Marker 1.

PHY151 H1F : Midterm 2016.

There are MANY different ways of solving this.

Solutions by J. Harlow.

Keep careful notes of partial credit.

1. $a = 3t + (t+1)^{-3}$ $x(0) = 0$, $x(1) = 0.25\ m$

Find $v(0)$.

Indefinite integral: $v = \int a\, dt + C_0$

where C_0 = an integration constant with units [m/s]

$$v = \int \left[3t + (t+1)^{-3}\right] dt + C_0$$

$$v = 3\int t\, dt + \int (t+1)^{-3}\, dt + C_0$$

$$\boxed{v = \frac{3t^2}{2} - \frac{(t+1)^{-2}}{2} + C_0} \qquad (*)$$

Indefinite integral: $x = \int v\, dt + C_1$

where C_1 = an integration constant with units [m]

$$x = \int \left[\frac{3t^2}{2} - \frac{(t+1)^{-2}}{2} + C_0\right] dt + C_1$$

$$x = \frac{3}{2}\int t^2\, dt - \frac{1}{2}\int (t+1)^{-2}\, dt + \int C_0\, dt + C_1$$

$$\boxed{x = \frac{1}{2}t^3 + \frac{(t+1)^{-1}}{2} + C_0 t + C_1}$$

Next, let's use the conditions $x(0) = 0$ and $x(1) = 0.25\ m$ to solve for C_0 and C_1.

cont....

1....continued $x(t=0) = 0 + \frac{1}{2} + 0 + C_1$

$$\Rightarrow \underline{C_1 = -0.5\ m}$$

$$x(t=1) = 0.25m = \frac{1}{2}(1)^3 + \frac{(2)^{-1}}{2} + C_0 + C_1$$

$$0.25 = 0.5 + 0.25 + C_0 - 0.5$$

$$\Rightarrow \underline{C_0 = 0\ m/s.}$$

So, (*) Becomes:

$$V = \frac{3t^2}{2} - \frac{(t+1)^{-2}}{2}$$

at $t=0$: $V_0 = 0 - \frac{(1)^{-2}}{2}$

$$\boxed{V_0 = -0.5\ m/s}$$

minus sign.

units

25 points available

→ Integrate without caring about constants → 15/25

→ Forget 1 constant but analysis is correct → 20/25

→ " units and constant → 20/25

→ integration wrong → 5/25.

→ Did everything good but made a boo-boo in the last constant → 23/25

No units

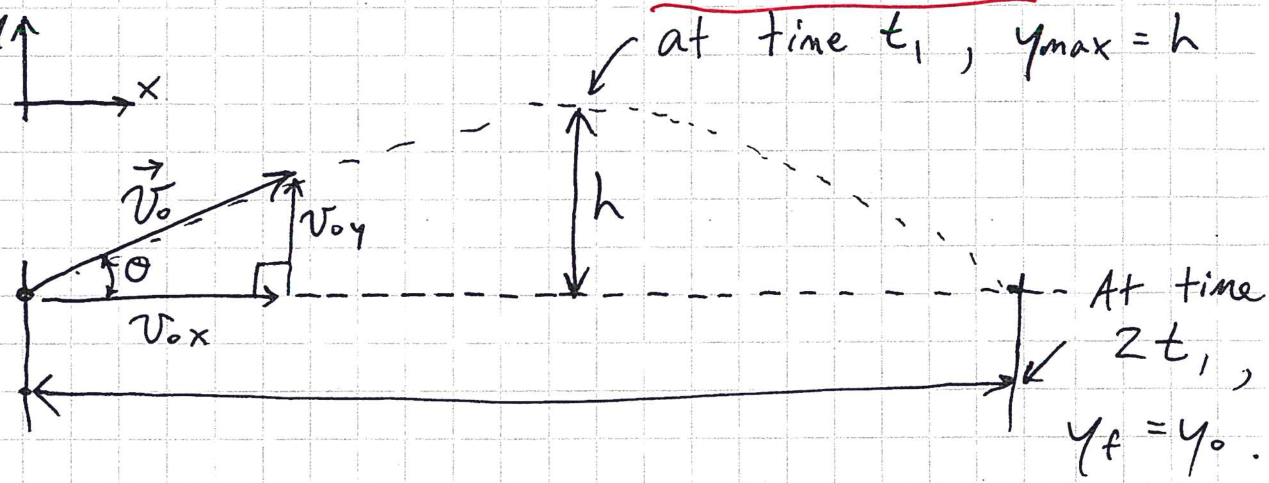
→ Made a mistake but analysis is correct → 20/25

→ wasn't careful with integration limits → 15/25

$\boxed{\text{! ! Scheme}}$

(3). Marker 2

2. (a).



<u>First half of trajectory $t=0$ to t_1:</u>

Initial y-component of velocity $= v_{oy} = v_0 \sin\theta$

Final y-component of velocity $= v_{o1} = 0$

Average y-component of velocity, assuming constant acceleration:

$$\bar{v}_y = \frac{v_{oy} + v_{o1}}{2} = \frac{v_0 \sin\theta}{2}.$$

Not many students will use this trick.

$$\bar{v}_y = \frac{\Delta y}{\Delta t} = \frac{h}{t_1}$$

$$\frac{v_0 \sin\theta}{2} = \frac{h}{t_1}$$

$$t_1 = \frac{2h}{v_0 \sin\theta} \qquad \text{(Eq. 2.1)}$$

<u>Whole trajectory $t=0$ to $2t_1$:</u>

Constant x-component of velocity $= v_x = v_0 \cos\theta$

$$v_x = \frac{\Delta x}{\Delta t} = \frac{L}{2t_1} = v_0 \cos\theta$$

$$t_1 = \frac{L}{2 v_0 \cos\theta} \qquad \text{(Eq. 2.2).}$$

<u>cont</u>....

2(a).... continued.

Combine Eqs 2.1 & 2.2:

$$t_1 = t_1$$

$$\frac{2h}{v_0 \sin\theta} = \frac{L}{2 v_0 \cos\theta}$$

$$\frac{h}{L} = \frac{\sin\theta}{4\cos\theta}$$

$$\boxed{\frac{h}{L} = \frac{1}{4}\tan\theta}$$

15 points

2 (b) Same.

10 points

Note, $h/L = \frac{1}{4}\tan\theta$ depends on initial launch angle only. It does not depend on g. So if you change g (by going to the moon), h/L remains the same!

brief, correct justification

a/ find V_y or time to reach h or h: 5

find V_x or time to reach L or L: 5

final ratio and convert to just θ: 5

5 points total if they did someth with Δy formula given in exam

2 points if they just wrote it down

2 if they did stuff, but used $\Delta y = h$, $\Delta x = L$

- good logic but bad answer: 3/5
- good answer but bad logic: 2/5
- only take point off once, if they use bad answer to get final ratio, that's fine
- good idea but bad setup: 2/5

b/ good answer: 5
good reasoning: 5 (give credit for partial answer)

- gave them 5/10 if they justified a wrong answer with the formula they got (assuming they got it so wrong that it actually supported the arg.)
  - was very rare

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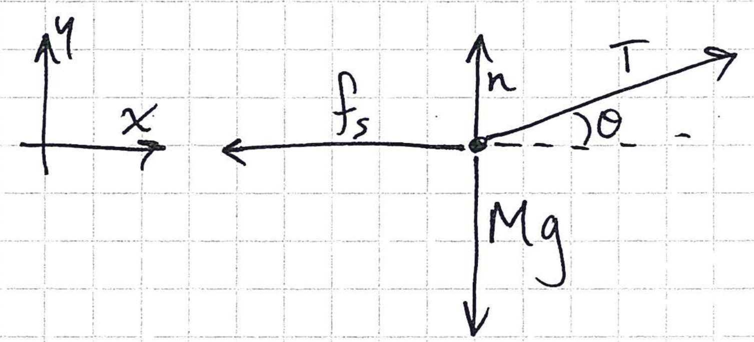
gave them 6 if they wrote formulas for h and L, then wrote down $\frac{h}{L}$

(5). Marker 3

3 (a). Free-body diagram of box:



n = normal force

If $T=0$, then the box does not move.

$\Rightarrow T_{min} = 0$.

T can be increased until $f_s = f_{s,max} = \mu_s n$

$(F_{net})_y = 0$ in equilibrium (no movement),

$\Rightarrow n + T\sin\theta - Mg = 0$, solve for n.

$$n = Mg - T\sin\theta$$

$(F_{net})_x = 0$ in equilibrium (no movement).

$$T\cos\theta - f_s = 0$$

$$\Rightarrow f_s = T\cos\theta$$

When $T = T_{max}$, $f_s = f_{s,max}$: $\mu_s\left[Mg - T_{max}\sin\theta\right] = T_{max}\cos\theta$

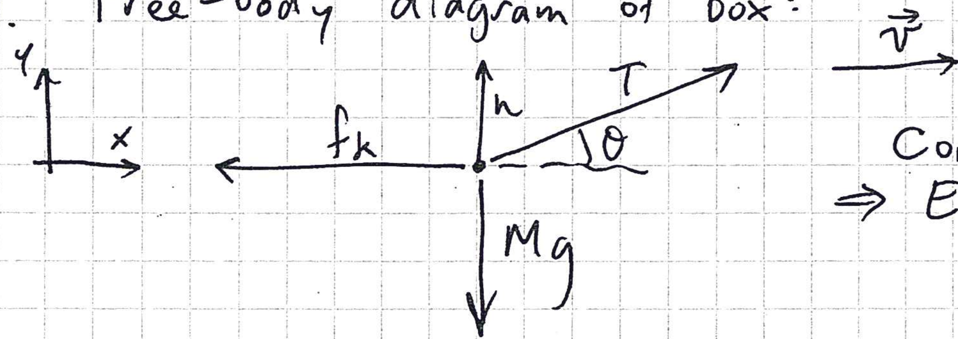
solve for T_{max}:

$$T_{max}(\mu_s \sin\theta + \cos\theta) = \mu_s Mg$$

$$T_{max} = \frac{\mu_s Mg}{\mu_s \sin\theta + \cos\theta}$$

Range: $\boxed{0 < T < \frac{\mu_s Mg}{\mu_s \sin\theta + \cos\theta}}$ 10 points

3 (b). Free-body diagram of box:



Constant speed
⇒ Equilibrium.
(zero acceleration.

Math is similar to Part (a).

$(F_{net})_y = 0 \Rightarrow \underline{n = Mg - T\sin\theta}$

$(F_{net})_x = 0 \Rightarrow f_k = T\cos\theta$, $f_k = \mu_k n$

$$\mu_k(Mg - T\sin\theta) = T\cos\theta,$$ solve for T

$$\boxed{T = \frac{\mu_k Mg}{\mu_k \sin\theta + \cos\theta}}$$ ← apply this tension to keep a steady speed.

15 points

Question 3.

$\cos\theta \to \sin\theta$ -5 (for the whole Question)

$+ \to -$ -1

result (small mistake) -2

not get a final result -4

include $\underset{\sim}{I}$ -3

$\left.\begin{matrix} -1 \\ -2 \\ -4 \\ -3 \end{matrix}\right\}$ each

if take $\underline{N = mg}$ get 6 at most.

part (a). if the final result not include $\underline{T > 0}$ -1

⑦

Marker 4

4 (a). Define inertial frame S. Two events in this frame have coordinates (x_1, t_1), (x_2, t_2).

if $x_2 = \gamma(x_1 - vt_1)$

Define $\Delta x = x_2 - x_1$

$\Delta t = t_2 - t_1$

$$\Delta s^2 = \Delta x^2 - c^2 \Delta t^2 \qquad \text{(Eq. 4.1)}$$

Define inertial frame S', moving in the $+x$ direction at speed v, relative to S.

Lorentz Transform the two events:

$$x_1' = \gamma(x_1 - vt_1) \qquad x_2' = \gamma(x_2 - vt_2)$$

if + instead of − 8

$$t_1' = \gamma\left(t_1 - \frac{vx_1}{c^2}\right) \qquad t_2' = \gamma\left(t_2 - \frac{vx_2}{c^2}\right)$$

if nearly → 2 or

if and 5 / 10

In Frame S':

if sub wrong 3

$$(\Delta s^2)' = (x_2' - x_1')^2 - c^2(t_2' - t_1')^2$$

$$(\Delta s^2)' = \gamma^2\left[(x_2 - vt_2) - (x_1 - vt_1)\right]^2 - c^2\gamma^2\left[\left(t_2 - \frac{vx_2}{c^2}\right) - \left(t_1 - \frac{vx_1}{c^2}\right)\right]^2$$

$$(\Delta s^2)' = \gamma^2\left[x_2 - x_1 - v(t_2 - t_1)\right]^2 - c^2\gamma^2\left[t_2 - t_1 - \frac{v}{c^2}(x_2 - x_1)\right]^2$$

$$(\Delta s^2)' = \gamma^2(\Delta x - v\Delta t)^2 - c^2\gamma^2\left(\Delta t - \frac{v}{c^2}\Delta x\right)^2$$

$$(\Delta s^2)' = \gamma^2(\Delta x^2 - 2v\Delta x\Delta t + v^2\Delta t^2) - c^2\gamma^2\left(\Delta t^2 - \frac{2v\Delta x\Delta t}{c^2} + \frac{v^2}{c^4}\Delta x^2\right)$$

↓ 6

$v = \beta c$ " β/c >1?

.... cont

4 (a) ... continued:

$$(\Delta s^2)' = \gamma^2 \left[\Delta x^2 - \cancel{2v\Delta x \Delta t} + v^2 \Delta t^2 - c^2 \Delta t^2 + \cancel{2v\Delta x \Delta t} - \frac{v^2}{c^2} \Delta x^2 \right]$$

$$(\Delta s^2)' = \left(\frac{1}{\sqrt{1 - v^2/c^2}} \right)^2 \left[\left(1 - \frac{v^2}{c^2}\right) \Delta x^2 + (v^2 - c^2) \Delta t^2 \right]$$

$$(\Delta s^2)' = \frac{1}{(1 - v^2/c^2)} \left[\left(1 - v^2/c^2\right) \Delta x^2 - \left(1 - v^2/c^2\right) c^2 \Delta t^2 \right]$$

$$(\Delta s^2)' = \Delta x^2 - c^2 \Delta t^2$$

Compare with Eq. 4.1:

$$(\Delta s^2)' = \Delta s^2$$ ← same in both frames. ■

4 (a): 10 points. small algebra mistake → 8

4 (b). Define present day Mount Paektu to be the origin $(x, t) = (0, 0)$.

Define "Event 1" to be the Crab Nebula Supernova Explosion.

Let's use units of ly (light years) for distance, and units of yr (years) for time.

In these units, $c = 1$.

$$(x_1, t_1) = (5000, -6000).$$

Define "Event 2" to be the volcanic eruption at Mt. Paektu.

$$(x_2, t_2) = (0, -1070)$$

We wish to find the speed v that a moving frame S' travels such that

$$t_1' = t_2'$$

cont...

Marker 5

4 (a) ... continued:

$$(\Delta s^2)' = \gamma^2 \left[\Delta x^2 - \cancel{2v\Delta x \Delta t} + v^2 \Delta t^2 - c^2 \Delta t^2 + \cancel{2v\Delta x \Delta t} - \frac{v^2}{c^2} \Delta x^2 \right]$$

$$(\Delta s^2)' = \left(\frac{1}{\sqrt{1 - v^2/c^2}} \right)^2 \left[\left(1 - \frac{v^2}{c^2}\right) \Delta x^2 + (v^2 - c^2) \Delta t^2 \right]$$

$$(\Delta s^2)' = \frac{1}{(1 - v^2/c^2)} \left[\left(1 - \frac{v^2}{c^2}\right) \Delta x^2 - \left(1 - \frac{v^2}{c^2}\right) c^2 \Delta t^2 \right]$$

$$(\Delta s^2)' = \Delta x^2 - c^2 \Delta t^2$$

Compare with Eq. 4.1:

$(\Delta s^2)' = \Delta s^2$ ← same in both frames.

4 (b). Define present day Mount Paektu to be the origin $(x, t) = (0, 0)$.

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Define "Event 2" to be the volcanic eruption at Mt. Paektu.

$$(x_2, t_2) = (0, -1070)$$

We wish to find the speed v that a moving frame S' travels such that

$$t_1' = t_2'$$

cont...

4 (b) continued.. Lorentz Transform the time coordinates:

$$t_1' = \gamma\left(t_1 - \frac{v x_1}{c^2}\right) = \frac{t_1 - v x_1}{\sqrt{1 - v^2}} \qquad (\text{since } c = 1)$$

Similarly: $$t_2' = \frac{t_2 - v x_2}{\sqrt{1 - v^2}}$$

For the events to be simultaneous in this frame:

$$t_1' = t_2'$$

$$\frac{t_1 - v x_1}{\sqrt{1 - v^2}} = \frac{t_2 - v x_2}{\sqrt{1 - v^2}}$$

$$t_1 - v x_1 = t_2 - v x_2$$

$$v(x_2 - x_1) = t_2 - t_1$$

$$v = \frac{t_2 - t_1}{x_2 - x_1}$$ ← [Note: this equation looks really weird! But it's okay because c = 1 ...

$$v = \frac{-1070 - (-6000)}{0 - 5000} = \frac{-4930}{5000} = -0.986$$

So the speed is 0.986 light years per year:

$$\boxed{v = 0.986\,c}\,, \quad \text{or} \quad v = 2.96 \times 10^8 \, m/s$$

4 (b) ← 15 points.

PHY151 H1F Midterm (10) Oct. 26, 2016

Uncertainty & Python Solution. Marker 6.

5. (a) $\bar{x}A : \sim 20$

$\bar{x}B : \sim 6$

$\bar{x}C : 15$

$\bar{x}D : 10$

: $\boxed{A > C > D > B}$ (4)

(b) By inspection $\boxed{C > D > B > A}$ (4)

(c) Recall from UM Module 2:

"The number of significant figures in an experimental uncertainty is 1 or 2, but never greater than this.".

5abc: 12 points So: $\boxed{31.4 \pm 0.2}$ (4)

or: 31.38 ± 0.19

] either answer is correct.

(d) 1. Set acceleration due to gravity to be $-9.8\ m/s$

2. Set time-step of numerical integration to be 0.1 seconds.

3. Set the initial time to be 0 s.

4. Set the initial height of the ball to be 3 m

5. Set the initial y-component of the velocity of the ball to be 0 m/s.

6. Start a loop in which $y \geq 0$.

7. Increment t by dt.

... cont

PHY151 H1F Midterm ⑩ Oct. 26, 2016

Uncertainty & Python Solution. Marker 7.

5.(a) $\bar{x}$ A : ~20

$\bar{x}$ B : ~6

$\bar{x}$ C : 15

$\bar{x}$ D : 10

: $\boxed{A > C > D > B}$

(b) By inspection $\boxed{C > D > B > A}$

(c) Recall from UM Module 2:

"The number of significant figures in an experimental uncertainty is 1 or 2, but never greater than this."

So : $\boxed{31.4 \pm 0.2}$

or : 31.38 ± 0.19

either answer is correct.

(d) 1. Set acceleration due to gravity to be -9.8 m/s

2. Set time-step of numerical integration to be 0.1 seconds. -½ marks if didn't explain what dt variable represent

3. Set the initial time to be 0 s.

4. Set the initial height of the ball to be 3 m

5. Set the initial y-component of the velocity of the ball to be 0 m/s.

took off 1 mark if units not mentioned (or values)

6. Start a loop in which $y \geq 0$ - ½ marks if didn't mention loop ~~[illegible]~~ (just stated 'while y-position ≥ 0')

7. Increment t by dt.

... cont

continued

5. (d) 8. Add $(v_y \cdot dt)$ to (11) the position.

9. Add $(a_y \cdot dt)$ to the y-comp. of the velocity.

10. Print t, y to the screen.

10 points.

(e) (0.1, 3.0)
(0.2, 2.902)
(0.3, 2.706)

3 points.

½ marks if one or the other is ok

5de: 13 points