2) the quantity $\Delta s^2 = (x_2 - x_1)^2 - c^2(t_2 - t_1)^2$ is the

same in all inertial frames.

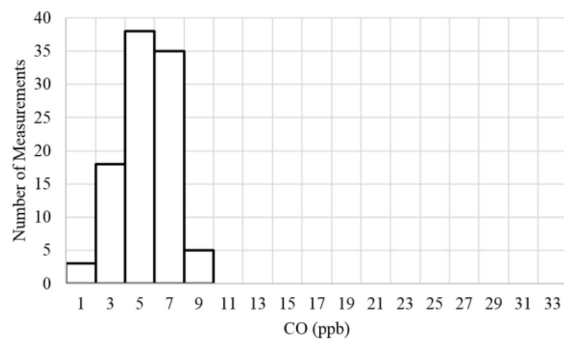
(b) (15 points) The Crab Nebula supernova explosion occurred 6000 years (1.9×10^{11} *s*) ago at a distance 5000 light years (4.7×10^{19} m) away (as measured in a reference frame moving with the earth). The volcano Paektu in Korea underwent an exceptionally violent eruption 1070 years (3.4×10^{10} *s*) ago. At what speed must an interstellar explorer have been traveling relative to earth so that the Crab Nebula explosion occurred *at the same instant* as the Paektu eruption?

5. Parts a, b and c of this problem are about uncertainty in physical measurements, and parts d and e (on the next page) are about computer programming.

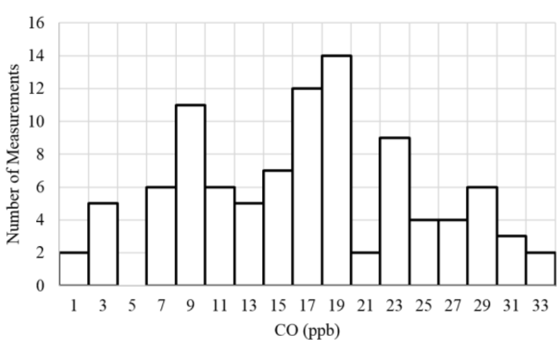
A researcher measures atmospheric carbon monoxide (CO) concentrations in ppb (parts per billion) above four different locations: Ajax, Brampton, Cobourg and Deep River. 100 measurements were made at each location, and the histogram of the four sets of measurements are shown below.



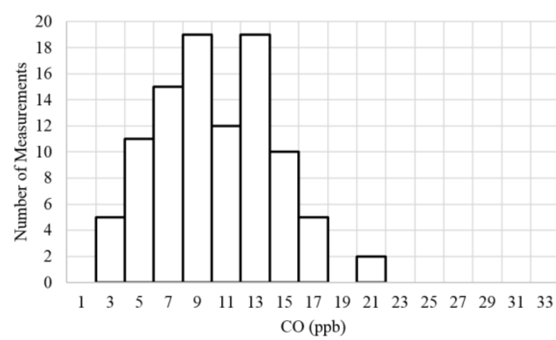
Ajax



Brampton



Cobourg



Deep River

a. (4 points) Rank, from largest to smallest, the **means** of the four distributions.
b. (4 points) Rank, from largest to smallest, the **variances** of the four distributions.
c. (4 points) At another location, the researcher makes a CO concentration measurement which reads as 31.3763 ppb, and the experimental standard uncertainty of this measurement is estimated to be 0.192 ppb. Using the style of "measurement ± uncertainty" and the correct number of significant figures in each, how should this researcher report her result?

(cont.

Carefully examine the working Python code, below, which a student has written to numerically integrate the time-dependent position of a ball dropped from rest at a height of 3.0 m above the ground.

```
1.  ay = -9.8
2.  dt = 0.1
3.  t = 0.0
4.  y = 3.0
5.  vy = 0.0
6.  while y >= 0:
7.      t = t + dt
8.      y = y + (vy*dt)
9.      vy = vy + (ay*dt)
10.     print(t, y)
```

d. (10 points) Describe briefly, in words, what you think each of the 10 lines of code are doing. Refer to each line by number, and specify the physical quantities involved, including the units of these quantities, when appropriate.

e. (3 points) When you run the program, it outputs nine pairs of numbers to the screen and then stops. What are the **first three** pairs of numbers it outputs?

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PHY151 Midterm Aid Sheet (you may detach this if you like)

<u>Derivatives & Integrals</u>: $f(t)$ and $g(t)$ are functions of time t, and b, c and n are constants,

$$f(t) = c(t-b)^n \Rightarrow \frac{df}{dt} = cn(t-b)^{n-1}$$

$$\frac{d(f+g)}{dt} = \frac{df}{dt} + \frac{dg}{dt} \qquad \frac{d(fg)}{dt} = \frac{df}{dt}g + f\frac{dg}{dt} \qquad \frac{d}{dt}f(g) = \left(\frac{df}{dg}\right)\left(\frac{dg}{dt}\right) \text{ (Chain rule)}$$

$$\int_{t_1}^{t_2} c(t-b)^n\, dt = \left[\frac{c(t-b)^{n+1}}{n+1}\right]_{t_1}^{t_2} = \frac{c(t_2-b)^{n+1}}{n+1} - \frac{c(t_1-b)^{n+1}}{n+1} \quad (n \neq -1)$$

• Kinematics:
$$v(t_f) = v(t_i) + \int_{t_i}^{t_f} a(t)\,dt; \qquad x(t_f) = x(t_i) + \int_{t_i}^{t_f} v(t)\,dt$$

for constant acceleration a_0:

$$\Delta v = \Delta t\ a_0; \quad v(t)^2 = v(t_i)^2 + 2\Delta x\ a_0 \qquad \Delta x = \Delta t\ v(t_i) + \frac{1}{2}\Delta t^2 a_0$$

projectiles: $\Delta y = \tan\theta\ \Delta x - \left(g/2v_0^2\cos^2\theta\right)\Delta x^2$

• Galilean Relativity:
$$x' = x - vt \qquad t' = t \qquad u' = u - v$$

• Special Relativity:
$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - vx/c^2\right) \qquad u' = \frac{u-v}{1 - uv/c^2} \qquad \gamma = 1\Big/\sqrt{1 - v^2/c^2}$$

• Conservation laws:
Momentum $P = \sum_{i=1}^{N} m_i v_i$; $\Delta P \equiv P_{after} - P_{before} = 0$ when no external forces are applied;

Kinetic Energy: $K = \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2$; $\Delta K = 0$ in a *perfectly elastic* collision.

For **two bodies**: $K = P^2/2(m_1+m_2) + \mu(v_1 - v_2)^2/2$, where $\mu = m_1 m_2/(m_1+m_2)$ is the reduced mass.

• Statistics and Error Analysis:
For repeated measurements $x_1, x_2, ..., x_N$,

mean (or average): $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ variance: $var = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$

standard deviation: $\sigma = \sqrt{var}$

Variance of a digital reading in which half the last digit is a: $var = a^2/3$.