PHY152 PRACTICE PROBLEMs FOR WEEK 10 PRACTICALS

Q1 Consider a light source emitting a light beam horizontally with wavelength $\lambda = 400.0$nm, and the light goes along the horizontal direction through the center of a diffraction grating of length 2.00 cm with 10,000 evenly distributed (or spaced) slits oriented perpendicular to the light beam. A screen is set 10.0 cm away from the grating and also oriented perpendicular to the light beam. What is the distance between central maximum and 1st order maximum on the screen?

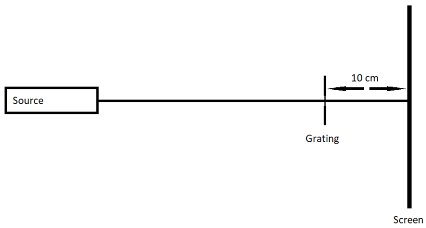

FIGURE 1. Figure for Q1

Q1 solution: The spacing of the grating is given by

$$d = 0.02/10000 = 2.0 \times 10^{-6}\text{m}$$

The angle θ between the central maximum and 1st order maximum is given by

$$d \sin(\theta) = m\lambda$$

with d = 2.00×10^{6} m, m = 1, $\lambda = 4.000 \times 10^{-7}$ m. plug in the numbers to give $\theta = 11.5\,\text{deg}$, and the distance between the central and 1st order maximum on the screen will be $10 \times \tan(\theta) = 2.04$cm.

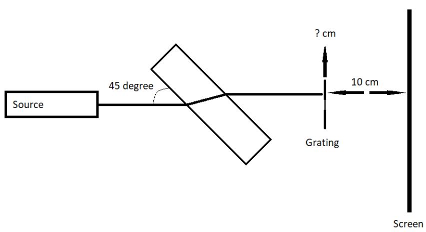

FIGURE 2. Figure for Q2

Q2 Now a block made of transparent material A with $h = 2$ cm thickness is placed between the light source and the grating with an angle of 45 degree with respect to the light's direction. The material A is known to have a index of refraction of 1.2. How far do you need to move the grating in the vertical so the light beam is centred on the grating again? Treat index of refraction of air to be 1 here.

Q2 solution: From the position of the block we know the incident angle θ_I would be 45 degree, and the refracted angle would be

$$\sin(\theta_I) = 1.2 \sin(\theta_F)$$

which gives $\theta_F = 36.1\,\text{deg}$. Therefore the position of the light path on the surface of the block where the light gets back into air would differ from its original path by

$$h(\tan(\theta_I) - \tan(\theta_F)) = 0.54\text{cm}$$

which converts to a displacement in the vertical direction by a factor of sin(45), which gives 0.38 cm, as the distance the grating needs to move (in the upward direction).

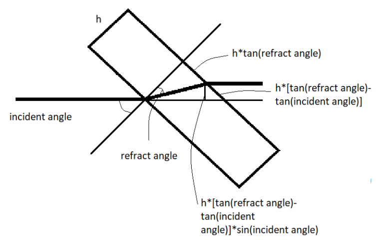

FIGURE 3. Figure for Q2 solution

Q3 The placement of the block has made the experiment space dusty, so instead of air now a tank of transparent liquid B is placed between the light source and the grating, with the block at the same position. B is known to have an index of refraction of 1.8. How far does the grating need to shift in the vertical direction from its position in Q1 so the light beam is centred on the grating again? Here assume the tank will have no effect on the light's path since light going in/out of it is normal to the tank's surface.

Q3 solution: Note here that the critical angle between A and B is

$$\theta_c = \arcsin(n_A/n_B) = \arcsin(1.2/1.8) \approx 41.8\,\text{deg}$$

which is smaller than 45 degrees. Therefore there will be no refraction into A and total reflection between the B/A surface. Therefore, there is no vertical position that the grating can be placed in for the light to be centred on it. The light will not get there.

Q4 Now a new light source with a different wavelength is used, which effectively increases the index of refraction for A and B by 0.3 each. Where should the grating be shifted vertically from its position in Q1 now in order for the light beam to be centred on the grating again?

Q4 solution: Using equations from Q3, The new indices would give a critical angle of

$$\arcsin(1.5/2.1) = 45.6\,\text{deg}$$

which is larger than the incident angle. Therefore the angle of refraction would be

$$\arcsin[\sin(45) \times 2.1/1.5] = 81.9\,\text{deg}$$

and using the same equation in Q2 will give the displacement where the light gets travels from the block to air approximately -12.0 cm. And since the direction of the block is unchanged, the $\sin(45)$ factor will remain the same and give a -4.24 cm shift in the vertical direction, which will be the distance the grating needs to be shifted.