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Quantum self-induced transparency in frequency gap media

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Abstract. – We study quantum effects of light propagation through an extended absorbing system of two-level atoms placed within a frequency gap medium (FGM). Apart from ordinary solitons and single-particle impurity band states, the many-particle spectrum of the system is shown to contain massive pairs of confined gap excitations and their bound complexes—gap solitons. Quantum gap solitons propagate without dissipation, and should be associated with self-induced transparency pulses in a FGM.

The self-induced transparency (SIT) pulses, predicted and observed in the pioneering work of McCall and Hahn [1], may be regarded as solitons of the Maxwell-Bloch model [2], describing classical radiation propagating in a single direction and coupled to an extended system of two-level atoms. The model is completely integrable [3] and the time evolution of an arbitrary radiation incident on an atomic system is described [4, 5] by the inverse scattering method [6].

In the case of a high-intensity pulse in ordinary vacuum, quantum corrections are negligibly small. Therefore the quantum version of the classical model—quantum Maxwell-Bloch (QMB) model—has been studied [7] only in the context of the superfluorescence phenomenon where quantum effects play a crucial role [8]. But the situation is drastically changed for frequency gap media (FGM), such as a frequency-dispersive medium [9], a photonic bandgap (PBG) material [10, 11], and a one-dimensional Bragg reflector [12], where classical, linear wave propagation inside a frequency gap is excluded [13, 14].

In this letter, we demonstrate the existence of nonclassical light propagation through an extended homogeneous [15] system of two-level atoms placed within a FGM. These light pulses are highly correlated quantum many-body states and are distinct from single-photon hopping conductivity [16] through the photonic impurity band created by the atoms. Because of a nonlocal polariton-atom coupling, an extension of the Bethe ansatz method [17] from the case of a single atom embedded in FGM [13, 14] to the case of an extended many-atom system requires a thorough analysis. The QMB model generalized to the case of FGM exhibits hidden integrability [13], provided that the characteristic times of the interatomic resonance dipole-dipole interaction (RDDI) and other collisional dephasing effects are much longer than

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the light pulse duration. Integrability of the model allows us to describe the time evolution of an arbitrary light pulse incident on the system in terms of the allowed soliton modes. Making use of the Bethe ansatz technique, we derive the Bethe ansatz equations (BAE), which completely determine the spectrum of the radiation plus medium plus atoms system. Here we consider the case of the atomic transition frequency ω_{12} lying deep inside a frequency gap of a frequency dispersive medium, for which the McCall-Hahn theory is inapplicable.

Unlike an attractive effective photon-photon coupling in empty space (or nondispersive media) [7] caused by scattering of photons on an atomic system, an effective polariton-polariton coupling in FGM is shown to be attractive only for polaritons of the lower polariton branch. Therefore bound many-polariton complexes (ordinary solitons) can be constructed only from polaritons of the lower branch. In the limit of a macroscopically large number of polaritons, these quantum complexes are nothing but SIT pulses (2π-pulses) of the classical theory slightly modified due to a nonlinear polariton dispersion.

Due to the existence of a frequency gap, the multiparticle spectrum of the system, apart from polaritons and ordinary solitons, also contains massive pairs of confined gap excitations, which do not exist out of pairs, and bound complexes of these pairs—quantum gap solitons. The energy-momentum dispersion relations for gap solitons are derived and the spatial sizes and the velocities of propagation inside the atomic system are evaluated as functions of the number of pairs and the atomic density.

In contrast to quantum gap solitons generated by a single atom, which propagate along a radial coordinate centered at the atom [14], quantum SIT pulses in a doped FGM propagate in a direction defined by a single wave vector. Furthermore, the gap SIT pulse, consisting of an even number of gap excitations, is distinct from (odd photon number) gap soliton hopping conduction inside the RDDI-mediated impurity band.

In the dipole, rotating-wave approximation [5] the Hamiltonian of the generalized QMB model can be written as $\hat{H} = \hat{H}_0 + \hat{V}$, where (setting $\hbar = c = 1$)

$$\hat{H}_0 = \omega_{12} \sum_{a=1}^{M} \left(\sigma_a^z + \frac{1}{2} \right) + \int_C \frac{\mathrm{d}\omega}{2\pi}\, \omega\, p^\dagger(\omega) p(\omega) \tag{1a}$$

represents the Hamiltonians of M identical two-level atoms and free polaritons, while the operator

$$\hat{V} = -\sqrt{\gamma} \sum_{a=1}^{M} \int_C \frac{\mathrm{d}\omega}{2\pi} \sqrt{z(\omega)} \left[\sigma_a^+ p(\omega) \mathrm{e}^{ik(\omega)x_a} + p^\dagger(\omega) \sigma_a^- \mathrm{e}^{-ik(\omega)x_a} \right] \tag{1b}$$

describes their coupling. The polariton operators $p(\omega)$ obey the commutator $[p(\omega), p^\dagger(\omega')] = 2\pi\delta(\omega-\omega')$, while the spin operators $\vec{\sigma}_a = (\sigma_a^x, \sigma_a^y, \sigma_a^z)$, $\sigma^\pm = \sigma^x \pm i\sigma^y$ describe atoms having the coordinates $\{x_a,\, a = 1, \ldots, M\}$ on the polariton propagation axis (the x-axis). The extent of the atomic system is large compared to the optical wavelength. The states between frequencies labeled as $\Omega_\perp$ and $\Omega_\parallel$ are forbidden for linear propagating polariton modes. Therefore, the integration contour C consists of two allowed intervals, $C = C_- + C_+$, where $C_- = (0, \Omega_\perp)$ and $C_+ = (\Omega_\parallel, \infty)$. The coupling constant $\gamma = 2\pi\omega_{12} d^2/S_0$, where d is the atomic dipole moment and S_0 is the cross-section of a light beam. The information about the medium spectrum is contained in the dispersion relation $k(\omega) = \omega n(\omega)$. The atomic form factor $z(\omega) = \omega n^3(\omega)/\omega_{12}$, where $n(\omega) = \sqrt{\varepsilon(\omega)}$ and $\varepsilon(\omega) = (\omega^2 - \Omega_\parallel^2)/(\omega^2 - \Omega_\perp^2)$ is the dielectric permeability of a frequency-dispersive medium. In the limit of empty space ($n(\omega) = 1$), the model Hamiltonian (1) obviously reduces to the QMB model, which describes the self-induced transparency effect with quantum corrections and has the McCall-Hahn theory as a classical limit.

The eigenvalues of the model (1) within the N-particle sector of the Hilbert space are found from the following Bethe ansatz equations:

$$\exp\left[ik_j L\right]\left(\frac{h_j - i\beta/2}{h_j + i\beta/2}\right)^M = -\prod_{l=1}^{N}\frac{h_j - h_l - i\beta}{h_j - h_l + i\beta}, \quad j = 1,\dots,N\,, \tag{2a}$$

where $\beta = \gamma/\omega_{12}$, $k_j \equiv k(\omega_j)$, $E = \sum_j \omega_j$ is the eigenenergy of the N-particle Schrödinger equation $(\hat{H} - E)|\Psi\rangle = 0$, and the "rapidity" $h_j \equiv h(\omega_j)$ is given by

$$h(\omega) = \frac{\omega - \omega_{12}}{\omega n^3(\omega)}. \tag{2b}$$

Here, we have placed the system in a box of length L and imposed periodic boundary conditions on the wave function at the points $\pm L$. BAE have a clear physical meaning: the first phase factor on the l.h.s. is acquired by a polariton wave function during free propagation between points $\mp L/2$, while the second one accounts for phase factors resulting from subsequent scattering on M atoms. Propagating between the points $\mp L/2$ the polariton is also scattered by the other $N-1$ polaritons, and its wave function acquires the phase factor given on r.h.s. in eq. ($2a$). Information concerning the nonlinear polariton dispersion is contained in the rapidity $h(\omega)$. In empty space, it is reasonable to neglect the resonance dipole-dipole interaction between atoms in self-induced transparency, since the dominant photon-atom interaction is pointlike and the excited atom decays by stimulated emission into optical pulse modes. In this case, light scattering from each of the M atoms is considered independently. In the FGM, there are no classical modes available for stimulated emission and the polariton-atom coupling is highly nonlocal: An excited atom (photon-atom bound state) exhibits nonlocal interaction with other atoms which are within the classical tunneling distance. This leads to coherent hopping conduction of a photon through the resulting impurity band [16]. If the characteristic time of the interatomic hopping is much longer than the pulse duration, RDDI-mediated transfer of energy between impurity atoms in an *arbitrary* direction can be neglected. Energy transfer occurs through the soliton band [14] rather than the impurity band [16]. Equation ($2a$) is obtained by including RDDI contributions only from virtual polaritons traveling in the *same* direction as the incident pulse. In this generalization of the single-atom soliton band to an M-atom soliton band, scattering from each atom is treated independently. Therefore, as well as in ordinary vacuum [7], all reference to the atomic coordinates drops out of the eigenvalue problem (2). This is equivalent to the independent atom (gas) approximation used by McCall and Hahn in ordinary vacuum. Accordingly, in ordinary vacuum ($n(\omega) = 1$), eqs. (2) reduce to the BAE of the QMB model which, for large N, describe the classical McCall-Hahn solution.

It is instructive to derive first the main results of standard SIT theory from eqs. (2) with the rapidity $h(\omega) \simeq (\omega - \omega_{12})/\omega_{12}$ corresponding to the case of empty space. As $L \to \infty$, eqs. (2) admit solutions in which N complex rapidities h_j are grouped into a number of "strings" containing $1 \le n \le N$ rapidities. A string with n rapidities is given by

$$h_j = H + i(\beta/2)(n + 1 - 2j), \quad j = 1,\dots,n, \tag{3}$$

where H is an arbitrary common real part ("carrying" rapidity). Due to the linear relationships between the rapidity, frequency and momentum, particle frequencies and momenta are also grouped into string structures, $k_j = K + i(\gamma/2)(n + 1 - 2j)$, $\omega_j = \Omega + i(\gamma/2)(n + 1 - 2j)$, where K and Ω are common real parts of momenta and frequencies, respectively. To avoid confusion, we use the term "string" for solutions of BAE in the h-space and the term "soliton" to refer to string's images in the ω- and k-spaces. In what follows, we consider for simplicity the case when all N particles are grouped into a single string, *i.e.* $N = n$. Inserting k_j and

ω_j in eq. (2a) and evaluating the product over $j = 1, \ldots, N$, we obtain the simple equation $\exp[iQnL] = 1$, where

$$Q(\Omega) = K - \frac{2\rho}{n} \arctan \frac{n\gamma}{2(\Omega - \omega_{12})} \tag{4a}$$

is the soliton momentum per photon, and the number of atoms is $M = \rho L$. Here ρ is the linear density of the number of atoms. Clearly $Q(\Omega)$ can be interpreted as the energy-momentum dispersion relation of a soliton of size n, where the second term describes a contribution of photon-atom scattering. The group velocity of soliton propagation $V = \frac{\mathrm{d}\Omega}{\mathrm{d}Q}$ is then given by

$$\frac{1}{V} = 1 + \frac{\gamma\rho}{(\Omega - \omega_{12})^2 + (n\gamma/2)^2} = \frac{1}{c} + \frac{2\pi}{\hbar c} \frac{\omega_{12} d_{12}^2 n_{\mathrm{a}}}{(\Omega - \omega_{12})^2 + (1/\tau_{\mathrm{s}})^2}. \tag{4b}$$

In the last expression d_{12} is the atomic dipole moment, τ_{s} is the duration of a soliton, n_{a} is the density of the atomic system, and for comparison we have restored the usual system of units. Equation (4b) is easily seen then to be identical to the corresponding expression in the classical SIT theory. The soliton duration is inversely proportional to the number of photons, $\tau_{\mathrm{s}} \simeq n^{-1}$ [7]. Therefore, only macroscopically “long” strings, $n \gg 1$, propagate in an absorbing atomic system without dissipation.

In FGM, eq. (3) is a solution of BAE if and only if the imaginary parts of rapidities h_j and corresponding momenta k_j have the same sign,

$$\mathrm{sgn}(\mathrm{Im}\, h_j) = \mathrm{sgn}(\mathrm{Im}\, k_j), \quad j = 1, \ldots, n. \tag{5}$$

It is easy to understand that the necessary condition (NC) (5) determines the frequency intervals, in which an effective particle-particle coupling is attractive, and hence admits bound many-particle complexes.

We start with the case when the real part of ω_j lies outside the gap. Let $\omega = \xi + i\eta$ and $\xi \in C$. Making use of the approach developed in [14], it is easy to show that the effective coupling is attractive only between polaritons of the lower branch, $\xi \in C_-$. Polaritons of the upper branch are described by one-particle strings with real positive rapidities and do not form any bound complexes. Bound many-polariton complexes (ordinary solitons) are quite similar to solitons of the QMB model, despite their inordinate behavior on different polariton branches. The dispersion relation of an ordinary soliton of size n is given by

$$q(\xi) = k(\xi) - \frac{2\rho}{n} \arctan \frac{\beta n}{2h(\xi)}, \tag{6a}$$

where $k(\xi) = \xi n(\xi)$. The group velocities inside, $V = \mathrm{d}\xi/\mathrm{d}q$, and outside, $v = \mathrm{d}\xi/\mathrm{d}k$, the atomic system are then related by

$$\frac{1}{V} = \frac{1}{v} + \frac{\rho\beta}{h^2(\xi) + (\beta n/2)^2} \frac{\mathrm{d}h(\xi)}{\mathrm{d}\xi}. \tag{6b}$$

Since ordinary solitons in the FGM are off-resonance to the atomic transition, the effect of the atomic system on their propagation is always weak, unlike the case of SIT in empty space.

Next we study the multiparticle excitations of the system with eigenenergies lying inside the frequency gap. We look for an image of a Bethe string when the real parts of particle frequencies ω_j lie inside the gap, $\xi \in G = (\Omega_\perp, \Omega_\parallel)$. To find the analytical continuations of the functions $k(\omega)$ and $h(\omega)$, an appropriate branch of the function $n(\omega)$ is fixed by the condition $n(\xi \pm i0) = \pm i\nu(\xi)$, where $\nu(\xi) = \sqrt{|\varepsilon(\xi)|}$. Soliton parameters ξ_j and η_j are determined by the equations

$$\mathrm{Re}\, h(\xi_j, \eta_j) = H, \quad \mathrm{Im}\, h(\xi_j, \eta_j) = \beta(l + 1/2 - j), \tag{7}$$

which, in the general case, should be solved numerically. Approximate solutions of eqs. (7) are easily found by the method of ref. [14]. One can show that in the model under consideration gap excitations exist only for $\xi \in (\omega_{12}, \Omega_{\|})$. A string with an even number of particles, $n = 2l$, describes a bound complex of l pairs of confined gap excitations—quantum gap soliton—with the eigenenergy and momentum per particle of

$$\epsilon = \omega_{12} + \frac{b}{a^3} H^2 \sim \omega_{12} + \frac{\omega_{12}^2}{\Delta} H^2 \,, \tag{8a}$$

$$q = -\frac{H}{a} |\kappa'(\omega_{12})| \sim -\frac{\omega_{12}^2}{\Delta} H \,, \tag{8b}$$

where $\kappa(\xi) = \xi\nu(\xi)$, $\kappa' = \mathrm{d}\kappa/\mathrm{d}\xi$, and $\Delta = \Omega_{\|} - \Omega_{\perp}$ is the gap size. The parameters a and b are found as coefficients in the Taylor series of the function $\phi(\xi) = [\xi\nu^3(\xi)]^{-1}$ at the point $\xi = \omega_{12}$: $\phi(\xi) \simeq a + b(\xi - \omega_{12})$. The soliton parameters are roughly estimated as $a \sim \omega_{12}^{-1}$, $b \sim (\omega_{12}\Delta)^{-1}$, and $\kappa'(\omega_{12}) \sim \omega_{12}\Delta^{-1}$. From the condition $q > 0$, one finds $H < 0$. The results obtained are valid only for quite a small kinetic term of soliton energy, $\epsilon_{\mathrm{kin}} = bH^2/a^3 \ll \Delta$, *i.e.* $|H| \ll \Delta/\omega_{12}$ when the gap soliton dispersion is described in the effective mass approximation. Thus, outside the atomic system, gap solitons are heavy massive excitations with a small group velocity $v = \mathrm{d}\epsilon/\mathrm{d}q \sim -H$. At arbitrary $|H|$ or for quite big solitons ($l \gg 1$), we have to solve eq. (7) numerically.

The spatial size of a pair, $\delta \simeq \kappa^{-1}(\omega_{12})$, is nothing but the penetration length of the radiation with the frequency $\omega_{12} \in G$ into the medium, and hence it lies on the scale of a few wavelengths. Since $\kappa(\xi)$ is monotonically decreasing function, the gap soliton size, in sharp contrast to the case of ordinary solitons, grows with the growth of the number of pairs l.

For a gap soliton, the dispersion relation inside the atomic system takes the form

$$Q(\epsilon_l) = q(\epsilon_l) - \frac{\rho}{l} \arctan \frac{\beta l}{H}. \tag{9a}$$

The group velocity of a gap soliton inside the atomic system, $V_l = \mathrm{d}\epsilon_l/\mathrm{d}Q$, is then found to be

$$\frac{1}{V_l} = \frac{1}{v} + \frac{\rho\beta}{H^2 + \beta^2 l^2} \frac{\mathrm{d}H}{\mathrm{d}\epsilon} \sim \frac{1}{v}\left(1 - \frac{\Delta}{\omega_{12}^2} \frac{\rho\beta}{H^2 + \beta^2 l^2}\right). \tag{9b}$$

In sharp contrast to the case of ordinary solitons, the particle-atom scattering speeds up a gap soliton: $V_l > v$, because $\mathrm{d}H/\mathrm{d}\epsilon$ is negative in the effective mass approximation, and $-\Delta/\omega_{12} < H < 0$. From the condition $V_l < 1$, it follows that $H < -\frac{\Delta}{\omega_{12}}(\rho\beta/\Delta)^{1/2}$. These two restrictions are mutually consistent only if $\rho\beta/\Delta < 1$. This is simply the condition that the polariton gap Δ must be larger than the interaction between the radiation and the atomic system which is characterized by the parameter $\rho\beta = 2\pi d_{12}^2 n_{\mathrm{a}}$. In other words, the atomic density n_{a} must be not so large to destroy the polariton gap itself.

In the absence of the impurity atoms, the Hamiltonian (1) is quadratic. However, the BAE allow correlated pairs of photons to propagate through a harmonic gap medium, even though single-photon propagation is forbidden. A 1D free field can be described either on the basis of free polaritons or in terms of interacting Bethe particles, such as solitons and gap solitons. While the Bethe basis is unnecessarily cumbersome in the absence of the impurity atoms, in the presence of atoms the only modification is an additional phase factor in the l.h.s. of eq. ($2a$) and a specific choice of $h(\omega)$ and β.

This scenario occurs in all integrable models, where an effective intermode coupling is generated by particle-impurity scattering [18]. In particular, in the QMB model, quantum solitons diagonalize the system Hamiltonian both in the presence and in the absence of atoms. To describe a light propagation through an atomic system, one can represent an incident pulse as a linear superposition of different soliton states. In the absence of atoms, all solitons propagate with the same velocity, therefore the initial shape of the pulse is preserved. In the presence of atoms, the soliton velocity depends on soliton parameters leading to a spatial separation of solitons in the propagation process.

In FGM, gap solitons are naturally contained in the spectrum of Bethe excitations of an undoped medium. Atoms with the transition frequency lying deep inside the gap speed up resonance solitons that should result in a spatial separation of gap solitons propagating in an extended doped medium. This represents an entirely new mechanism for nondissipative subgap energy transfer in a FGM. These *quantum* solitons involve only a small (even) number of photons and are entirely distinct from high-intensity *classical* solitary waves in a doped PBG material [19]. Furthermore, they suggest that a doped FGM with suitable optical pumping may act as a source of novel quantum-correlated states of light.

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