

## PHY152H1S – Practical 6: Oscillations

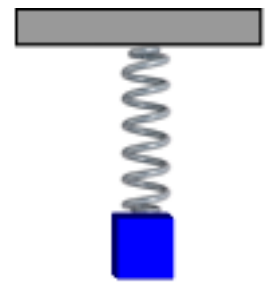
### Don't forget:

- List the NAMES of all participants on the first page of each day's write-up. Note if any participants arrived late or left early.
- Put the DATE (including year!) at the top of every page in your notebook.
- NUMBER the pages in your notebook, in case you need to refer back to previous work.

Note that the activities below have numbers which refer to numbers in the Oscillations Module at <http://faraday.physics.utoronto.ca/Practicals/>.

### Activity 4 (30 Minutes)

Mount the supplied spring on the support and hang the supplied 200 g mass from it. Gently lift the mass a small amount and release it, so that it bobs up and down. The oscillations should be small enough so that the motion is smooth and the spring is always vertical and stretched. Position the Motion Sensor under the mass and pointing up at it, as shown. You will connect the Motion Sensor to the U of T DAQ Device Digital Channel 0.



Be aware that the Motion Sensor has a switch which adjusts the width of the beam; one of the switch settings will probably result in better data than the other. Recall that the Motion Sensor can only measure distances greater than 0.15 m. This means that for the given coordinate system, if the Motion Sensor is at  $x = 0$  the bottom of the mass must always have a value of  $x > 0.15\text{m}$ .



- Set the vertical position of the mass-spring position so that when the mass is oscillating the minimum distance from the Motion Sensor is close to but greater than 0.15 m. Try to have the mass moving only up and down.
- Set the Motion Sensor for the wide beam (stick man).
- After starting the *MotionSensor* software (Located in the Labview Shortcuts folder on the Desktop), choose the Setup/Calibrate tab and set the sample rate to about 60 samples per second. You may wish to try a different sampling rate to see if it improves the quality of your data.
- Collect data for a few oscillations. “Start Detector” and “Collect Data” both act as toggle switches. The sequence for collecting data is:
  1. Click **Start Detector** to start the clicking sounds.
  2. Click **Collect Data** to refresh the screen and start plotting distance measurements.
  3. Click **Collect Data** again to stop plotting distance measurements.
  4. Click **Start Detector** again to stop the clicking sounds.

Now the Activities:

- A. Does the motion appear to be Simple Harmonic? What are the period,  $T$ , and amplitude,  $A$ , of the motion? Include uncertainty estimates in your numbers.
- B. From  $T$  and  $A$  please estimate:
  - $f$
  - $\omega$
  - $v_{\max}$
- C. The mass,  $m$ , of the hanging mass should be stamped on it (200 g), or you can measure it using the digital scale in the room. From this and your measurement of  $\omega$ , compute the spring constant of the spring,  $k$ .
- D. If you hang 200 g at equilibrium, the spring stretches to a certain length. If you connect a 200 g mass to the 200 g mass, and allow these 300 g to come to equilibrium, the spring will be a greater length. From these two equilibrium measurements, compute the spring constant  $k$  using Hooke's Law  $F_s = k(\Delta x)$ . Does this agree with your measurement from part C?

### Activity 10 (60 minutes)

In Section 14.2 of your text, the pendulum is solved using a small-angle approximation:  $\sin \theta \approx \theta$ . Figure 14.11 shows the error incurred using the small-angle approximation, which grows large when  $\theta$  is more than about  $30^\circ$ . In Activity 10 below, we wish to explore a real pendulum for which the small-angle approximation is not valid. We concentrate on the how the period of the pendulum varies with the maximum amplitude.

Newton's Laws for a simple pendulum of length  $L$  gives us a differential equation (eq.14.12 from your text):

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \sin \theta = 0 \quad (1)$$

The solution using the small-angle approximation is simple harmonic motion, with an approximate period:

$$T_0 = 2\pi \sqrt{\frac{L}{g}} \quad (2)$$

With considerable effort, from Eqn (1) one can determine that the period of the pendulum with a maximum amplitude of  $\theta_{\max}$  is given by:

$$T = \frac{\sqrt{2}}{\pi} T_0 \int_0^{\theta_{\max}} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_{\max}}} \quad (3)$$

This is not terribly helpful, since the integral involves *elliptical integrals* which are not analytically solvable, so one must either numerically approximate to find the value of  $T$  or look up the integral in some table. These sorts of integrals are so common that considerable effort has been expended to develop computer algorithms that are fast and accurate. CASIO publishes one algorithm online at <http://keisan.casio.com/exec/system/1224598524>. This was used to give some values of  $T/T_0$  in Table 1.

**Table 1**

$\theta_{\max}$ (degrees)	$T/T_0$
5	1.00048
10	1.00191
20	1.00767
30	1.01741
45	1.03997
60	1.07318
75	1.11896
90	1.18034
180	$\infty$

Here you will side-step trying to solve Eqn (3) by taking data on how the period of a real physical pendulum varies with its maximum amplitude. You will use a Pasco Rotary Motion Sensor, the U of T Data Acquisition Device (DAQ), and the *RMS* program. The *RMS* program not in the Labview Shortcuts folder, unfortunately. To find it open the Computer folder, then click on “public (\\Feynman)”, “Modules”, “Oscillations”. There are three files called RMS there, but one of them is an Application with a size of about 2.6MB. Click on the RMS application.

The Rotary Motion Sensor is mounted on a support, and has a rod with a mass on it mounted on it. The rod and mass will be the physical pendulum you will study. Also part of the rotating system are the plastic disc on which the rod is mounted and the axel connected to the Rotary Motion Sensor. Connect the Rotary Motion Sensor to the U of T DAQ Device Digital Channel 0.

## Setup

When you start *RMS* the position of the Rotary Motion Sensor defines the angle to be zero. Make sure that the pendulum is stationary and start *RMS*.

Occasionally the hardware and software gets confused about the resolution of the measurement.

1. Rotate the pendulum by one full rotation. The angle on the graph should read  $360^\circ$  either a positive or negative number.
2. If the angle is not correct, disconnect the Rotary Motion Sensor from the DAQ the plug it back in.
3. Stop and restart the *RMS* program and check that one full rotation reads  $360^\circ$ .
4. Stop *RMS*.

## Trial Run

It is a good idea to do a trial run first. You should do a run with a maximum amplitude  $\theta_{\max} \approx 1 \text{ radian} \approx 60^\circ$ .

1. Make sure the pendulum is stationary.
2. Start *RMS*.
3. Rotate the pendulum to  $60^\circ$ . You can read the angle on the graph.
4. Release the pendulum. The graph will show the angle of the pendulum as a function of time.
5. After about 10 oscillations stop *RMS*.
6. You should notice two vertical black lines near the left side of the graph. These are cursor 0 and cursor 1. Drag the cursor 0 to one of the early oscillations. The horizontal line of the cursor will track the data.
7. Drag the cursor 1 to one of the later oscillations.
8. At the bottom of the graph the  $\Delta x$  field is the time between the two cursors in seconds.  $y_1$  and  $y_2$  are the angles of the first and second cursors respectively, in radians, and  $\Delta y$  is the change in the angle.

Thus you may calculate the value of the period and the maximum amplitude.

## Data Collection and Analysis

Collect data for the period for at least 5 values of the maximum amplitude. Your goal will be to compare with Table 1.

Use *CreateDataSet* to create a dataset of values for the period and maximum amplitude, including their errors, and save it into your Team's area. You will want the values of the maximum amplitude to be the *Independent (x) Variable*, and the values of the period be the *Dependent (y) Variable*.

You can then use *ViewDataSet* to view your data. You may wish to print the window and staple it into your lab book.

Does the data look reasonable? Does it appear to be consistent with the values of Table 1?

## If You Have Time

You have determined the period of the pendulum for measured values of the maximum amplitude. Here we explore how to use that data to determine the value of the period for maximum amplitudes that have not been measured.

Recall the relation between the period and the maximum amplitude is a complicated function involving elliptical integrals:

$$T = f(\theta_{\max}) = \frac{2}{\pi} T_0 \int_0^{\theta_{\max}} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_{\max}}} \quad (3)$$

However, it might be reasonable to approximate the function as a polynomial:

$$T = a(0) + a(1)\theta_{\max} + a(2)\theta_{\max}^2 + a(3)\theta_{\max}^3 + a(4)\theta_{\max}^4 + \dots \quad (5)$$

- A. Can you eliminate the odd coefficients  $a(1)$ ,  $a(3)$ , etc. from the series using a physical argument? If yes, what argument can you use?
- B. Use the *PolynomialFit* program to find the best fit the data. Do not include any terms that are physically unreasonable. You will find that adding some terms to the polynomial will not improve the quality of the fit: your goal is to find the minimum number of terms in the polynomial that provides a good fit to the data. Any fitted parameters  $a(i)$  that are zero within errors should not be included in the fit.
- C. From the best fit, what is the value of  $T_0$ ?
- D. How well does the best fit do in duplicating the values of Table 1? Does it do a better job for small angles than for large ones?

This Student Guide was originally written by David M. Harrison, Dept. of Physics, Univ. of Toronto in May 2008  
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