Practical Problem Set 3: Solutions

January 28, 2018

20.65

Method 1: one can use Coulomb's law to calculate the force by charge Q on each charge ±q of the dipole, then calculate the net force and the net torque.

Method 2: (a) Use

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}, \tag{1}$$

where **E** is the electric field generated by charge Q,

$$\mathbf{E} = k\frac{Q}{x^2}\hat{\mathbf{x}}, \tag{2}$$

here $\hat{\mathbf{x}}$ is towards the right in the figure, and **p** is the dipole moment,

$$\mathbf{p} = 2aq\hat{\mathbf{y}}, \tag{3}$$

here $\hat{\mathbf{y}}$ is upwards (the direction of the dipole) in the figure. Thus, the magnitude of the net torque is

$$\tau = k\frac{2Qqa}{x^2} \tag{4}$$

whose direction is pointing inward the page.

(b) Instead of calculating the forces from charge Q on each charge of the dipole and then calculating the net force by adding two force vectors, one can first calculate the force by the dipole on the charge Q. Since the electric field by the dipole is already known (textbook, p.364), the force exerted on charge Q can be directly calculated without any composition of forces, and then the force from the charge on the dipole can be obtained based on Newton's third law.
Using the limit $x >> a$, the electric field generated by the dipole at the location of Q is (textbook, p.364):

$$\mathbf{E}' = -\frac{kp}{x^3}\hat{\mathbf{y}} \tag{5}$$

Then, the electric force will be

$$\mathbf{F}' = \mathbf{E}'Q = -\frac{kpQ}{x^3}\hat{\mathbf{y}} \tag{6}$$

Due to Newton's third law, the net force on the dipole by charge Q will be

$$\mathbf{F} = -\mathbf{F}' = \frac{kpQ}{x^3}\hat{\mathbf{y}} = \frac{2kaQq}{x^3}\hat{\mathbf{y}} \tag{7}$$

(c) upward. The direction can be understood as the the electric field by the dipole on charge Q is downwards, so as the electric force. Then Newton's third law leads to the upward force on the dipole.

20.73
(a)The area of such a ring can be calculated in this way,

$$\begin{aligned} dA =& A(r+dr) - A(r) \\ =& \pi(r + d^2r) - \pi r^2 \\ =& 2\pi r\ dr + \pi\ dr^2 \\ \approx& 2\pi r\ dr \end{aligned} \tag{8}$$

(b) the charge on an infinitesimal ring is

$$dq = dA \cdot \sigma = 2\pi\sigma r\ dr \tag{9}$$

(c) Following Example 20.6, the infinitesimal electric field dE of this ring at point on the disk axis is

$$\begin{aligned} dE =& \frac{kx\ dq}{(x^2+r^2)^{3/2}} \\ =& \frac{2\pi kx\sigma r\ dr}{(x^2+r^2)^{3/2}} \end{aligned} \tag{10}$$

Note that a positive x yields a positive dE, which means the electric field generated by this small ring will be in the direction of $\hat{\mathbf{x}}$ on the positive $x-$axis. On the negative $x-$axis, a negative x yields a negative dE which means the electric field will be in the direction of - $\hat{\mathbf{x}}$.

(d)The magnitude of the net electric field on the axis can be calculated by adding all the contributions from such small rings. Let say we consider the field along the positive

$x-$axis, then we know all the dE will be positive as $x > 0$ in eq(10). The magnitude of the net electric field on the axis will be

$$\begin{aligned}
E =& \int_0^R dE \\
=& 2\pi kx\sigma \int_0^R \frac{r\ dr}{(x^2+r^2)^{3/2}} \\
=& \pi kx\sigma \int_0^R \frac{d(r^2)}{(x^2+r^2)^{3/2}} \\
=& \pi kx\sigma \int_0^{R^2} \frac{dt}{(x^2+t)^{3/2}} \\
=& -2\pi kx\sigma \left[(x^2+t)^{-1/2}\right]\Big|_0^{R^2} \\
=& 2\pi k\sigma \left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]
\end{aligned} \tag{11}$$

eq(11) is also true for the negative $x-$axis, while the negative value indicates the direction of the electric field will be along opposite direction of $\hat{\mathbf{x}}$.

20.79 the electric field strength at $x = -L$ can be expressed as

$$\begin{aligned}
E =& \int_0^L dE \\
=& \int_0^L \frac{k\ dq}{r^2}
\end{aligned} \tag{12}$$

Note that $dq = \lambda dx$ and $r = x + L$ $(x > 0)$,

$$
\begin{aligned}
E =& \int_0^L dE \\
=& \int_0^L \frac{k\lambda dx}{(x+L)^2} \\
=& \frac{k\lambda_0}{L^2} \int_0^L \frac{x^2 dx}{(x+L)^2} \\
=& \frac{k\lambda_0}{L^2} \int_0^L \frac{(x+L)^2 - 2L(x+L) + L^2}{(x+L)^2} dx \\
=& \frac{k\lambda_0}{L^2} \int_0^L \left[1 - 2\frac{L}{x+L} + \frac{L^2}{(x+L)^2}\right] dx \\
=& \frac{k\lambda_0}{L^2} \left\{ x\Big|_0^L - 2Lln(x+L)\Big|_0^L - \frac{L^2}{x+L}\Big|_0^L \right\} \\
=& \frac{k\lambda_0}{L}(\frac{3}{2} - 2ln2)
\end{aligned}
\tag{13}
$$

and the direction is along the - x-axis.