1. Wolfson 14.59. Note: You will need to use the tension within a spring here. Make sure you understand where it comes from via Hooke's Law and symmetry.

   SOLUTION:

   We use equation 14.6 from Wolfson: $v = \sqrt{F/\mu}$. To use this, we must find the tension in the spring. If a spring is stretched, then the two fixtures at either end reactively apply (via Newton's Third Law) a force $|\vec{F}| = k\Delta L$, where k is Hooke's constant, and ΔL is the difference in length from the spring's equilibrium length. Since every point has net force zero along the spring, and both ends have the same tension, then every point inside must also have this same tension, else equilibrium would be broken. So, we have:

$$v = \sqrt{\frac{k\Delta L}{\mu}} = \sqrt{\frac{kL\Delta L}{m}} \tag{1}$$

   Now, say that $L_2 = 2L$, $v_2 = 3v$ and that L_0 is the equilibrium length. We then have that:

$$\frac{v_2}{v} = 3 = \frac{\sqrt{\frac{kL_2\Delta L_2}{m}}}{\sqrt{\frac{kL\Delta L}{m}}} = \sqrt{\frac{2L(2L-L_0)}{L(L-L_0)}} = \sqrt{\frac{2(2L-L_0)}{(L-L_0)}} \tag{2}$$

   We can solve this for L_0 in terms of L:

$$3 = \sqrt{\frac{2(2L-L_0)}{L-L_0}} \tag{3}$$

$$9(L-L_0) = 4L - 2L_0 \tag{4}$$

$$L_0 = \frac{5}{7}L \tag{5}$$

2. Consider a flute, which we will assume we can model as an air column with both ends open.

   (a) With all holes covered, the flute has an effective length of 65cm. What is the fundamental frequency of the flute in this state?

(b) Suppose the flutist wants to play a G note (roughly 392 Hz). Describe how they could do this, and calculate the effective change in length.

(c) Now, suppose that the flutist is playing along with an organ, which can be viewed as a half-open tube. The organ player is also playing a G note. Are the effective lengths the same?

(d) Finally, the flutist wishes to match to the organ's third overtone. Some flutists are able to play a note by putting more relative weight on an overtone, done by using a technique called overblowing, which involves the manipulation of the supplied air. In this instance, might the flutist achieve their goal by doing this, or will they be forced to change their fingering? Explain.

SOLUTION:

(a) For the doubly-open air column, we have that the fundamental frequency corresponds to $\lambda = 2L$. Thus:

$$f_1 = \frac{v}{2L} = \frac{343\text{m/s}}{2 * 0.65\text{cm}} = 264Hz. \tag{6}$$

(b) By removing some fingers from holes, the closed length of tube that makes up the flute can be shortened. In this case, we have a length of:

$$L = \frac{343\text{m/s}}{2 * 392\text{Hz}} = 0.4375\text{m} \tag{7}$$

resulting in a change in length of about 21 cm.

(c) The effective lengths are not the same, due to the organ functioning as a half-open pipe, and the flute as a fully open pipe:

$$f = \frac{v}{4L_{\text{organ pipe}}} = \frac{v}{2L_{flute}} \implies L_{\text{flute}} = 2L_{\text{organ pipe}} \tag{8}$$

(d) The third overtone is the fourth harmonic (the first being the fundamental frequency). We can discuss things more generally here. First note that, using part c:

$$f_{m,\text{organ}} = \frac{(2m-1)v}{4L_{\text{organ}}} = \frac{(2m-1)v}{2L_{\text{flute}}}, \quad m = 1, 2, 3, ... \tag{9}$$

and we also have

$$f_{n,\text{flute}} = \frac{nv}{2L_{\text{organ}}}, \quad n = 1, 2, 3, ... \tag{10}$$

So we can see that the organ's harmonics above G each coincide with a harmonic of the flute. Though, the flute has harmonics that the organ lacks. So to answer the original question, yes, the flutist could use their overblowing technique to sync with one of the organ's overtones.

3. A police car passes by you. As it approaches, the siren wails at a pitch of 1062 Hz, and after passing you, at a pitch of 945 Hz. What is the frequency emitted by the siren, and what is the speed of the car?

   SOLUTION:

   Using the Doppler equation for moving source (both before and after the car passes) and equating the emitted frequencey, we have that:

$$f = f_1\prime(1 + \frac{u}{v}) = f_2\prime(1 - \frac{u}{v}) \tag{11}$$

$$f_1\prime + f_1\prime\frac{u}{v} = f_2\prime - f_2\prime\frac{u}{v} \tag{12}$$

$$(f_1\prime + f_2\prime)\frac{u}{v} = f_2\prime - f_1\prime \tag{13}$$

$$u = v\frac{f_2\prime - f_1\prime}{f_1\prime + f_2\prime} \tag{14}$$

$$\tag{15}$$

   Plugging in the numbers and solving for u:

$$u = v\frac{f_2\prime - f_1\prime}{f_1\prime + f_2\prime} = (343\text{m/s})\frac{1062 - 945}{1062 + 945} = 20\text{m/s} \tag{16}$$

   Now, we can plug this back into one of the Doppler equations to solve for f:

$$f = f_1\prime(1 + \frac{u}{v}) = 945(1 + \frac{20}{343}) = 1000Hz \tag{17}$$