Practice problem set 4

1 Chapter 21, Problem 74

A solid sphere of radius R carries a uniform volume charge density ρ. A hole of radius $R/2$ occupies a region from the center to the edge of the sphere, as shown in Fig. 21.38. Show that the electric field everywhere in the hole points horizontally and has magnitude $\rho R/6\epsilon_0$. *Hint:* Treat the hole as a superposition of two charged spheres with opposite charges.

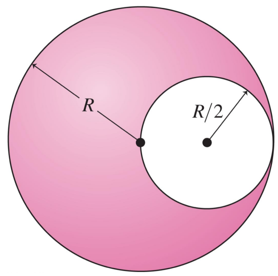

Figure 1: Charge configuration for problem 1. The solid sphere of radius R is shown in pink; also visible is the spherical hole of radius $R/2$ extending from the center of the sphere to its edge.

2 Flatland

Coulomb's law is a consequence of Gauss' law, one of the four fundamental laws of electromagnetism. In this problem, we will use Gauss' law to think about electric fields in a hypothetical two-dimensional (2D) space.

(a) Does Coulomb's law take the same form in 2D space? Use Gauss' law to find the electric field due to a point charge q existing in 2D space.

(b) Use Gauss' law to find the electric field due to an infinite, thin rod with uniform linear charge density λ existing in 2D space.

(c) Repeat part **(b)**, this time using your result from part **(a)** and integrating over the charged rod. You may find it useful to know that

$$\frac{\mathrm{d}}{\mathrm{d}u}\arctan u = \frac{1}{1+u^2}. \tag{1}$$

3 Gauss' Law for Gravitation

An analogy can be made between electric fields and graviational fields, which both obey r^{-2} laws:

$$\vec{E} = \frac{kq}{r^2}\hat{r}, \qquad \vec{g} = -\frac{GM}{r^2}\hat{r}. \tag{2}$$

In particular, just as we did for electric fields, we could explore gravitational field lines, gravitational flux through surfaces, and so on.

(a) Based solely on comparison between the two laws appearing in Eq. 2, deduce the gravitational analogue of Gauss' law. Remember that Gauss' law for electric fields is

$$\oint_S \vec{E}\cdot \mathrm{d}\vec{A} = \frac{1}{\epsilon_0} q_{\text{enclosed}}, \tag{3}$$

where S is a closed surface, and $k = 1/(4\pi\epsilon_0)$.

(b) Take the Earth to be a ball of uniform mass density ρ. What is the gravitational field as a function of radius within the ball?