# PHY151 Practice Problem Solutions Week 7

## Question 1

(a) $F_x = ax - bx^3$ and $F_x = -\frac{dU}{dx}$, so $U(x)$ can be found by integrating:

$$\int \frac{dU}{dx} dx = -\int (ax - bx^3) dx \tag{1}$$

$$U(x) = -(\frac{1}{2}ax^2 - \frac{1}{4}bx^4) + C \tag{2}$$

Set $U(0) = 0$:

$$0 = -(\frac{1}{2}a(0)^2 - \frac{1}{4}b(0)^4) + C \tag{3}$$

$$C = 0 \tag{4}$$

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \tag{5}$$

(b) For an exact solution, use the quadratic formula to solve for $x^2$ when $U(x)$ = -1 J:

$$0 = \frac{1}{4}bx^4 - \frac{1}{2}ax^2 + 1 \tag{6}$$

$$x^2 = a/2 \pm 2\sqrt{a^2/4 - b}/b \tag{7}$$

Plugging in values for a and b, the positive solutions are x = 0.66 m and x = 2.1 m.

## Question 2

Centripetal acceleration is caused by some combination of the normal and gravitational forces on the car throughout the loop. At the top of the loop, gravitational centripetal acceleration is at a maximum, while the velocity (and corresponding required centripetal acceleration) of the car is at a minimum, so if the car can make it past the top of the loop, it can safely travel the rest.

The minimum top-of-loop velocity is given by $a_c = g = v^2/R$. ($F_N = 0$ N). This yields $v = \sqrt{Rg}$. This gives kinetic energy $E_k = mRg/2$. The total energy at the top of the loop:

$$E_{tot} = E_k + E_g = mRg/2 + mg(2R) = \frac{5}{2}mRg \tag{8}$$

This can be equated to the initial energy in the spring to find the minimum compression distance:

$$kx^2/2 = \frac{5}{2}mRg \tag{9}$$

$$x = \sqrt{5mRg/k} \tag{10}$$

Plugging in values gives $x = 2.9$ m.

## Question 3

The initial energy in the system is given by $E_{tot} = kx^2/2$. This energy is converted into kinetic energy when the block is launched. To stop the block, work equal to the kinetic energy must be done on the block by the frictional force:

$$\Delta E_k = W \tag{11}$$

$$-kx^2/2 = \vec{F_f} \cdot \vec{\Delta d} \tag{12}$$

$$-kx^2/2 = -\mu_k mg\Delta d \tag{13}$$

$$\Delta d = \frac{kx^2}{2\mu_k mg} \tag{14}$$

Plugging in values gives $\Delta d = 1.4$ m.