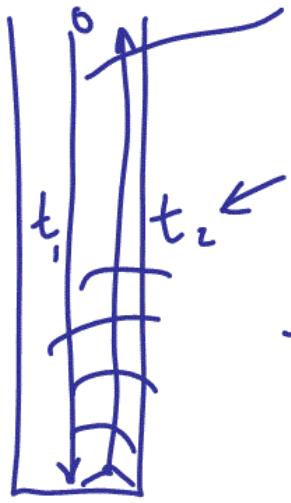
Example: How deep is the well? [Advanced version]

You drop a rock into a dark well, and you hear the sound of it hitting the bottom exactly 2.50 seconds later. The speed of sound in the well is 343 m/s. How deep is the well?



$v_0 = 0$, $a = 9.8\ m/s^2$

sound travels upward at $v_s = 343\ m/s$.

Two time intervals sum to 2.5 s:

$$t_1 + t_2 = 2.50\ s \quad (1)$$

Rock in freefall: ($v_0 = 0$)

$$d = v_0 t_1 + \frac{1}{2} a t_1^2$$

$$d = \frac{1}{2} g t_1^2 \quad (2)$$

Sound traveling up:

$$v_s = \frac{d}{t_2} \quad (3)$$

3 equations, 3 unknowns: t_1, d, t_2

Strategy: eliminate t_1 and t_2, solve for d.

Use (2) to solve for t_1

Use (3) to solve for t_2

→ Plug these both into (1), solve for d.

(2) $\Rightarrow \frac{2d}{g} = t_1^2 \Rightarrow t_1 = \pm\sqrt{\frac{2d}{g}}$ (only + solution makes physical sense)

(3) $\Rightarrow t_2 = \frac{d}{v_s}$

(1) $\Rightarrow t_1 + t_2 = 2.5\ s$

$$\pm\sqrt{\frac{2d}{g}} + \frac{d}{v_s} = 2.5$$ Solve for d.

$$\pm\sqrt{\frac{2d}{g}} = 2.5 - \frac{d}{v_s}$$ square both sides

$$\frac{2d}{g} = \left(2.5 - \frac{d}{v_s}\right)^2$$

$$\frac{2d}{g} = 2.5^2 - 2(2.5)\frac{d}{v_s} + \frac{d^2}{v_s^2}$$

$$\frac{d^2}{v_s^2} - \frac{5d}{v_s} - \frac{2d}{g} + 2.5^2 = 0$$

$$\left(\frac{1}{v_s^2}\right)d^2 - \left(\frac{5}{v_s} + \frac{2}{g}\right)d + 2.5^2 = 0$$

Quadratic: Set $a = \frac{1}{v_s^2} = \frac{1}{343^2} = 8.4999 \times 10^{-6}$

$$b = -\left(\frac{5}{v_s} + \frac{2}{g}\right) = -\left(\frac{5}{343} + \frac{2}{9.8}\right)$$

$$b = -0.21866$$

$$c = 2.5^2 = 6.25$$

$$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$d = 25.96$ m ~~or~~ or 28.615 m

which one?

(2) $\Rightarrow t_1 = \pm\sqrt{\frac{2d}{g}} = \pm 2.417$ or ± 2.4166

(3) $\Rightarrow t_2 = \frac{d}{v_s} = 74.914$ or 0.083426

$= 2.5$ if t_1 is negative → unphysical

$= 2.5$ if t_1 is positive → correct.

$$\boxed{d = 28.6 \text{ m}}$$