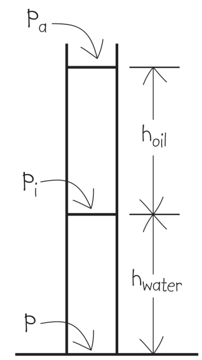
PHY152 – Practice Problem Set #1 Solutions

Winter 2018

1. Wolfson 15.26

We have an open tube filled with water on the bottom and oil on the top of water. The pressure pushing down on the oil at the top of the tube is the atmospheric pressure, p_{atm}, (see figure below).



The absolute pressure at the oil-water interface, p_i, is equal to

$p_i = p_{atm} + \rho_{oil} g h_{oil}$.

and the pressure at the bottom of tube (or the water), p, is equal to

$p = p_i + \rho_{water} g h_{water} = p_{atm} + \rho_{oil} g h_{oil} + \rho_{water} g h_{water}$.

Therefore, the gauge pressure at the bottom is

$$\begin{aligned}\Delta p &= p - p_{atm} = (\rho_{oil} h_{oil} + \rho_{water} h_{water}) g \\ &= [(0.82\text{x}10^3 \text{ kg/m}^3)(0.05 \text{ m}) + (1.00\text{x}10^3 \text{ kg/m}^3)(0.05 \text{ m})](9.8 \text{ m/s}^2) \\ &= 890 \text{ hPa.}\end{aligned}$$

2. Wolfson 15.48

Archimedes' principle for floating objects tells us that the weight of the beaker equals the weight of water displaced by one-third of its volume, so the maximum weight of rocks the beaker can carry and still float is equal to the weight of water displaced by two-thirds of the beaker's volume. This is

$n m_R g \leq \frac{2}{3} V \rho_w g$,

where m_R is the mass of each rock (= 12 g) and n is the number of rocks.

Solving for n gives,

$$n \leq \frac{2V\rho_w}{3m_R} = \frac{2\pi(2.5 \text{ cm})^2(14 \text{ cm})(1.0 \text{ g/cm}^3)}{3(12 \text{ g})} = 15.3.$$

Therefore, 15 is the maximum number of rocks that can be placed in the beaker before sinking it.

3. Wolfson 15.33

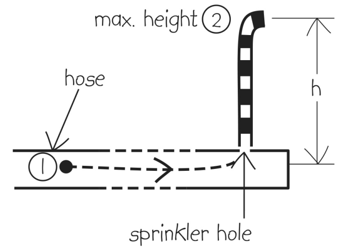
Using the principle of conservation of mass, we can find the speed of fluid flow in the narrow section given the flow speed in the wide section (v_1 = 1.8 m/s) and the diameters of each section (d_1 = 2.5 cm and d_2 = 2.0 cm). Also, we know the continuity equation for an incompressible liquid is vA = constant.

For this pipe, this is $v_1 A_1 = v_2 A_2$. So, solving for v_2, the flow speed in the narrow section, we find

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{\pi {d_1}^2}{\pi {d_2}^2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = (1.8 \text{ m/s})\left(\frac{2.5 \text{ cm}}{2.0 \text{ cm}}\right)^2 = 2.8 \text{ m/s}.$$

As expected, the speed increases in the narrow section.

4. Wolfson 15.55



We can use Bernoulli's equation allows us to find the maximum height reached by the water coming out from the hose. This is a case where making a sketch will help (see figure at right).

The flow of water in the hose can be described by

$p + \frac{1}{2}\rho v^2 + \rho g y =$ constant.

So, at point 1, the pressure, velocity, and height of the water in the hose are

$p_1 = p_{atm} + \Delta p_1 = p_{atm} + 140$ kPa, $v_1 \approx 0$, and $y_1 = 0$.

At the highest point attained by the jet of water emerging from the hole (point 2), $p_2 = p_{atm}$, $v_2 \approx 0$, and $y_2 = h$.

By equating the result of Bernoulli's equation at points 1 and 2, we can find h as follows

$$p_1 = p_2 + \rho g h$$

$$p_{atm} + \Delta p_1 = p_{atm} + \rho g h$$

$$h = \frac{\Delta p_1}{\rho g} = \frac{140 \text{ kPa}}{(1.0\text{x}10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 14 \text{ m.}$$

At the maximum height, all the work done by pressure has been converted to potential energy of the fluid. Energy is conserved in the process (this ignores dissipative forces such as air resistance).