Solution to PHY 152 Practice Problem Set 2

1.a. By symmetry, many of the electric forces due to individual charges on the central charge cancel. Nonzero contributions come from the charge $q \equiv +5\,\mathrm{nC}$ at the middle of the left boundary, and the charges q and $2q$ at the lower left and upper right corner respectively. Let $r = 10\,\mathrm{cm}$, the net electric field at the center is

$$\begin{aligned}\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\hat{\mathbf{x}} + \frac{(q-2q)}{2r^2}\frac{1}{\sqrt{2}}(\hat{\mathbf{x}}+\hat{\mathbf{y}})\right) \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\left((1-\frac{\sqrt{2}}{4})\hat{\mathbf{x}} - \frac{\sqrt{2}}{4}\hat{\mathbf{y}}\right)\end{aligned}$$

The net electric force on the central charge is then

$$\begin{aligned}\mathbf{F} &= 2q\mathbf{E} \\ &= 2.9\times 10^{-5}\,\mathrm{N}\,\hat{\mathbf{x}} - 1.6\times 10^{-5}\,\mathrm{N}\,\hat{\mathbf{y}}\end{aligned}$$

2.a. First, it is obvious that $-q$ experiences no net electric force. By symmetry, the electric force on one of the two $+Q$'s is zero if and only if the other is zero. Therefore, we can just focus on one $+Q$ and calculate its value such that the electric field it sees is zero. Denote the distance between $-q$ and $+Q$ by r,

$$\frac{1}{4\pi\epsilon_0}\left(\frac{-q}{r^2} + \frac{Q}{(2r)^2}\right) = 0$$

which yields $Q = 4q$.

2.b. For a point on the y-axis above all three charges, the electric field is

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(y-\sqrt{3}a/2)^2} + \frac{2q}{y^2+(a/2)^2}\frac{y}{\sqrt{y^2+(a/2)^2}}\right)$$

pointing in y-direction. When $y \gg a$, this expression reduces to

$$\begin{aligned}E &\approx \frac{1}{4\pi\epsilon_0}\left(\frac{q}{y^2} + \frac{2q}{y^2}\frac{y}{\sqrt{y^2}}\right) \\ &= \frac{1}{4\pi\epsilon_0}\frac{3q}{y^2}\end{aligned}$$

which is the electric field of a point charge $3q$.

3. Consider the situation when there is only one balloon. It stays at vertical ($\theta = 0$) when there is no net external force. The balloon floats because buoyance force is greater than its weight, i.e. it experiences a net force pointing upward,

$$\begin{aligned}F_{\mathrm{up}} &= F_{\mathrm{b}} - W \\ &= \rho_{\mathrm{air}}Vg - \rho_{\mathrm{He}}Vg\end{aligned}$$

where V is the volume of the balloon and we have used Archimedes' principle. When you try to tilt the balloon from equilibrium by angle θ, it experiences a restoring force $F_R = F_{\mathrm{up}}\sin\theta$ perpendicular to the direction of string. In the case when θ is small, $\sin\theta \approx \theta$ and the direction of the F_R is almost horizontal. Suppose now that we have two balloons as shown in the figure. Since they carry identical charge, the repulsive electric force F_E tries to push them apart, while F_R tries to pull them inward. At equilibrium, assuming θ is small,

$$\begin{aligned}F_E &\approx F_R \\ \frac{1}{4\pi\epsilon_0}\frac{Q^2}{(2L\sin\theta)^2} &\approx F_{\mathrm{up}}\sin\theta \\ \theta^3 &\approx \frac{1}{4\pi\epsilon_0}\frac{Q^2}{4L^2(\rho_{\mathrm{air}}-\rho_{\mathrm{He}})Vg} \\ \theta &\approx \left(\frac{1}{4\pi\epsilon_0}\frac{3Q^2}{16L^2(\rho_{\mathrm{air}}-\rho_{\mathrm{He}})\pi r^3 g}\right)^{1/3}\end{aligned}$$