Quantum Optical Spin-Glass State of Impurity Two-Level Atoms in a Photonic Band Gap

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We describe the collective optical properties of impurity two-level atoms in a photonic band gap interacting by resonance dipole-dipole interaction (RDDI) and coupled to a localized cavity mode. The random impurity atom positions are modeled by means of a Gaussian random distribution of RDDI's with variance J and atomic line fluctuation with variance δ. We demonstrate the occurrence of a new collective atomic steady state, the optical analog of a spin-1/2 dipolar glass, and an associated Bose-glass state of photons in the cavity mode.

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Spin-glass systems have been widely studied in condensed matter physics [1–4]. In the simplest example, they consist of a collection of classical spins with quenched random, frustrated interactions whose low-temperature state is a frozen disordered one. This is a state with zero macroscopic magnetization while the local spontaneous magnetization at a given site is nonzero. Glassy behavior may also arise in quantum systems. Interacting electrons in a highly disordered metal may, as a consequence of localization effects, freeze into a Fermi-glass rather than a Fermi-liquid state [5]. It has been suggested that bosons with strong repulsive interactions in a disordered medium may form a Bose glass rather than a Bose-Einstein condensate (superfluid) in thermodynamic equilibrium at low temperature [6]. In this Letter, we show that under suitable nonequilibrium boundary conditions photons interacting with impurity two-level atoms in a photonic band-gap (PBG) material may tend to a novel collective steady state, the optical analog of a Bose glass. This state is intermediate between incoherent light arising from a thermal source and the coherent light arising from conventional laser emission. Corresponding to this novel optical state, the impurity atoms acquire a steady-state polarization (dipole moment). The phase of this polarization, however, varies randomly form atom to atom and the resulting collective steady state is the optical analog of a quantum spin glass. This state arises from coherent (but random) resonance dipole-dipole interaction (RDDI) between atoms. Because of the relatively long-range nature of RDDI, this state is the quantum optical analog of a classical neural network [7,8] and may have applications to optical information storage and quantum computers [9].

In this paper, we demonstrate that PBG materials [10–13] provide a suitable environment for the emergence of novel macroscopic quantum coherence involving photons and atoms. Band-gap to center frequency ratios $\Delta\omega/\omega_0$ of 20% have been experimentally demonstrated [10]. For visible frequencies ω_0, the resulting electromagnetic energy gap $\hbar\Delta\omega \simeq 0.4$ eV provides substantial protection from spontaneous emission and other incoherent effects [11,14,15]. PBG materials are distinct from conventional optical microcavities [16] whose dimensions are comparable to the optical wavelength λ, in which wave propagation effects are precluded. In a PBG material, energy transport is possible over length scales long compared to λ. For instance, when the atomic transition frequency ω_a is well within the gap, RDDI becomes the dominant interaction mechanism between atoms leading to photon hoping conduction (energy transfer) [17]. The inhibition of spontaneous emission and the preservation of propagative effects are the key elements in the manifestation of macroscopic quantum coherence in a PBG material.

We consider a collection of N two-level atoms within a PBG interacting with a single resonant, localized dielectric, defect mode. The influence of a continuum of modes when the resonant atomic frequency is near the band edge will be discussed elsewhere. Far inside a PBG ($|\omega_a - \omega_0| \ll \Delta\omega$), where spontaneous emission is nearly absent, The interaction between atoms and the electromagnetic field may be described (in the interaction picture) by the model Hamiltonian:

$$H = \sum_i^N \frac{\hbar\delta_i}{2}\sigma_i^z + g\sum_i^N(\sigma_i^\dagger a + a^\dagger\sigma_i) + \sum_{i\neq j}^N J_{ij}\sigma_i^\dagger\sigma_j\,, \tag{1}$$

Here $\sigma_i^\dagger$ and σ_j describe atomic excitation and deexcitation of the ith atom, respectively; σ^z described the atomic inversion; a and $a^\dagger$ are the annihilation and creation operators for photons in the resonant dielectric defect mode, respectively; and $\delta_i = \omega_i - \omega_a$ is the atomic frequency shift (from its average value ω_a) caused by the random, static field in the photonic crystal (inhomogenous atomic line broadening). Here the magnitude of coupling constant g is related to the volume $\xi_{\rm loc}^3$ of the defect mode, where $\xi_{\rm loc}$ is the localization length. In particular [17,18], $g = \hbar(\omega_a\mu/\hbar c)(2\pi\hbar c^2/\omega_a\xi_{\rm loc}^3)^{1/2}$. Here ω_a is the atomic resonant frequency and $\mu \sim ea_0$ is the atomic dipole moment. $J_{ij} = J_{ji}$ denotes the RDDI between atoms i and j. The energy scale for RDDI is given

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by $J_{ij} \sim (\hbar\omega_a)(a_0/R_{ij})^3$, where a_0 is the atomic Bohr radius and R_{ij} is the atomic separation [14]. For a gas of cold atoms, with $10 < R_{ij} < 1000$ Å, in the void region of a PBG material, J_{ij} is small compared to the energy scale $\hbar\Delta\omega$ of the PBG, but large compared to any form of radiative relaxation For those atomic densities, the range of RDDI is much larger than R_{ij} and considerable collective enhancement of RDDI-mediated photon hopping conduction occurs [17].

The detailed, microscopic evaluation of J_{ij} as a function of atomic distance r_{ij} and atomic configurations may be found in Ref. [19] for ordinarily vacuum and in Refs. [14,20] for a PBG material. In a realistic description, the RDDI term in (1) would be replaced by a *traceless* tensor interaction for virtual-photon-mediated atomic excitation transfer between two sets of triply degenerate atomic orbitals. The *transverse* part of this tensor, which becomes important on long length scales in ordinary vacuum, is exponentially suppressed deep within a PBG [14]. We simulate the effects of this traceless tensor interaction with the simpler two-level atom system by allowing J_{ij} to be a Gaussian random variable with zero mean value. That is, the statistical distribution $P(J_{ij}) = (2\pi J^2)^{-1/2} e^{-J_{ij}^2/2J^2}$ is a symmetric Gaussian with standard deviation J for all interactions J_{ij}. While deviations from Gaussian behavior may appear in real systems [21], our numerical calculations have shown that our final results remain qualitatively the same for a variety of symmetric distributions. We note that in the absence of a localized defect mode ($g = 0$) and for $\delta_i = 0$ the Hamiltonian (1) is the quantum spin-1/2 version of the Sherrington-Kirkpatrick (SK) model of spin glasses. However, we consider this problem under nonequilibrium boundary conditions [22] with optical pumping, rather than under thermal equilibrium boundary conditions as has been done previously in spin-glass theory [1–4].

In analogy to spin-glass theory [1,8], we characterize the atomic system in terms of the order parameters $m = N^{-1}\sum_{i=1}^{N}[\langle\sigma_i\rangle]_c$, $q = N^{-1}\sum_{i=1}^{N}[\langle\sigma_i^\dagger\rangle\langle\sigma_i\rangle]_c$. Here $\langle\cdots\rangle$ denotes the quantum expectation value and $[\cdots]_c$ denotes the configuration average over the random atomic positions. Here m is the global polarization density of the atomic system and q is the so-called Edwards-Anderson order parameter [1], describing local, spontaneous, atomic polarization. For the optical system, $m \neq 0$, $q \neq 0$ corresponds to a superradiant (ferromagnetic) state, $m = 0$, $q = 0$ corresponds to an incoherent (paramagnetic) state, and $m = 0$, $q \neq 0$ corresponds to the intermediate (spin-glass) state. As discussed in [8], m and q, defined above, also characterize the local order parameters $m_i = [\langle\sigma_i\rangle]_c$ and $q_i = [\langle\sigma_i^\dagger\rangle\langle\sigma_i\rangle]_c$. Analogously, we characterize the cavity mode by the field amplitude $m_c = [\langle a\rangle]_c$ and Bose-glass order parameter $q_c = [\langle a^\dagger\rangle\langle a\rangle]_c$. The state with $m_c = 0$ and $q_c \neq 0$ is the Bose-glass state. This state is distinct from the coherent state with $m_c \neq 0$, $q_c \neq 0$ and from the incoherent state with $m_c = q_c = 0$.

For comparison purposes, we begin with the case of low excitation density where an approximate analytical solution for the optical spin-glass problem exists. In the case of low excitation density we utilize the standard Holstein-Primakoff approximation [4]: $\sigma_i^\dagger = b_i^\dagger(1 - b_i^\dagger b_i)^{1/2} \simeq b_i^\dagger$. Here the Fermionic spin algebra $\{\sigma_i, \sigma_i^\dagger\} = 1$ is replaced by the bosonic commutation relation $[b_i, b_i^\dagger] = 1$ in the dilute $(b_i^\dagger b_i) \ll 1$ limit. b_i and $b_i^\dagger$ can be considered as annihilation and creation operators for the excited state of the ith atom. In the absence of inhomogeneous line broadening ($\delta_i = 0$), the Hamiltonian (1) leads to Heisenberg equations of motion for the Holstein-Primakoff operators in the form $dx_i/dt = -ig\alpha - i\sum_j J_{ij}x_j$ and $d\alpha/dt = -ig\sum_i x_i$, where $x_i = \langle b_i\rangle$ and $\alpha = \langle a\rangle$. It follows that $Nq + q_c$ is a constant of motion; that is, at a long time limit, q and q_c are nonzero if we impose initial coherence $[q(0)$ or $q_c(0) \neq 0]$. The parameters m and m_c can be calculated form the solution of the equations of motion. We use the spectral representation [23] $J_{ij} = \sum_\lambda^N J_\lambda\langle i|\lambda\rangle\langle\lambda|j\rangle$, where J_λ and $\langle\lambda|i\rangle$ are the orthonormal eigenvectors, respectively. The eigenvalue density $\rho(J_\lambda)$ satisfies a semicircular law [24] for the case of $N \gg 1$: $\rho(J_\lambda) = (2\pi\tilde{J}^2)^{-1}(4\tilde{J}^2 - J_\lambda^2)^{1/2}$, where $\tilde{J} = J\sqrt{N}$. For the case $g = 0$, the solution of the equations of motion takes the form $x_\lambda(t) = \sum_i\langle\lambda|i\rangle x_i(t) = e^{-iJ_\lambda t}x_\lambda(0)$. Using these facts, the parameter $m(t)$ can be written as $m(t) = (b_0/\tilde{J}t)J_1(2\tilde{J}t)$, where $b_0 = b_i(0)$ and $J_1(\xi)$ is the Bessel function of the first kind. Clearly, the function $m(t)$ displays oscillatory behavior with a collective time scale factor of $\sqrt{N}$, and tends to zero in the steady-state limit; that is, the atomic system tends to the optical quantum spin-glass state.

For $g \neq 0$, a simple *approximate* solution can be derived by replacing $\sum_i x_i$ in the second equation of motion by $-igNx_i$. This leads to an overestimate of $m(t)$ by assuming that each atomic dipole is the same on average and that when they act in phase. For $g \gg J$, it can be shown that $m(t) \cong [b\cos(gN^{1/2}t) - i(\alpha/N^{1/2})\sin(gN^{1/2}t)] \times (2/\tilde{J}t)J_1(\tilde{J}t)$. Once again, $m(t)$ displays oscillatory behavior with a collective time scale factor of $N^{1/2}$, and tends to zero in the steady-state limit while q remains nonzero. However, we note here that on the time scale t, short compared to that required for RDDI-mediated photon hopping conduction between atoms ($gN^{1/2} \geq t^{-1} \gg JN^{1/2}$), $(2/\tilde{J}t)J_1(\tilde{J}t) \cong 1$. That is, the atomic system exhibits persistent, oscillatory, macroscopic polarization (ferroelectric) before it tends to the glassy state at the steady-state limit. Likewise $\alpha(t)$ tends to zero at the long time limit while q_c remains nonzero; that is, the cavity mode tends to a Bose-glass state in the steady-state limit. We have verified this qualitative picture by numerical simulation of the equations of motion.

In the case of high excitation density, the possibility of spontaneous symmetry breaking arises. In particular, if the initial state has population inversion (and infinitesimal initial polarization), we find that in the steady-state limit a macroscopic value of the glass order parameter can build up from an infinitesimal seed. The Holstein-Primakoff approximation does not adequately describe this spontaneous glass formation. To recapture this effect, we consider the Heisenberg equations of motion resulting from the Hamiltonian (1):

$$\frac{d}{dt}\langle\sigma_i\rangle = -i\delta_i\langle\sigma_i\rangle + ig\langle\sigma_i^z a\rangle + i\left\langle\sigma_i^z \sum_{j(\neq i)} J_{ij}\sigma_j\right\rangle, \tag{2a}$$

$$\frac{d}{dt}\langle\sigma_i^z\rangle = -2ig\langle\sigma_i^\dagger a\rangle - 2i\left\langle\sigma_i^\dagger \sum_{j(\neq i)} J_{ij}\sigma_j\right\rangle + \text{ c.c.}, \tag{2b}$$

$$\frac{d}{dt}\langle a\rangle = -ig\sum_i\langle\sigma_j\rangle. \tag{2c}$$

These equations may be rendered tractable by assuming that each subsystem behaves in a stochastically uncorrelated way with respect to the other [22]. This is equivalent to a mean-field approximation [1] which ignores certain quantum correlation effects in the system. We have verified by numerical simulation that for low excitation density the mean-field approximation and Eq. (4) give quantitatively similar results. Following Ref. [22], we have also included the Langevin noise of the atomic system by giving the initial atomic polarization random values with Gaussian statistics. In this case, numerical simulation reveals a modified dynamics of m and q, but the steady-state value of these order parameters remains substantially unchanged. Fluctuations in the system may, however, play a crucial role in determining quantum statistical properties such as photon number distribution of the cavity mode and quantum fluctuations of the atomic system. This problem will be discussed elsewhere. In the mean-field approximation, the expectation values of operator products in Eq. (7) can be factorized. This leads to a closed set of differential equations which can be integrated for each set of the Gaussian random numbers J_{ij} and δ_i. We than take a configurational average over a large number ($\sim 2 \times 10^3$) of sets of the random number J_{ij} and sets of random numbers δ_i. For illustration purposes we set the variances of these random numbers J and δ respectively, to be equal to g.

In Fig. 1(a) we plot the macroscopic polarization m (solid curves) and the Edwards-Anderson order parameter q (dashed curves) as a function of gt for the initial condition in which atoms are mostly populated in the excited state $|2\rangle$ and the initial atomic coherence $\langle\sigma(0)\rangle$ is infinitesimal. Such a state can be created by interaction of atoms with an external pulse [18]. The defect mode is assumed to be initially in the vacuum state. Clearly, at the outset, the photon hopping conduction between atoms

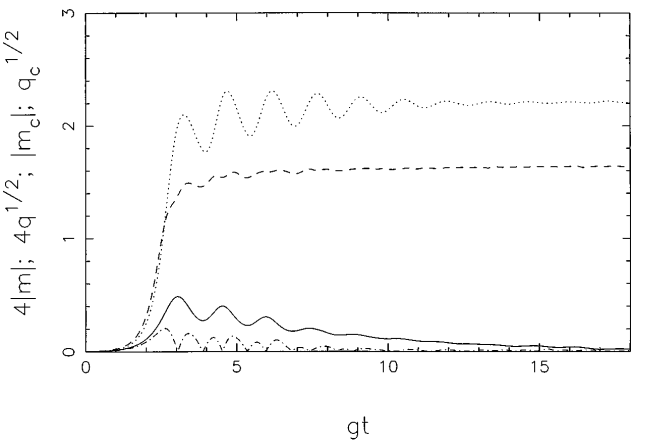

FIG. 1. Macroscopic atomic polarization $|m|$ (dash-dotted curve), spin-glass order parameter $q^{1/2}$ (dashed curve), coherent state amplitude $|m_c| = |[\langle a\rangle]_c|$ (solid curve), and Bose-glass order parameter $q_c^{1/2} = [\langle a^\dagger\rangle\langle a\rangle]_c^{1/2}$ (dotted curve) as a function of scale time gt for $N = 10$, $J = \delta = g$. All atoms are initially in a coherent superposition of states (mostly excited) with $\langle\sigma(0)\rangle = 10^{-3}$ and the defect mode is initially in the vacuum state.

is minimal because atoms are mostly in the excited state and there is no "hole" for photon hopping conduction to take place. At the outset, the dominant process is superradiance [22]. That is, excited atoms emit photons into the defect mode. As a result, the macroscopic polarization $m(t)$ as well as $q(t)$ are built up. This is followed by photon hopping conduction between atoms, which leads to decay of the macroscopic polarization to zero in the steady-state limit. In Fig. 1(b) we plot m_c (solid curve) and q_c (dashed curve) as a function of gt for the same parameters. Clearly, photons in the defect mode tend to the Bose-glass state with $m_c = 0$ and $q_c \neq 0$. Unlike incoherent light for which the electric field autocorrelation $G^{(1)} \propto [\langle E^\dagger(t+\tau)E(t)\rangle]_c \to 0$ as $\tau \to \infty$, this function remains finite in the Bose glass at long times. Unlike coherent light, however, the expectation value of the electric field $[\langle E(t)\rangle]_c$ is identically zero in the glass state.

The optical spin-glass and Bose-glass states can be also obtained in the case when all atoms are initially in the ground state and the photons occupying the defect mode initially in a coherent state [Figs. 2(a) and 2(b)]. In this case, we also find that if the mean number of photons in the defect mode is much larger than the number of atoms or when the coupling constant $g \gg J$, the macroscopic atomic polarization persists for a much longer time before giving way to the glassy state.

In conclusion, we have shown that a system of impurity atoms inside a PBG can evolve into a new collective state, the optical analog of a quantum spin glass. Photons in the accompanying resonant dielectric mode evolve into a steady-state Bose glass. Programmable, classical spin glasses have applications in computing and optimization

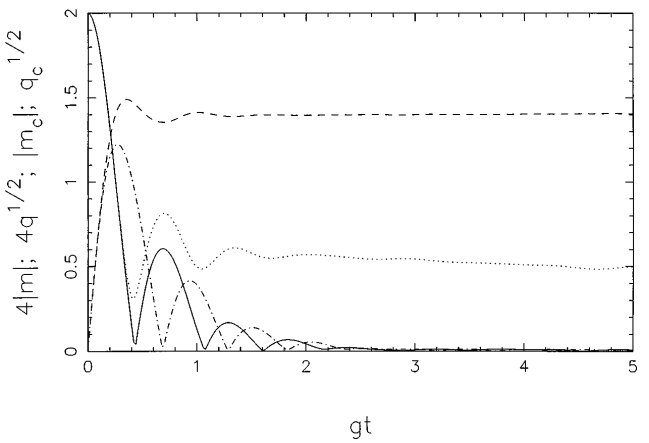

FIG. 2. The same as in Fig. 1 but for $N = 20$, all atoms are initially in the ground state, and the defect mode is initially in the coherent state with $\langle a(0)\rangle = 2$.

problems. This requires that the interaction parameters J_{ij} can be controllably altered by external input. For the case of atoms in a photonic band gap these RDDI matrix elements are determined by the atomic positions which may be controlled through laser cooling [25]. Cooling may be achieved with laser beams that couple to a third atomic level lying outside of the PBG. The effects of RDDI on the cooling process have been discussed in Ref. [26]. Photoexcitation of the atomic system may be realized through (nonlinear) electronic Raman scattering from the third level. Initial state preparation and control of the dynamical evolution of the glassy state may be achieved by choosing the "McCall-Hahn area" of the exciting pulse and subsequent pulses [27]. These techniques have been successfully employed in photon echo experiments [18]. In this case it is possible that the photonic band gap may provide an ideal environment for the quantum analog of a neural network and exhibits its associated quantum computing capabilities [9].

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