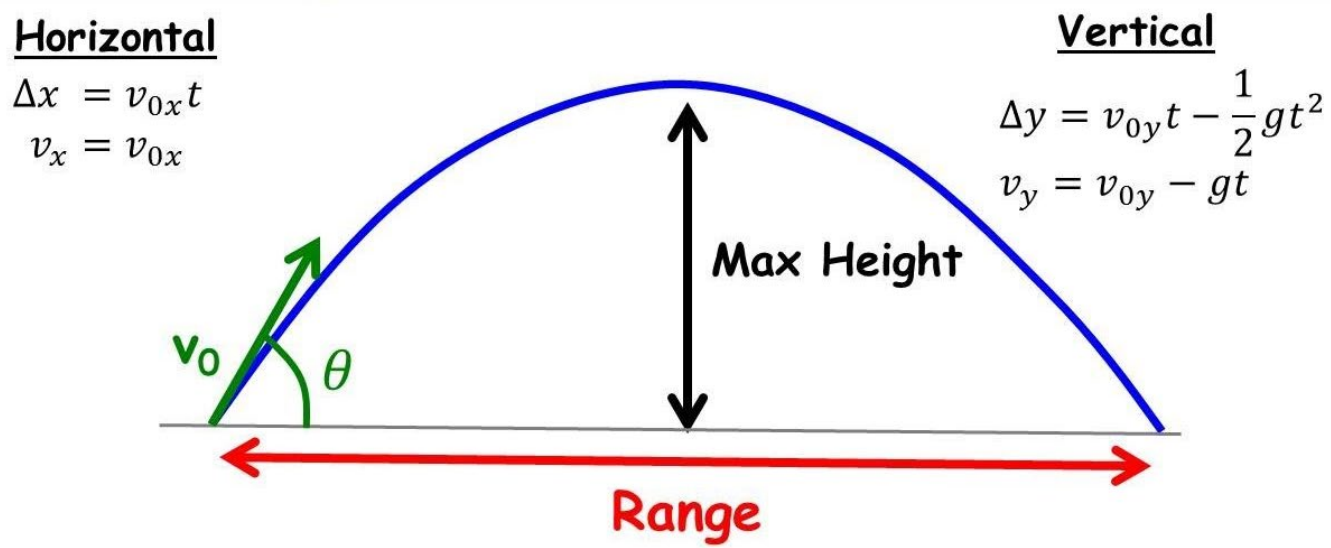
# PHY151H1F Practicals 5 Intro Video Slides

- When analyzing projectiles in an introductory physics course, we are often asked to "neglect air resistance".
- When air resistance is neglected, the acceleration of the projectile is a constant value throughout its motion, so the differential equations describing the motion can be solved **analytically**. In other words, you can write down a simple formula describing the $x$ and $y$ positions as functions of time, completely describing the trajectory.



# Numerical Integration

- Sometimes in physics it is difficult or impossible to write down an analytical solution to a differential equation you are trying to solve.
- For example, projectile motion with air resistance. In this case, the drag force of air on the projectile is a function of velocity, so the acceleration of the projectile is not constant.
- However, if we choose a small enough time interval, $\Delta t$, the acceleration is almost constant over that interval.
- We can estimate the trajectory by stepping through the motion of the projectile in steps of $\Delta t$.
- This can take many thousands of calculations in order to get a good answer.
- Computers are great at doing thousands of calculations!

# Freefall

- Even though an analytical solution exists, this week you will start by writing a Python code to numerically integrate freefall *without* air resistance.
- We will look at how to add air resistance afterwards.
- This week, let's consider the following question:

> If a small, steel ball is dropped from rest from a height of 20 m, how long does it take for it to hit the ground? Neglect air resistance. Use g = 9.80 m/s$^2$.

- You should be able to figure this out without a Python code.

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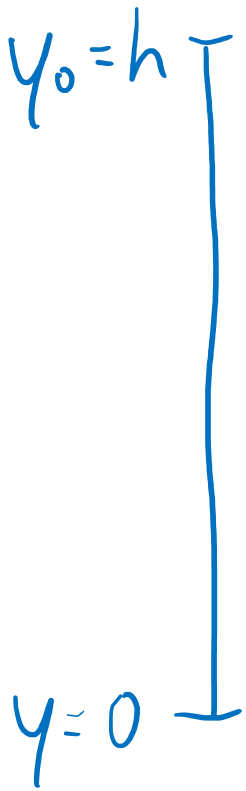


$v_{0y} = 0$ , $a_y = -g = -9.8\ m/s^2$

Use : $y = y_0 + \cancelto{0}{v_{0y}}\, t + \frac{1}{2} a_y t^2$

$$0 = h - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 = h$$

$$t^2 = \frac{2h}{g}$$ , choose + solution

$$t = \sqrt{2h/g}$$

# "Constant" Acceleration

- Let's assume that we choose a small enough time interval $\Delta t$ so that the acceleration is effectively constant.
- When acceleration is constant, you can write it in terms of change in velocity and time interval:
- $a_y = \dfrac{\Delta v}{\Delta t}$
- You can then rearrange this equation for the change in velocity:
- $\Delta v = a_y \Delta t$
- which can be used to write the velocity at time $t + \Delta t$ in terms of the velocity at time $t$ :
- $v(t+\Delta t) = v(t) + a_y\Delta t$

# “Constant” Velocity

- Next, let’s assume that, furthermore, we have a small enough time interval $\Delta t$ so that the velocity is effectively constant.
- When velocity is constant, you can write it in terms of displacement and time interval:
- $v = \frac{\Delta y}{\Delta t}$
- You can then rearrange this equation for the displacement:
- $\Delta y = v\Delta t$
- which can be used to write the position at time $t + \Delta t$ in terms of the position at time $t$ :
- $y(t+\Delta t) = \cancel{x}y(t) + v\Delta t$

# Numerical Integration "Brute Force" Method

- Strategy:
- First, set up the initial conditions in your code, and the time-step:
- $y_0 = 20$ m
- $t_0 = 0$ s
- $v_0 = 0$ m/s
- $\Delta t = 0.01$ s
- Next, you make a **while statement** which will start the code on a loop, which ends when your while statement is satisfied.
- You wish to find the time when the ball hits the ground, so make it while $y > 0$ m.

# While Loop

- Before the loop, set: $y = y_0$ , $t = t_0$ , $v = v_0$.
- While ($y > 0$):
  - Update $t$ to become $t + \Delta t$.
  - Knowing position and velocity, compute acceleration, $a_y$. (In our case this is constant)
  - Update $v$ to become $v + a_y\Delta t$
  - Update $y$ to become $y + v\Delta t$
  - Output current $t$ and $v$ and $y$, if you wish.

- When the while loop is all done, output the final value of $t$. This will be when the ball hits the ground.

# Terminal Velocity

- Due to air resistance, many objects which fall straight down eventually reach a state in which the upward drag force from the air cancels the downward force of gravity.
- If this is the case, then the net force on the falling object is zero, meaning the acceleration is zero, meaning the velocity is constant
- This is called terminal velocity, $v_{term}$.
- Since the acceleration starts at $a_y = -g$, and approaches $a_y = 0$ as the object speeds up, one crude way of estimating the acceleration is:
- $a_y = -g\left(1 - \frac{|v|}{v_{term}}\right)$